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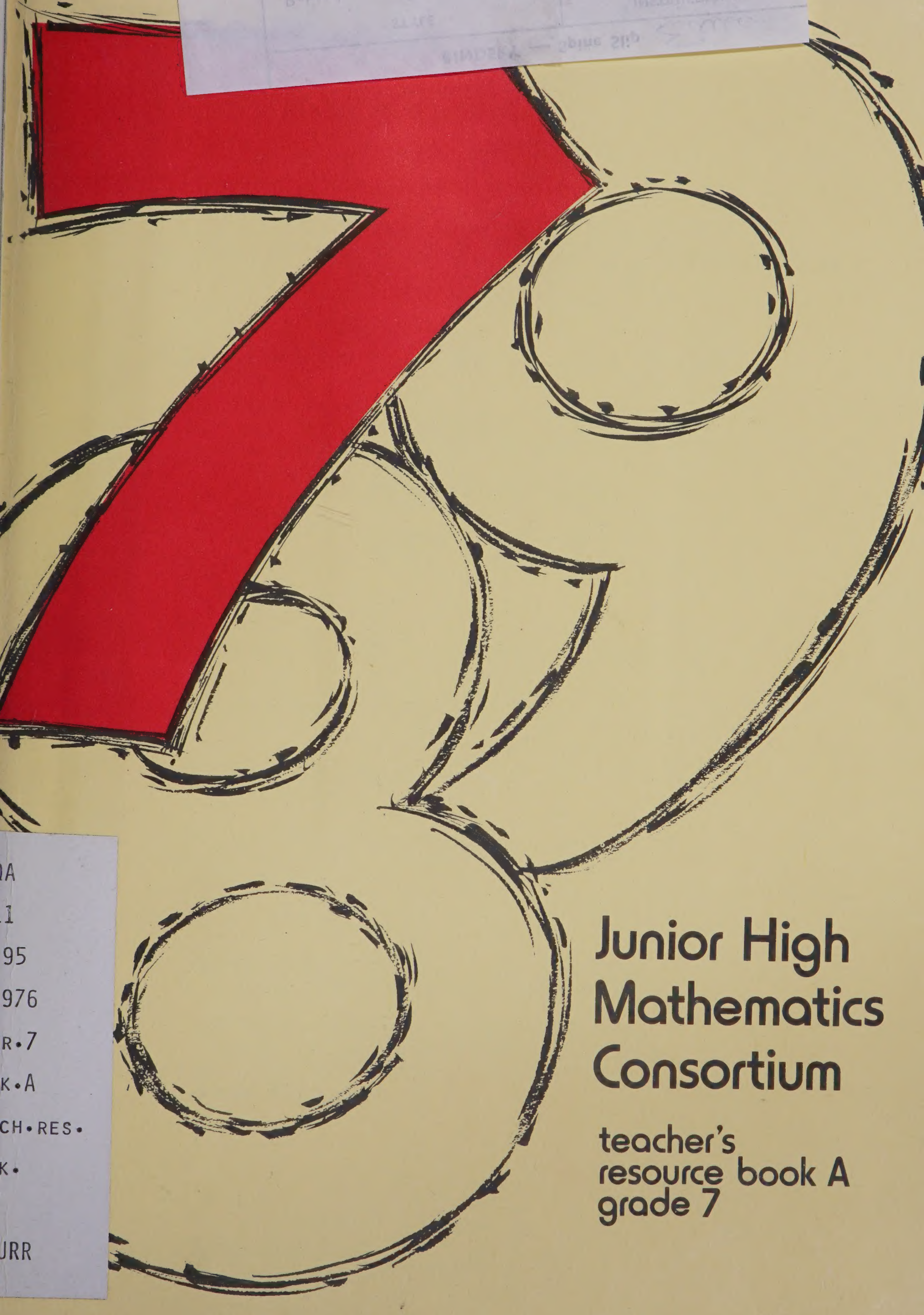

Junior High Mathematics Consortium

teacher's resource book A grade 7

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JUNIOR HIGH MATHEMATICS CONSORTIUM

TEACHER'S RESOURCE BOOK A

LEVEL SEVEN

Unit I Set Theory
Unit II Whole Numbers
Unit III Rational Numbers

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Junior High Mathematics Consortium, 1976

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JUNIOR HIGH MATHEMATICS CONSORTIUM

MATHEMATICS PROGRAM

This Junior High Mathematics Program has been prepared through the cooperative efforts of the twenty member systems in the Junior High Mathematics Consortium. All the components of the program are related to performance objectives and provide the teacher with a variety of teaching strategies, activities and resources to facilitate the instructional process. The teachers have the opportunity to determine the modes of instruction that best suits them and their students.

For each objective within the program, there is a bank of test items. The items in the bank have been field tested to determine difficulty and reliability and validated by a committee of teachers and curricular coordinators.

The program was made possible through the assistance and cooperation of many individuals. Special thanks and acknowledgments are extended to:

the many teachers who helped design, develop, pilot and revise the program materials;

the publishers for referencing their materials to the performance objectives: Addison-Wesley, Gage Publishing and Holt Rinehart;

the Department of Education, Province of Alberta for partially funding the project;

the administrators, supervisors, consultants and curricular associates from each of the cooperating systems for providing time, money and expertise to the program.

2698140

ACKNOWLEDGEMENTS

The components of this program have been developed by teachers from the following school systems:

1. Edmonton Public School District #7
2. Edmonton Separate School District #7
3. County of Leduc #25
4. County of Parkland #31
5. St. Albert Protestant Separate School District #6
6. County of Strathcona #20
7. Calgary School District #19
8. Calgary Roman Catholic Separate School District #1
9. Foothills School Division #38
10. County of Vulcan #2
11. County of Ponoka #3
12. Red Deer School District #104
13. Red Deer Roman Catholic Separate School District #17
14. Mountain View County #17
15. Falher Consolidated School District #69
16. Fort McMurray Roman Catholic Separate School District #32
17. Bonnyville School Division #46
18. County of Beaver #9
19. County of St. Paul #19
20. St. Paul School District #2228

LABS

LEVEL 7

UNIT I

SET THEORY

OBJECTIVE: *To develop an understanding of set.*

SUGGESTED DEVELOPMENT FOR THE LAB APPROACH:

TIME: The lab can be done in one class period. (40 - 60 minutes)

MATERIALS:
4 half-sheets of construction paper of different colours (red, blue, green, yellow) per station.
1 pair of scissors per station.
1 piece of yarn approximately 2 metres long per station
4 pieces of yarn approximately 80 cm long per station
1 pen or pencil per student
4 sheets of looseleaf per student

ORGANIZATION: Organize the room into "lab stations". Number each station (1,2,3, etc...) Group students according to the number of stations you want. Assign a station to each group. (Two or three students per group is desirable).

DIRECTIONS TO STUDENTS:

In this lab you will need 1 pen or pencil and 4 sheets of looseleaf. On one sheet of looseleaf answer all questions asked: and on the other three draw any diagrams required. Go to your assigned station. Using the materials at the station follow all instructions on the lab sheets. When you complete the lab, bring it to me for checking.

PURPOSE: To develop an understanding of set.

METHOD:

1. Cut out from each coloured sheet: 1 triangle, 1 square and 1 rectangle.
2. Spread out the 12 figures on a cleared desktop, and keep them spread out throughout the lab.
3. Enclose all the 12 figures within the long piece of yarn, and keep them enclosed throughout the lab.
4. Draw a diagram of what you observe, on one of your sheets of looseleaf. Within each figure indicate the colour by using R for red, B for blue, G for green and Y for yellow.
5. Label this diagram, DIAGRAM 1.
6. Group together separately all the triangles, all the squares and all the rectangles.
7. Enclose each of these groups with a short piece of yarn.
8. Draw a diagram of what you observe, on another sheet of looseleaf, indicating the colours as before.
9. Label this diagram, DIAGRAM 2.
10. Group together separately, all the red figures, all the blue figures, all the green figures, and all the yellow figures.
11. Enclose each of these groups with a short piece yarn.
12. Draw a diagram of what you observe, on a third sheet of looseleaf, indicating the colours as before.
13. Label this diagram, DIAGRAM 3.

OBSERVATIONS: A) OBSERVE DIAGRAM 2

1. How many small groups do we have in diagram 2?
2. How are the figures in each group similar?
3. How are these groups different?
4. List all the different groups.

B) OBSERVE DIAGRAM 3

1. How many small groups do we have in diagram 3?
2. How are the figures in each group similar?
3. How are these groups different?
4. List all the different groups.

CONCLUSIONS: A) FROM OBSERVATIONS OF DIAGRAM 2.

1. Why can't we have a square with the triangles?
2. What rule can you make about forming the groups in diagram 2?

B) FROM OBSERVATIONS OF DIAGRAM 3.

1. Why can't we have a blue figure in with the red figures?
2. What rule can you make about forming the groups in diagram 3?
3. What would you call these groups?

In mathematics we call a collection of items which are similar, a SET; and each item in the set, an ELEMENT.

4. Do you think that it would be a good idea to call the groups in diagram 2 and diagram 3, sets?

ACTIVITIES

LEVEL 7

UNIT I

SET THEORY

Tic-Tac-Toe

NUMBER OF PLAYERS: Two

RULES:

1. Shuffle the 18 playing cards.
2. Flip a coin to determine who plays first.
3. First player draws card and places a chip on the appropriate square on the game board.
4. Second player does same thing.
5. If a space is already covered, the player cannot play, it is the next player's turn.
6. Play continues on until one player has a straight line. He scores one point.
7. The winner is the first player to reach 7.
8. If no Tic - Tac - Toe occurs, there is no point awarded.

CARDS:

BELONGS TO

IS AN
ELEMENT OF

THE SET
OF DOGS
WITH FOUR
HEADS

THE SET
OF PEOPLE
WITH 27½
THUMBS

A= {△, ○, □}
B= {1,2,3}

C = {🯅, *, ?}
D = {a,b,c}

X = {1,2,3}
Y = {2,1,3}

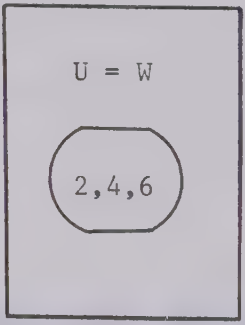
S = {cow,dog}
T = {dog,cow}

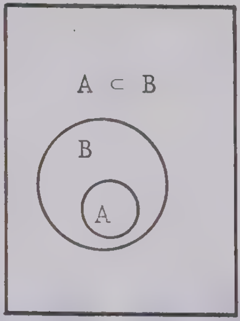
O= {1,2,..7}

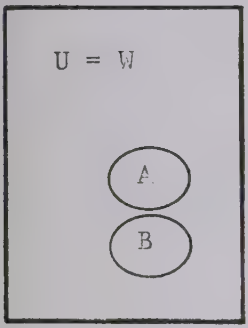
I= {a,b,..z}

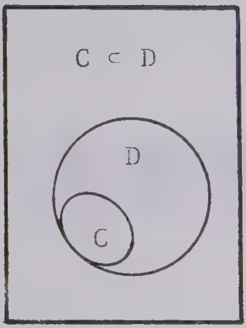
F= 2,4,6...

R= 5,10,15...









A SET
CONTAINING
ALL POSSIBLE
REPLACEMENTS
FOR A
GIVEN SET

YOU
MISS
A
TURN

GAME BOARD

UNIVERSAL SET	{ }	∈
EQUIVALENT SETS	EQUAL SETS	FINITE SET
INFINITE SET	VENN DIAGRAM	SUBSETS

APPLICATIONS

LEVEL 7

UNIT I

SET THEORY

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Lyng, Meconi, Lwick, Career Mathematics: Industry and the Trades, Houghton Mifflin, Boston, 1974

Schor, Meng, Insights and Skills Parts 1, 2 and 3, Globe Book Co., New York 1973

Stein, Practical Applications in Mathematics, Allyn and Bacon Inc., Boston, 1972

Witherding, Margaret F., From Fingers to Computers, Franklin Publications Inc., Chicago, 1970

VIDEO TAPES

ETV Math Series produced in Ontario Tape #1 Part D

Approximating & Estimations
Good for Grade 8 Measurement Applications

Tape #3 Part B

So You Want to Buy a Car
(Application in credit buying Grade 9 level)

Tape #5 Part A

Art from Computers
Useful as maxirational unit for applying Math to Art any grade level.

SET THEORY APPLICATIONS

LEVEL 7

OBJECTIVES | UNIT I | Reference Section

SET THEORY

Students should be able to:

1. Use symbols such as $\{\ \}$, $\in$, $\emptyset$, ... when referring to sets.
2. Tabulate five sets from their everyday life. (Limit: five elements) — D
3. Select the sets that contain only the elements from a given universe. — E
4. List all the subsets, (including the empty set), for any given set. (Limit: four elements)

*5. Use Venn diagrams to represent a set and one of the subsets.

6. Determine from a list of sets which sets are equal.

*7. Determine from a list of sets which sets are equivalent.

8. Determine if a given set is finite or infinite.

SET THEORY APPLICATIONS UNIT I

A. A student did a survey in a class of 35 students and recorded the following data:

Characteristic	Number
Glasses	11
Blond Hair	6
Left-Handed	5
Glasses and Left-Handed	3
Blond Hair and Glasses	4
Left-Handed and Blond	3
Blond Hair, Left-Handed, Wears Glasses	2

1. Use a Venn Diagram to illustrate the results of the survey.

2. Conduct a survey in your class based on the number of students who ski, play hockey and play a musical instrument. Record your results and make a Venn diagram to illustrate your survey.

3. Conduct a survey in your community (outside of the school).

B. In Language, the word 'and' corresponds to the Math idea of 'intersection'. The word 'or' corresponds to the Math idea of 'union'.

Suppose 'A' represents the sentence, "the person is wearing green" and 'B' represents the sentence "the person wears glasses" and 'C', "the person is wearing runners".

Name the people in your class who fit the following criteria:

A and B = ______________________________

A and C = ______________________________

B and C = ______________________________

A and B and C = ______________________________

A or B = ______________________________

A or C = ______________________________

(A or B) and C = ______________________________

(A or B) and A = ______________________________

A and (B and C) = ______________________________

(A and B) and C = ______________________________

A or B or C = ____

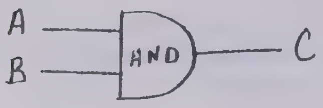
(Not A) or B = ____

(Not A) and A = ____

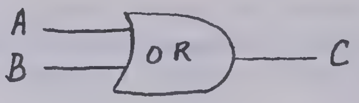
(Not A) or (Not B) = ____________________________________

(Not A) and (Not B) = ___________________________________

C. Electronic computers use integrated circuits to reproduce the idea of "or" (union) and "and" (intersection), in electric circuits. These circuits are represented in electrical diagrams in the following manner.

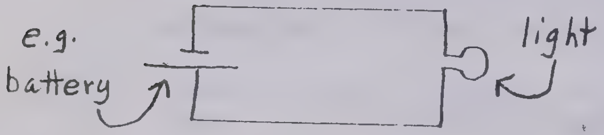


To get electricity at "C" you must have a current at "A" and "B".

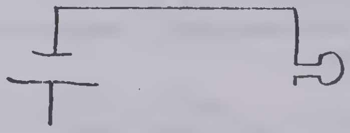


To get electricity at "C" you only need electricity at "A" or "B".

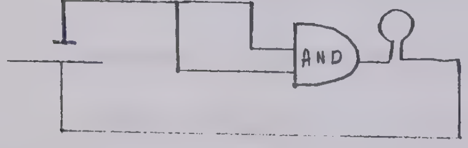
1. Would the light come on in each of the following circuits?

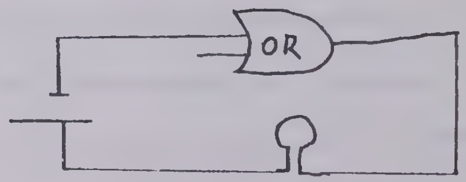


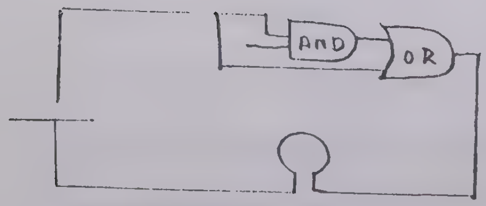
Yes

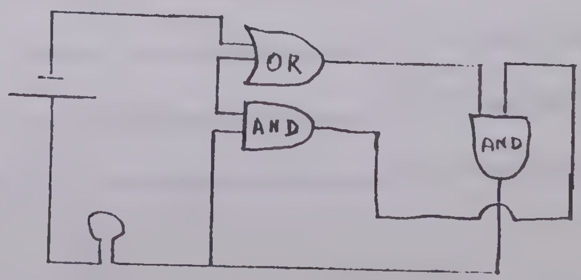


No

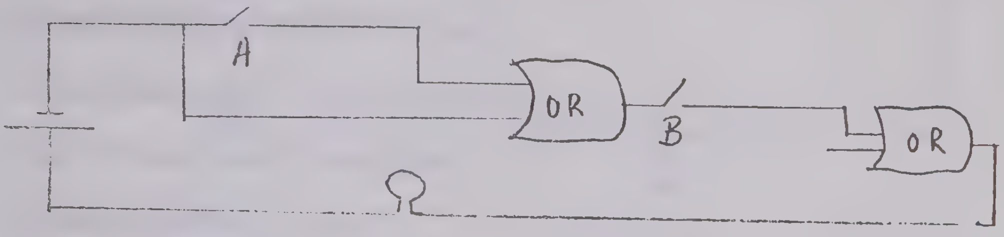


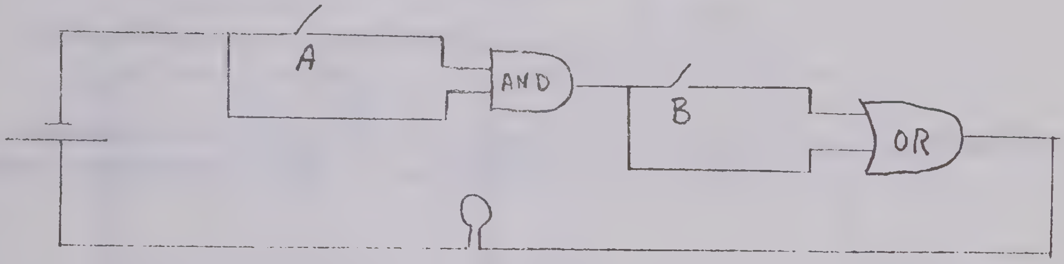






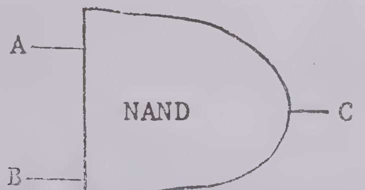
2. The following symbol represents a switch ___/ ________. In the following diagrams which switch must be on to make the light work?





3. Design circuits of your own using "AND" and "OR" integrated circuits. Try to think of uses for these circuits, e.g., a switch at the top and bottom of a stairwell controls a basement light.

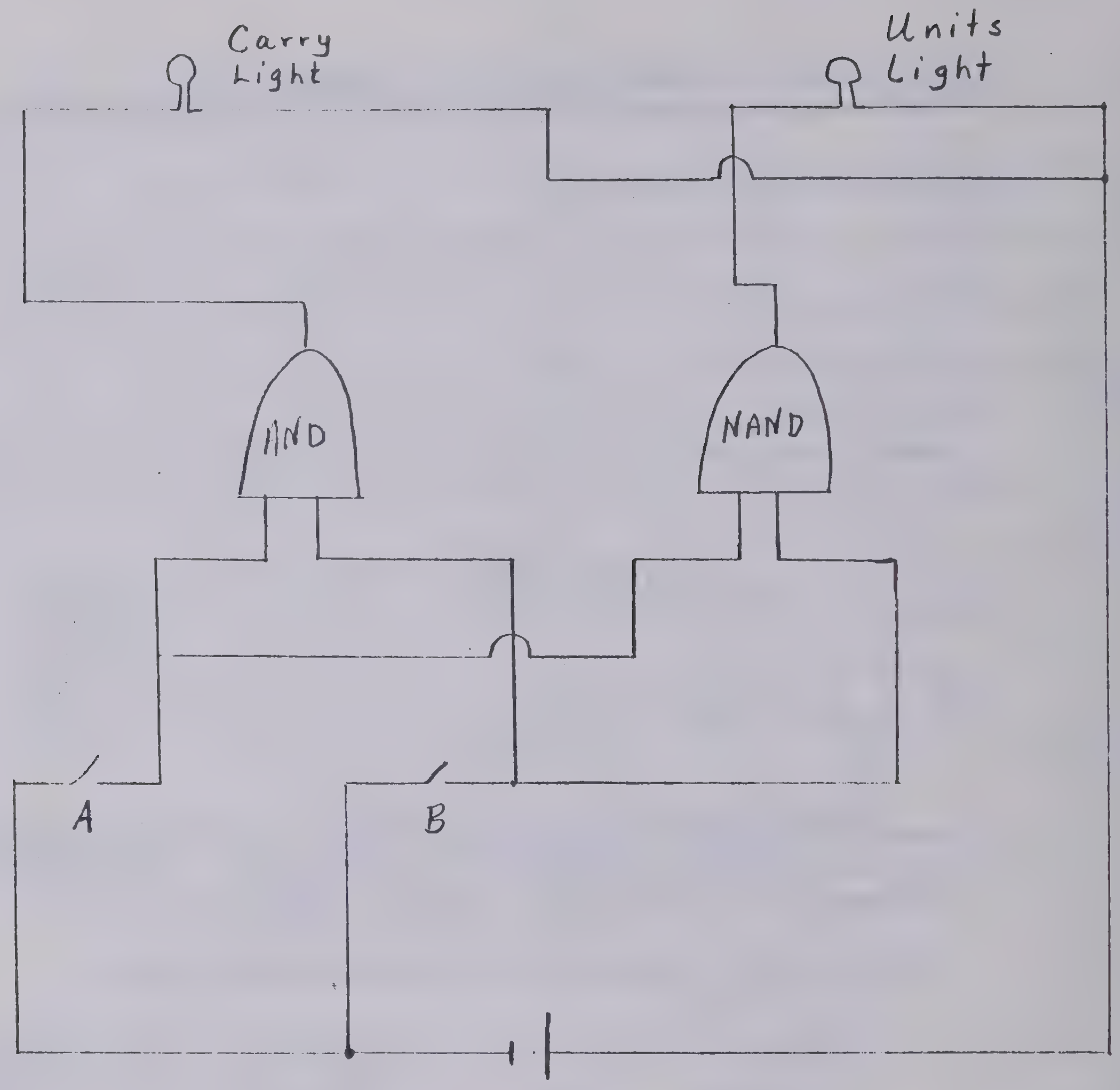
4. The NAND logic circuit works in the following manner:



The current passes through the circuit to C if there is electricity at A or B but not both.

Binary addition is performed by an electrical circuit called a binary adder. One way to make a binary adder is shown below. This performs the operations shown in the addition table.

	BINARY ADDER			
ADDITION TABLE	A	B	CARRY LIGHT	UNITS LIGHT
0 + 0 = 0	off	off	off	off
0 + 1 = 1	off	on	off	on
1 + 0 = 1	on	off	off	on
1 + 1 = 1	on	on	on	off



To extend this adder, join this circuit to itself through the carry light. Try to build a binary adder or a binary counter.

5. Switches arranged in series act like the "AND" logic and switches in parallel like the "OR" logic. The SRA have a kit called "Math Applications Kit". Look up the card under "Occupations" #42, and try the suggested activities.

6. There are many books on logic and electrical circuits. Prepare a report from one of these books on this topic.

D. Use the ideas you have learned to depict some of the following:

1. Classification of animals.
2. Classification of blood types.
3. Classification of ethnic groups.
4. Classification of countries.
5. Classification of plants.
6. Classification of motor vehicles.
7. Classification of buildings, etc.

E. An additional activity can be found in the SRA Math Applications Kit under Occupations, #35.

HISTORY

LEVEL 7

UNIT I

SET THEORY

REFERENCES

The Committee recommends the following references as primary sources of information for Junior High School teachers and students. We suggest that those books labelled (T) be available as teacher references and those labelled (S) be available in quantities of 3 - 5 for class use. Many of these books may be in your library now and extra copies may be borrowed from the Library Service Centre.

These references are numbered (1 - 14) for referral in the following outline:

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   510 AD 59 g (S)

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519 K58 King, Amy C. and Read, Cecil B. Pathways to Probability, Holt, Rinehart and Winston, Inc., New York, 1963.

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512 N213 N.C.T.M. Historical Topics in Algebra. National Council of Teachers of Mathematics, Washington, D.C., 1971.

Newman, James R. The World of Mathematics. (Vol. 1, 2, 3, 4) Simon and Schuster, New York, 1956.

Smith, D.E. History of Mathematics. (Vol. 1,2) Dover Publications, Inc., New York, 1958.

920 T849 Turnbull, H.W. The Great Mathematicians. New York University Press, New York, L969.

Black, Gerald J. Canada Goes Metric. Doubleday Canada Ltd., Toronto, 1974.

Posters

1. Walch, J.W. (Publisher) "Posters on Famous Mathematics". Available on loan from the Library Service Centre.

2. I.B.M., Timeline "Men of Mathematics", available from I.B.M. Library, Calgary. Ask for Item #5050003. (Free)

Busts

"Mathematicians of the Century", available from Moyer. Available on loan from the Library Service Centre. (Price $48.00)

Movies

CK 10591 "Possibly So Pythagoras". Available on loan from Instructional Aids Department.

CK 538 "Donald Duck in Math Magic Land". Instructional Aids.

Games

1. Euclid. Western Educational Activities. For advanced students.

The resource list on Posters, Busts, Movies and Games was taken from Men of Mathematics - A Resource Unit developed by J. Barnes.

E. T. V. Math Series (Produced in Ontario). Available from Central Office.

Tape #3 part (a) Square Root: Newton's Method. (Time 20 min., 275 ft.)

Useful for introducing square roots in grades 8 or 9.

Tape #5 part (b) History of Computers.

Useful as a motivational unit.

Tape #5 part (f) Number Systems.

Useful for introducing number theory, grade 7.

Tape #6 part (a) History of Numerals

Useful in grade 7 whole numbers.

Tape #6 part (b) History of π.

Grade 9 geometry.

Tape #6 part (c) From Time to Time

Development of calendar.

Tape #6 part (f) History of India(n) Mathematics

Laid the basis for our present number system and useful in History of Math in an option.

Tape #7 part (a) Inverse Variation

Grade 9 functions.

Tape #7 part (b) Graphs

Grade 8 coordinate system (Descarte).

Tape #9 part (a) Fibonacci Sequence

Grade 8 real numbers.

Tape #9 part (b) The Divine Proportion: Golden Section

Grade 9 geometry.

Tape #9 part (c) Map Making

Useful for upper ability students in grade 9 solid geometry.

Tape #10 part (c) What are Numbers

History of development of number systems. Useful as an introduction to grade 7 number systems.

LEVEL 7

SET THEORY

OBJECTIVES

Set Theory

Students should be able to:

	Objective	Reference Activities
1.	Use symbols such as $\{\ \}$, $\in$, $\emptyset$, ..., when referring to sets.	8
2.	Tabulate five sets from their everyday life. (Limit: five elements)	
3.	Select the sets that contain only the elements from a given universe.	2, 3
4.	List all the subsets, (Including the empty set) for any given set. (Limit: four elements)	
* 5.	Use Venn diagrams to represent a set and one of its subsets.	4, 6
6.	Determine from a list of sets which sets are equal.	
* 7.	Determine from a list of sets which sets are equivalent.	
8.	Determine if a given set is finite or infinite.	1, 3, 5
9.	Develop an appreciation of mathematical historians.	7

UNIT I: SET THEORY

RESOURCES

Georg Cantor (1845 - 1918) originated Set Theory and developed theories on infinity.
Reference #12, Pages 121-123
Reference #10, Pages 224-229
Reference #3, Pages 29-31 and
Reference #11, pp. 14

Aristotle (384 - 322 B.C.) - Developed symbolic logic.
Reference #14, Pages 105-109

Zeno (about 300 B.C.) - Fascinated by the idea of infinity.
Reference #5, Page 45
Reference #6, Pgs. 58-60
Reference #4, Pages 24-25

John Venn (1834 -1923) - English logician whose name is used for Venn diagrams.
Reference #12, Pages 120-121
Reference #5, Page 171

Evariste Galois (1811 - 1832) - Unified different branches of math by development of set theory, group theory and symbolic logic.
Reference #5, Pages 172-173
Reference #11, Page 12
Reference #4, Pages 362-377

Lewis Carrol (1832 - 1898) - Development of symbolic logic.
Reference #5, Pages 174-175
Reference #2, Pages 4-5

Euler (1707 - 1783) - Euler's circles were used to show sets and subsets.
Reference #4, Pages 139-152

George Boole (1815 - 1864) - Developed Boolean Algebra, and algebraic treatment of symbolic logic.
Reference #4, Pages 433-477
Reference #5, Page 170

UNIT I: SET THEORY

ACTIVITIES

1. Read about "The Founder of Set Theory" in Math Enrichment (reference 11) page 14.

   (a) Who founded set theory?

   (b) How did he define a finite set? Infinite set?

   (c) What other contribution did he make to mathematics?

2. Read pages 4 and 5 in Readings in Mathematics (reference #2) and try the questions at the end of page 5.

3. (a) Explain why Achilles (a fast runner) was unable to beat the Tortoise. Refer to:

   Mathematics (reference 5) page 45 or
   Mathematics: Man's Key to Progress (reference 6) pages 58-60.

   (b) Complete the activity on page 60 of

   Mathematics: Man's Key to Progress.

4. Aristotle discovered a relationship between mathematics and logic. Explain.

   References: The Mathmen (reference 14) pages 105-111.

5. Make a poster to display some ways that people have thought about infinity. References:

   Readings in Math Book II (reference 3) pages 30-31
   Of Men and Numbers (reference 10) pages 224-229
   Mathematics: Man's Key to Progress (reference 6) pages 59-60

6. In "Story of Math for Young People" (reference 12) read pages 120-121.

   After whom were the Venn diagrams named?

   What nationality was he?

   Make up a diagram showing intersection from your reading.

   Make up a diagram showing union.

7. Group theory has been called the supreme art of mathematical abstraction. Galois was involved in this theory as well as a great tragedy.

   Tell the story of the tragedy.

Reference #11, page 12	Mathematics Enrichment
Reference #5, page 173	Life Science Library Mathematics.

8. Make a truth table similar to Boole's in Life Science Library, Mathematics, page 170 (reference #5).

LEARNING PACKAGE

LEVEL 7

UNIT I

SET THEORY

UNIT I - SET THEORY

PERFORMANCE OBJECTIVES

Students should be able to:

1. Use symbols such as $\{\ \}$, $\in$, $\emptyset$, ..., when referring to sets.
2. Tabulate five sets from their everyday life.
   (Limit: five elements.)
3. Select the sets that contain only the elements from a given universe.
4. List all the subsets, (including the empty set), for any given set.
   (Limit: four elements.)

*5. Use Venn diagrams to represent a set and one of its subsets.

6. Determine from a list of sets which sets are equal.

*7. Determine from a list of sets which sets are equivalent.

8. Determine if a given set is finite or infinite.

DEVELOPMENT AND EXERCISES

UNIT: I — LEVEL: 7

Sets — OBJECTIVE NUMBER: 1 & 2

OBJECTIVE: 1. Use symbols such as $\{\ \}$, $\in$, $\emptyset$, $\cdots$, when referring to sets.
2. Tabulate five sets from their everyday life.
(Limit: five elements)

NOTE: Transparency masters have been included through-out the unit. They could also be used as student notes.

SUGGESTED DEVELOPMENT: I. First develop the idea of a set by using words like: class, group, team, family, school, club, scout pack, etc. Give the students some specific examples.

eg. The set of aces in a deck of cards.
The set of coaches for the wrestling team.
The set of pupils in grade 7, in your room.

Have students give some other examples orally and then formulate the definition.

SET: A collection of things which are distinguishable from each other.

ELEMENTS: The objects that make up the set.

II. Introduce the idea of symbols by talking about a secretary and her knowledge of shorthand. List the following, along with their meaning:

$A = \{1,2\}$	A is the set whose elements are one and two.
$\in$	Belongs to or is an element of. eg. $B = \{2,4,6,8\}$ $4 \in B$
$\{\ \}$ or $\emptyset$	A set that has no elements. (Null set or empty set). eg. The set of blue-haired boys in the room.
$\ldots$	On and on, or and so on.

III. Now that the students have these symbols, they can now tabulate various sets. To tabulate means simply, list the elements within the brace brackets $\{ \ \}$.

eg. Tabulate the set of boys in the first row.

$\{\text{Bob, Don, Dave, John, Tony, Harry}\}$

IV. DESCRIBING SETS

Have the students use words to describe sets. Emphasize that the description should be specific enough to describe the particular set and the universe from which it came.

eg. $\{0,1,2,3\}$ should be described as the set of whole numbers less than four, not a set of four numbers.

SETS

SETS MAY BE DESCRIBED IN WORDS OR TABULATED

DESCRIPTION: SET A IS THE SET OF WHOLE NUMBERS LESS THAN 6

TABULATION: TO LIST BETWEEN BRACES

$A = \{1, 2, 3, 4, 5\}$

SETS 7 OBJ 1, 2, 3

SET SYMBOLS

$A = \{0, 1, 2, 3\}$

A IS THE SET WHOSE ELEMENTS ARE 0, 1, 2, 3

$2 \in A$

2 IS AN ELEMENT OF SET A

$B = \{\ \}$ OR $B = \varnothing$

SET B IS AN EMPTY SET

$B = \{0, 1, 2, 3 \ldots\}$

B IS THE SET WHOSE ELEMENTS ARE 0, 1, 2, 3 AND SO ON

SETS 7 OBJ 1, 2, 3

DEFINITIONS

SET	ELEMENT
A COLLECTION OF THINGS WHICH ARE DISTINGUISHABLE FROM EACH OTHER	AN OBJECT THAT BELONGS TO A SET

EMPTY SET
A SET THAT HAS NO ELEMENTS

SETS 7 OBJ 1, 2, 3

EXERCISES OBJECTIVE NO. 1 - 2

I. WRITE THE MEANING FOR EACH OF THE FOLLOWING SYMBOLS:

1. ... and so on
2. $\{a, b\}$ the set of a,b
3. $\in$ belongs to ; is an element o
4. $\emptyset$ the empty set

To Tabulate is to List in Braces

II. TRANSLATE INTO WORDS:

1. Volkswagen $\in$ B Volkswagen is an element of set B.
2. d $\in$ A "d" belongs to set A
3. C = $\emptyset$ Set C is an empty set.

III. DESCRIBE EACH OF THE FOLLOWING:

1. C = $\{60, 61, 62, \ldots 68\}$
   The set of whole numbers from 60 through 68
2. A = $\{a, b, c, \ldots z\}$
   The set of the English alphabet from a
3. B = {Volkswagen, Toyota, Datsun}
   A set of three ~~imported~~ cars.
4. D = {Paris, Washington, Ottawa, Tokyo}
   A set of national capitals.
5. E = {B. C., Alta., Sask.}
   A set of Canadian provinces
6. F = {St. Paul, Leduc, Spruce Grove, Ft. McMurray}
   A set of Alberta towns.
7. G = {Oilers, Toros, Blazers, Jets}
   A set of names of professional hockey teams
8. H = $\{3, 6, 9, \ldots\}$ The set of multiples of three.
9. I = $\{2, 4, 6, 8\}$ The set of even numbers which are less than ten.

IV. TABULATE:

1. The set of the first five natural numbers
   $\{1, 2, 3, 4, 5\}$
2. The set of 5 of your favorite T.V. programs answers may vary
3. The set of whole numbers between 14 and 15 $\emptyset$
4. The set of girls seven feet tall $\{\ \}$
5. The set of the first 3 numbers divisible by 5 $\{5, 10, 15\}$

V. WRITE THE NUMBER OF ELEMENTS IN EACH OF THE FOLLOWING SETS:

1. A = $\{x, y, z\}$ 3
2. B = $\{3, 7\}$ 2
3. C = $\{\ \}$ 0
4. D = $\{0\}$ 1
5. E = $\{\square, \nabla, \circ, \Delta\}$ 4
6. F = $\{3, *, L, \uparrow\}$ 4
7. G = $\{6, 7, 8, \ldots 17\}$ 12

DEVELOPMENT AND EXERCISES

UNIT:	I	LEVEL:	7
	Sets	OBJECTIVE NUMBER:	3

OBJECTIVE: Select the sets that contain only the elements from a given universe.

SUGGESTED DEVELOPMENT: Begin a short discussion about things in the classroom. Tabulate some sets concerning them.

eg. (a) Set of boys.
(b) Set of books.
(c) Set of writing instruments.

We call the set of all the possible elements in a particular example a universe. The universe for the above is the set of things in the classroom.

Do some more orally, making sure you have the students describing the universe.

eg. (a) Universe - students in school
Set - students in your class

(b) Universe - tools
Set - carpenter's tools

EXERCISES OBJECTIVE NO. 3

I. SELECT THE SETS THAT CONTAIN ONLY THE ELEMENTS FROM THE UNIVERSE OF NORTH AMERICAN CARS.

A = {Chev., Toyota, Pontiac, Buick} no
B = {Ford, Meteor, Chev.} yes
C = {Datsun, Volkswagen, Austin} no
D = {Ford, Mustang, Pinto} yes
E = {Vega, Rambler, Olds, Camaro} yes

II. SELECT THE SETS OF CITIES THAT CONTAIN ONLY THE ELEMENTS FROM THE UNIVERSE OF THE CITIES EAST OF EDMONTON.

A = {Winnipeg} yes
B = {Halifax, Vancouver, Victoria} no
C = {Regina, Winnipeg, Montreal, Toronto} yes

III. SELECT THE SETS THAT CONTAIN FOOTBALL TEAMS IN THE CANADIAN FOOTBALL LEAGUE.

X = {Edmonton, Calgary} yes
Y = {Vancouver, Regina, Winnipeg} yes
Z = {Montreal, Toronto, Hamilton} yes
S = {Leduc, Red Deer, Grande Prairie} no
T = {Edson, Halifax, Calgary, Ponoka} no

IV. SELECT THE SETS THAT CONTAIN ONLY ODD NUMBERS.

R = {1, 2, 3, 4} no
O = {3, 6, 9} no
N = {2, 4, 6, 8} no
S = { } no
Q = {1, 3, 5, 7} yes

Secret Message

V. PLACE THE LETTER OF THE SETS IN COLUMN 2 IN THE APPROPRIATE BLANKS IN COLUMN 1. IF YOU DO IT CORRECTLY, THERE WILL BE A SECRET MESSAGE FOR YOU.

UNIVERSE

M 1. Food
A 2. Records
T 3. Cars
H 4. Movie Stars
C 5. T. V. Programs
E 6. Hockey Players
N 7. 10-Speeds
T 8. vehicles with 4 wheels
E 9. Skaters
R 10. Holidays
O 11. Football Player
F 12. Books
T 13. Machines With Motors
H 14. Male Hollywood Stars
E 15. Hockey Forwards
U 16. Beds
N 17. 2-Wheel Machines
I 18. Singers
V 19. Letters Of The Alphabet
E 20. People Who Score Goals
R 21. Days Schools Are Closed
S 22. Girls' Clothes
E 23. People Who Use Hockey Sticks

SETS

A = {LP, 45}
C = {Sesame Street, Disneyland}
E = {Bobby Hull, Gordie Howe, Bobby Orr}
F = {Novels, Pocket}
H = {Paul Newman, Steve McQueen}
I = {Cat Stevens, Dionne Warwick}
M = {Hot Dogs, Hamburgers}
N = {CCM, Peugoet, Raleigh}
O = {George McGowan}
R = {Labour Day, May 24th, Christmas}
S = {Dress, Skirt, Nylons}
T = {Ford, Chev., Buick}
U = {Bunks, Double, Single}
V = {a, b, c, d, e, f, g}

DEVELOPMENT AND EXERCISES

UNIT:	I	LEVEL:	7
	Sets	OBJECTIVE NUMBER:	4

OBJECTIVE: List all the subsets including the empty set for any given set. (Limit: 4 elements)

SUGGESTED DEVELOPMENT: Present the students with

$$A = \{\text{Xmas, Easter, May 24th}\}$$

To list all the subsets of a given set use the ALL to NOTHING approach by making a chart with spaces for all the subsets with decreasing numbers of members.

Eg. #1 $\{X, E, M\}$

3 members	$\{X, E, M\}$
2 members	$\{X, E\}$ $\{X, M\}$ $\{E, M\}$
1 member	$\{X\}$ $\{E\}$ $\{M\}$
0 members	$\emptyset$ or $\{\ \}$

Eg. #2 $\{1, 2, 3, 4\}$

4 members	$\{1, 2, 3, 4\}$
3 members	$\{1,2,3\}$ $\{1,2,4\}$ $\{1,3,4\}$ $\{2,3,4\}$
2 members	$\{1,2\}$ $\{1,3\}$ $\{1,4\}$ $\{2,3\}$ $\{2,4\}$ $\{3,4\}$
1 member	$\{1\}$ $\{2\}$ $\{3\}$ $\{4\}$
0 members	$\emptyset$ or $\{\ \}$

This makes it impossible to omit the improper subset (the set with all the elements) and the empty set (the set with no elements).

Proper Subset : Any subset of a given set except the set itself.

Improper Subset: The set itself.

Once the students have the idea of how to list subsets, extend it to see if they can choose which sets are subsets of a given universe. Make sure you include the improper subset and the empty set.

eg. $U = \{\text{triangle, square, rectangle,}$
$\text{trapezoid}\}$

$A = \{\text{triangle}\}$

$B = \{\text{square, circle}\}$

$C = \{\text{rectangle, triangle, trapezoid}\}$

$D = \{\ \}$

$E = \{\text{trapezoid, rectangle, square,}$
$\text{triangle}\}$

We could make these statements about the above sets:

$A \subset U$, $C \subset U$, $E \subset U$, $A \subset C$

$B \not\subset A$, $B \not\subset U$, $D \subset U$, ...

EXERCISES OBJECTIVE NO. 4

I. LIST ALL THE SUBSETS OF:

1. A = {1,2,3} – {1,2,3}, {1,2}, {1,3}, {2,3}, { }, {1}, {2}, {3}
2. B = {△, ○, □, ⏢} ② {△, ○, □, ⏢} {△,○,□} {△,○,⏢} {△,□,⏢} {○,□,⏢} {△,○} {△,□} {△,⏢} {○,□} {○,⏢} {□,⏢} {△} {○} {□} {⏢} { }
3. C = {dog, cat} ③ {dog, cat} {dog} {cat} { }
4. D = {Bob} ④ {Bob} { }
5. E = { } ⑤ { }

II. LIST THE PROPER SUBSETS OF:

1. F = {%, #} {%} {#} { }
2. G = {6,7,8} {6, 7} {6, 8} {7, 8} {6} {7} {8} { }

*III. LIST THE SUBSETS OF V = {a,e,i,o,u}

IV. A = {2,4,5,7,8} C = { } E = {5,7,8,2}
B = {2,4} D = {6,7,8,5} F = {4}

THE FOLLOWING STATEMENTS REFER TO THE 6 SETS TABULATED ABOVE. THEY ARE EITHER TRUE OR FALSE. PLACE 'T' or 'F' IN THE APPROPRIATE BLANKS.

1. $B \subset A$ T
2. $C \subset A$ T
3. $D \subset A$ F
4. $E \subset A$ T
5. $F \subset A$ T
6. $F \subset B$ T
7. $F \subset E$ F
8. $F \subset D$ F
9. $C \subset F$ T
10. $B \subset E$ F
11. $E \subset B$ F
12. $A \subset E$ F
13. $C \subset D$ T
14. $E \subset D$ F
15. $F \subset F$ T

V. a) CROSS OUT THE SETS WHICH ARE NOT SUBSETS OF R = {2, 4, 6}.
THE REMAINING SUBSETS SHOULD SPELL YOUR FAVORITE WORD.

~~C = {0,1}~~ ~~D = {8,12}~~ ~~X = {104,106}~~
M = {6,4,2} A = { } T = {6} MATH
~~B = {1,2}~~ ~~E = {3,6,4}~~ ~~Y = {2,4,8}~~ H = {4}

b) WRITE THE RELATIONSHIP BETWEEN 'R' AND IT'S SUBSETS USING CORRECT NOTATION. eg. $H \subset R$

$M \subset R$ $R \subset M$ $A \subset R$
$T \subset R$ $H \subset R$

* { } IS A SUBSET OF EVERY SET *

*

A SET IS NOT * A PROPER SUBSET OF ITSELF *

SUBSETS

DEFINITION: A SET, ALL OF WHOSE MEMBERS ARE MEMBERS OF ANOTHER SET, IS A SUBSET OF THAT SET

$A = \{5, 6, 7\}$ $B = \{5, 6\}$ $C = \{7, 8\}$

SET B IS A SUBSET OF SET A	SET C IS NOT A SUBSET OF SET A
$B \subset A$	$C \not\subset A$

THE EMPTY SET IS A SUBSET OF EVERY SET

A SET IS A SUBSET OF ITSELF

SETS 7 OBJ 4

LISTING SUBSETS

TO LIST ALL THE SUBSETS OF A GIVEN SET, LIST FROM ALL TO NOTHING

SET A = { 1, 2, 3 }

THE SUBSETS OF A ARE:

{ 1, 2, 3 }, { 1, 2 }, { 1, 3 }
{ 2, 3 }, { 1 }, { 2 }, { 3 }, { }

B = { □, ○, △, ☆ }

THE SUBSETS OF B ARE:

{□, △, ○, ☆}, {□, △, ○}, {□, △, ☆}
{□, ○, ☆}, {△, ○, ☆}, {□, △}
{□, ○}, {□, ☆}, {△, ○}, {△, ☆}
{○, ☆}, {□}, {△}, {○}, {☆}, { }

DEVELOPMENT AND EXERCISES

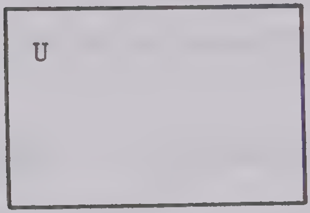
UNIT: I | LEVEL: 7

Sets | OBJECTIVE NUMER: 5

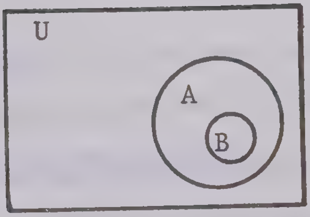
OBJECTIVE: * Use Venn diagrams to represent a set and one of its subsets.

NOTE: This objective will be extremely useful when we wish to picture the relationship between various number systems.

SUGGESTED DEVELOPMENT: One of the simplest ways to demonstrate notions of sets is through the use of diagrams. There is a variety of methods available but we will only concern ourselves with Venn diagrams.



The rectangle represents the Universe.



$B \subset A$

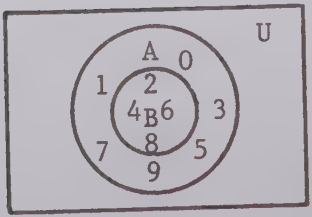
The circular regions represent the subsets of U.

Circular region B represents a subset of A.

EXAMPLE: U = Whole Numbers

$A = \{0, 1, 2, \ldots 9\}$

$B = \{2, 4, 6, 8\}$



EXERCISES OBJECTIVE NO. 5

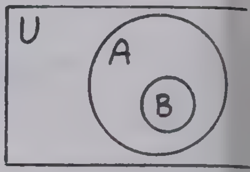
I. MAKE A VENN DIAGRAM FOR SETS A AND B

U = set of all teachers in the school

A = set of all math teachers in the school

B = set of all Grade 7 math teachers in the school

I



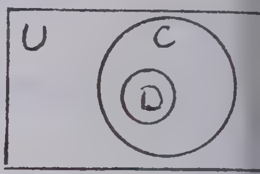
II. MAKE A VENN DIAGRAM FOR SETS C AND D.

U = set of all grade 7 students in Alberta

C = set of all grade 7's in your room

D = set of all female grade 7's in your room

II



III. USE VENN DIAGRAMS TO SHOW THE RELATIONSHIP BETWEEN THE FOLLOWING SETS. WRITE THE ELEMENTS IN THE PROPER CIRCLES.

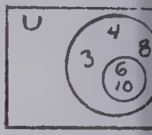
1. $\{3, 4, 6, 8, 10\}$ and $\{6, 10\}$ U = $\{1, 2, 3 \ldots\}$
2. $\{W, S\}$ and $\{W, S, T, X\}$ U = Alphabet
3. $\{2, 5, 6, 7\}$ and $\{7, 5, 2, 6\}$ U = $\{1, 2, 3, \ldots\}$
4. $\{$chocolate, milk, shake$\}$ and $\{$shake, chocolate, milk$\}$

   U = English words

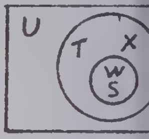
5. The set (U) of letters in the English alphabet; the set (A) of vowels in the English alphabet.
6. The set (M) of the months of the year beginning with J. Universe is the set of months in one year.

III

1.



2.



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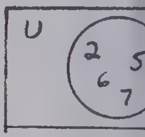

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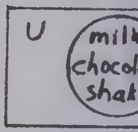

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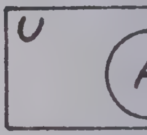

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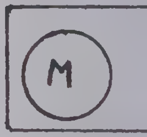
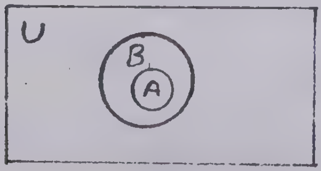


IV. DRAW A VENN DIAGRAM REPRESENTING $A \subset B$.

IV



VENN DIAGRAMS

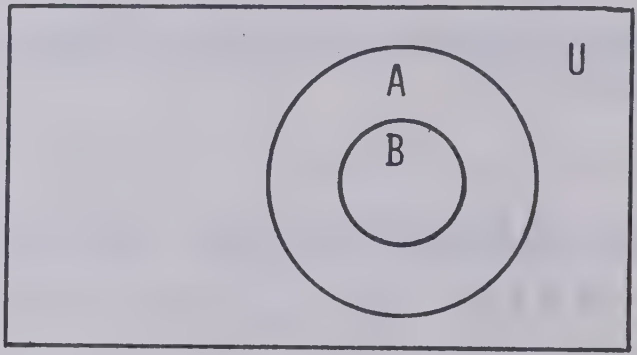
A UNIVERSE, SET AND SUBSET CAN BE REPRESENTED BY VENN DIAGRAM

A UNIVERSE IS REPRESENTED BY A RECTANGLE

A SET OR SUBSET IS REPRESENTED BY A CIRCLE

VENN DIAGRAMS

U IS THE SET OF
WHOLE NUMBERS



$A = \{1, 2, 3, 4\}$

$B = \{2, 3\}$

$B \subset A$

DEVELOPMENT AND EXERCISES

UNIT: I — LEVEL: 7

Sets — OBJECTIVE NUMBER: 6 & 7

OBJECTIVE: 6. Determine from a list of sets which ones are equal.

*7. Determine from a list of sets which sets are equivalent.

SUGGESTED DEVELOPMENT: Place two equivalent sets on the board.

EQUIVALENT SETS: Two sets which have the same number of elements.

SYMBOL: $\sim$ shows equivalence.

$$\begin{tikzcd}
X = & \{5, \arrow[d, leftrightarrow] & 7, \arrow[d, leftrightarrow] & 12\} \arrow[d, leftrightarrow] \\
Y = & \{\bigcirc, & \square, & \triangle\}
\end{tikzcd}$$

Point out that there is a one to one correspondence between the two sets.

Now place two equal (congruent) sets on the board.

$$\begin{tikzcd}
A = & \{10, \arrow[drr, leftrightarrow] & 100, \arrow[dl, leftrightarrow] & 1{,}000\} \arrow[dl, leftrightarrow] \\
B = & \{100, & 1{,}000, & 10\}
\end{tikzcd}$$

They have the same number of elements and the elements are the same for both sets.

NOTE: Equal sets are equivalent sets, but equivalent sets are not equal.

EQUAL AND EQUILVALENT SETS

EQUAL SETS

SETS THAT HAVE EXACTLY
THE SAME ELEMENTS ARE EQUAL

$A = \{4, 5, 6\}$ $B = \{5, 4, 6\}$

$$A = B$$

SET A IS EQUAL TO SET B

EQUIVALENT SETS

SETS THAT HAVE THE
SAME NUMBER OF ELEMENTS

C = { ■, ▲, ★ } $D = \{1, 2, 3\}$

$$C \sim D$$

SET C IS
EQUIVALENT TO SET D

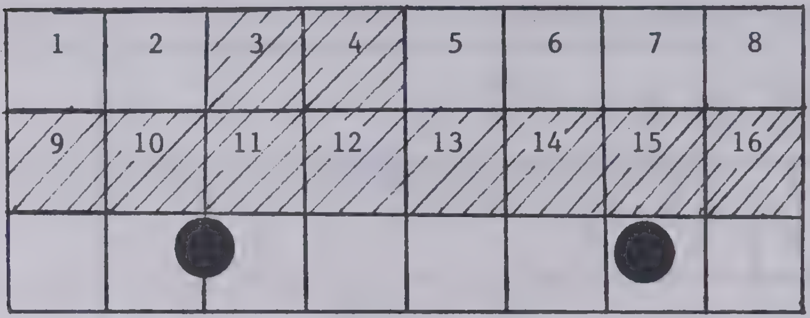
EQUAL SETS ARE
EQUIVALENT SETS
BUT
EQUIVALENT SETS
ARE NOT ALWAYS EQUAL

I. Place an X in the column or columns to show when the sets are equal, equivalent, or neither equal nor equivalent.

		EQUAL	EQUIV.	NEITHER
	{3, 6, 9, 12}, {6, 9, 12, 3}	X	X	
1.	{a, b, c}, {2, 9, 5, 4}			X
2.	{○, △, □, ◇}, {boy, girl, dog, cat}		X	
3.	{a, e, i, o, u}, {u, a, i, o, e}	X	X	
4.	{Bob, Bill, Sue}, {Shelly, Gregg, Terry}		X	
5.	{heads in your room}, {people in your room}		X	
6.	{records, food, sleep}, {dance, holidays, shows}		X	
7.	{4}, {1, 2, 3, 4}			X
8.	{football, hockey, baseball}, {baseball, hockey, football}	X	X	
9.	{dress, pattern, cooking}, {sewing, recipe, dish}		X	
10.	{10, 11, 12}, {10, 11, 12, ...}			X

ALL EQUAL SETS
ARE EQUIVALENT

II. What do you do if you are having difficulty with math?



There is a box in the above grid for each question below. If the question has 2 equal sets, shade the same numbered box in the grid as the question. When you have finished, you will have a picture for the answer to the original question.

X 1. {○ △ □} {○ ▱ □}

X 2. {1, 2, 3} {2, 3, 4}

✓ 3. {1, 2, 3} {3, 2, 1}

✓ 4. {a, b, c} {b, a, c}

X 5. {boy} {b, o, y}

X 6, { } {o}

X 7. {dog, cat} {pets}

X 8. {bikes, records} {2}

✓ 9. {a, b, c, f} {f, a, c, b}

10. {Sam,Sal,Joe} {Joe,Sam,Sal} ✓

11. {ice cream, hamburgers} {ice cream, hamburgers} ✓

12. {T.V., radio, shows} {radio, T.V., shows} ✓

13. { } { } ✓

14. {10, 20, 30} {30, 10, 20} ✓

15. {dollar, penny, quarter} {quarter, penny, dollar} ✓

16. {math, teacher, great} {great, math, teacher} ✓

KEEP ON TRUCKIN'

DEVELOPMENT AND EXERCISES

UNIT: I

Sets

LEVEL: 7

OBJECTIVE NUMBER: 8

OBJECTIVE: Determine if a given set is finite or infinite.

SUGGESTED DEVELOPMENT: Place a number of finite sets on the board.

$$X = \{2, 4, 6, 8\}$$

$$Y = \{\circ, \square, \triangle\}$$

$$Z = \{0, 1, 2, 3, 4\}$$

Ask how many elements in each set.

If a whole number can be named that tells how many elements are in a set, then the set is called finite.

Place the following sets on the board:

$$A = \{1, 2, 3, \ldots\}$$

$$P = \{5, 10, 15, \ldots\}$$

Ask how many elements in these two sets.

INFINITE SET: A set whose elements are unlimited in number.

FINITE AND INFINITE SETS

FINITE SET

SET WITH A DEFINITE
NUMBER OF ELEMENTS

$A = \{1, 2, 3\}$ - 3 ELEMENTS

$B = \{0, 1, 2, \ldots 100\}$ - 101 ELEMENTS

INFINITE SET

SET WHOSE ELEMENTS
NUMBER MORE THAN ANY NUMBER
WE COULD NAME

$C = \{0, 1, 2, 3, \ldots\}$

$D = \{5, 10, 15, \ldots\}$

$C = \{1, 3, 6, 9\}$ $D = \{1, 2, 3, \ldots\}$

$A = \{51, 107\}$ $O = \{10, 100, 1000, \ldots\}$

$T = \{6, 12, 18\}$ $G = \{2, 4, 6, \ldots\}$

I. Tabulate the next six elements of set D. $\{4,5,6,7,8,9\}$

II. Tabulate the next three elements of set O. $\{10\,000,\ 100\,000,\ 1\,000\,00$

III. Tabulate the next six elements of set G. $\{8, 10, 12, 14, 16, 18\}$

IV. Label each of the following sets "F" if they are finite, and "I" if they are infinite:

F (1) $\{2,4,6,8,10,12\}$

I (2) $\{0,1,2,3,4,\ldots\}$

F (3) The set of all the days of the week.

I (4) The set of all the numbers greater than 20.

V. Refer to the sets at the top of the page.

(1) How many elements in set C above? 4

(2) How many elements in set A above? 2

(3) How many elements in set T above? 3

(4) The three sets C, A, T are finite sets.

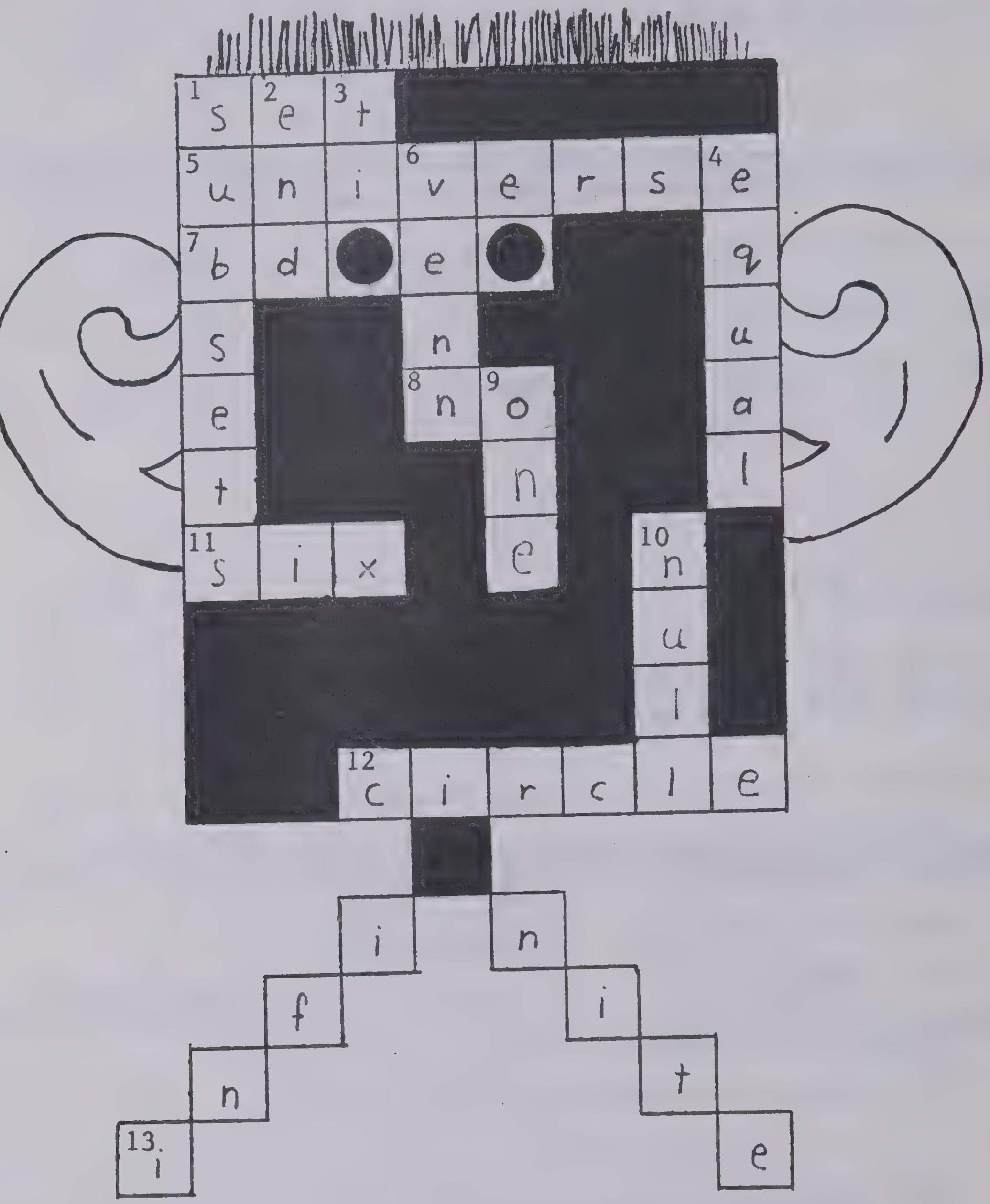
IV. Tabulate the elements of these infinite sets.

(1) The set of whole numbers greater than 10. $\{11, 12, 13, \ldots\}$

(2) The set of odd numbers. $\{1, 3, 5, \ldots\}$

(3) The set of numbers that end in 5. $\{5, 15, 25, \ldots\}$

I.



ACROSS:

1. A collection of things that have something in common.
5. A set that is usually represented by a rectangle.
7. C comes between them.
8. Not yes
11. $B=\{*, \square, \Delta, \nabla, \rho, o\}$
    There are ___ elements in B.
12. A set is usually represented as a _____ within a Venn diagram.
13. You cannot count the elements of this set.

DOWN:

1. Part of a set (plural).
2. A finite set of elements has an ____________.
3. It spelled backwards.
4. $\{a, b, c\}$, $\{c, a, b\}$
   These sets are __________.
6. The name of the diagrams that illustrate sets.
9. $A = \{0\}$
   This set has ____ element.
10. Another name for an empty set.

LABS

LEVEL 7

UNIT II

WHOLE NUMBERS

OBJECTIVE: *Solve conditions for equality using an informal approach for the following:*

a) $A + X = B$

b) $AX = B$

c) $AX + B = C$

d) $A + BX = C$

TIME: one class period

MATERIALS: metresticks, wire or string stand (or suitable substitute, see diagram) a quantity of items (2 ½ inch common nails or metal flat washers) having the same weight. 2 styrofoam cups.

ORGANIZATION: Organize the room into "Lab stations". Assign each group to a station. Have a set of the materials at each station, this set will include variables x, y, t, p, q, s, 2 of w, 4 of d, 3 of m, 4 of v, 2 of n, 3 of q, 2 of r, 2 of u, 5 of a, 7 of b.

The variables are made of a quantity of items eg. a number of nails taped together in different shapes so that the students will not recognize the number of nails present in each package.

The number of items (nails or washers) are:

$x = 4$, $y = 3$, $t = 7$, $p = 8$, $g = 6$, $5 = 6$, $w = 5$, $b = 2$, $d = 3$, $m = 4$, $v = 2$, $n = 3$, $q = 3$, $r = 5$, $u = 2$, $a = 2$.

Label taped variables according to the above letters. ie. x will be a taped variable composed of 4 nails.

**NOTE:* A group session with the teacher is needed after the 1st table of data. This session will deal with problems students may have with conditions of the type $AX = B$. eg. $2W = 10$; students will take off one w at a time and will consider it subtracting. The discussion would bring out the point that division is repeated subtraction.

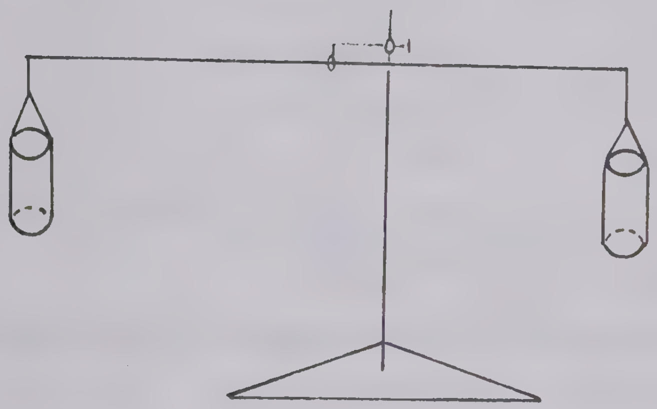
Direction to students: Go to your assigned stations, using the material at the station follow the instructions on the lab sheet.

ATH LAB: Solving conditions

URPOSE: To solve conditions

ATERIALS: 1 metrestick, stand, wire, or string,
2 cups, a number of items labelled, x, y, t, p, g, s, 4 of d,
7 of b, 5 of a, 2 or r, 2 of u, 3 of m, 4 of v, 2 of n, 3 of q.

LEFT SIDE = RIGHT SIDE



Metre Stick Cup Balance

ETHOD:

1. Set up apparatus as in diagram.
2. Balance apparatus ie. so that the left side is equal to the right side.
3. For Trial 1 put 3 single nails and an X package on the left side and 7 single nails on the right side. Keep the apparatus balanced when X is alone by doing the same thing to each side.

Record the results for this condition. Solve the following conditions in the same manner and record the results.

RESULTS: I.

Trial	Left side = Right side	What did you do to each side to keep it balanced?	Write a condition to show this	Variable equals
1.	3 + x = 7			
2.	y + 5 = 8			
3.	t + 2 = 9			
4.	p - 5 = 3			
5.	g - 2 = 4			
6.	s - 5 = 1			
7.	2 x w = 10			
8.	3 x b = 6			
9.	4 x d = 12			

You should now attend a group session with your teacher and other members of the class who are ready.

What conclusions did you arrive at in the group session?

(a) When the condition involves adding?

(b) When the condition involves subtracting?

(c) When the condition involves multiplying?

II. Using the same method as above solve the following conditions and record results in the table below:

Trial	Left side = Right side	What did you do to each side to keep it balanced?	Write a condition to show this	Variable equals
10.	3 x m = 12			
11.	4 x v = 8			
12.	2 x n = 6			

Check your answers to #10, 11 and 12 with your teacher. If they are correct try solving the conditions given below.

1. 3 x q + 2 = 11

   (a) Divide both sides by 3. Can you do this and still have have it balance?

(b) Subtract 2 from each side. Does it still balance?

Write a condition to show this.

(c) Which should be done first, the division or the subtraction?

(d) Continue solving the condition you now have $3 \times q = 9$.

Write a condition to show what you did.

(e) What does the variable equal?

Solve the following conditions and record the results in the table below.

Trial	Left side = Right side	What did you do first?	Write a condition to show This	What did you do next?	Write a condition to show this	Variable equals
2	$2 \times r + 1 = 11$					
3	$2 \times r + 3 = 7$					

CONCLUSION: When solving a condition of the type $2 \times r + 1 = 11$,

(a) What gets done first?

(b) What gets done second?

III. Solve the condition $3b + 4b = 14$

(a) Rewrite the condition to show how many b's there are.

(b) Solve the new condition and write another condition to show what you did.

(c) What is the variable equal to?

ACTIVITIES

LEVEL 7

UNIT II

WHOLE NUMBERS

Place Value Matchup

TEACHER INSTRUCTIONS: Duplicate and distribute to class. Let them prepare the activity.

Two Can Play:

TO GET READY:

Cut out the cards. Put the *NUMBER CARDS* facedown in a pile. Put the *PLACE VALUE CARDS* facedown in another pile.

HOW TO PLAY:

Draw one card from each pile. The *PLACE VALUE CARD* tells you how to find your score.

EXAMPLE: You draw these cards -

2 346 519 047	Your score is 3 times the digit in the *TENS* place.

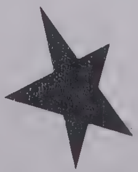
NUMBER CARDS

1 824 693 853	7 160 529 438
6 251 847 309	3 615 280 970
2 308 415 767	8 473 062 195
9 732 154 686	4 097 936 521
5 986 701 042	6 549 378 214

The digit 4 is in the tens place. Your score is 3 x 4 = 12 points.

Take turns until all cards have been played. Then find your total score. The player with the highest score is the winner.

PLACE VALUE CARDS

Your score is 2 times the digit in the *ONES* place.	Your score is 1 times the digit in the *TENS* place.	Your score is 1 times the digit in the *HUNDREDS* place.	Your score is 3 times the digit in the *THOUSANDS* place.
Your score is 3 times the digit in the *TEN THOUSANDS* place.	Your score is 4 times the digit in the *HUNDRED THOUSANDS* place.	Your score is 2 times the digit in the *MILLIONS* place.	Your score is 1 times the digit in the *TEN MILLIONS* place.
Your score is 3 times the digit in the *HUNDRED MILLIONS* place.	Your score is 2 times the digit in the *BILLIONS* place.		

Team Work

OBJECT OF THE GAME:

To add, subtract, multiply and divide whole numbers with speed and accuracy.

PLAY:

Each row works as a team. The teacher prepares computational questions (involving the four basic operations) on a ditto sheet. The computational sheet is placed face down in front of the first person of each row. When the teacher says "Start", the first person in each row does any question and passes the sheet onto the person behind. This person does any other question and passes it onto the next person. This continues until all questions on the sheet are done. A sheet is completed when it is returned to the first person in each row and placed face down. The teacher notes the order the rows finish. When all sheets are completed, papers are exchanged and the answers checked. (The teacher may read the answers or put them on the blackboard or overhead).

RULES:

Each player must do a question when it is his/her turn. Players may check the work of their team mates.

SCORING:

The completion of each sheet constitutes a round. If there are five rows the team that finishes first would be given a bonus of five points. The team finishing second gets four points, and so on. Add to this total the number of questions answered correctly by each team. The team that compiles the highest total wins the round. The team that accumulates the most points after three rounds is the winner of the game.

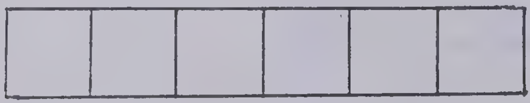
Digi - Place

OBJECT OF THE GAME:

To place digits in the appropriate place value slots to make the largest possible number.

PLAY:

The entire class including the teacher is involved. The play explained here is for a six-digit number but the teacher may wish to vary this from time to time. A set of ten-digit cards, one for each digit, is prepared and placed in a hat, box or any other open container. Each student is then asked to draw six squares as illustrated, each square representing one place value.



The teacher draws one card and holds it up for all students to see. The student may place this digit in any one of his squares, keeping in mind that he is trying to make the largest possible number. Another card is drawn and the student again places this digit in another square. This process continues until six digits have been called.

RULES:

Each digit called must be used. Once a digit has been placed it cannot be changed.

SCORING:

The student with the largest number is the winner. The teacher may wish to introduce a scoring system for the top five numbers such as 10 - 7 - 5 - 3 - 1.

Terms Trial

Players: 2

METHOD:

Shuffle the deck and turn it face down.
First player draws a card and turns it face up in front of him.
Second player does the same.
Each player takes turns drawing cards until he has a combination of two out of three terms for any one operation.

FOR EXAMPLE:

a) Suppose one player has a minuend of 12 and a difference of 3. Then, the missing term is subtrahend of 9. The player may then mark an "X" over the 9 on the game sheet. He should then discard the two cards he has used.

b) Suppose the other player has a divisor of 3 and a dividend of 9. The missing term is quotient 3. The player marks "O" on the 3 on the game sheet.

c) If a player has a subtrahend 15 and a minuend 12 drawn in that order, the player may not reverse positions to be able to use a difference of 3.

The first player to get three X's or three O's in a straight line is the winner.

When all of the cards have been used, the discards may be reshuffled and used again. A player must use any combination which they draw. Cards cannot be saved up and played all at once.

ADDEND 15	ADDEND 3	ADDEND 9
ADDEND 12	SUM 24	SUM 27
SUM 21	SUM 18	SUM 15

SUM 12	MINUEND 12	MINUEND 27
SUBTRAHEND 12	SUBTRAHEND 3	SUBTRAHEND 9
SUBTRAHEND 15	DIFFERENCE 3	DIFFERENCE 9

DIFFERENCE 12	DIFFERENCE 24	DIFFERENCE 18
DIFFERENCE 15	MULTIPLICAND 9	MULTIPLICAND 6
MULTIPLICAND 3	MULTIPLIER 3	MULTIPLIER 6

MULTIPLIER 9	PRODUCT 9	PRODUCT 18
PRODUCT 27	DIVISOR 9	DIVISOR 6
DIVISOR 3	DIVIDEND 9	DIVIDEND 27

DIVIDEND 18

QUOTIENT 9

QUOTIENT 3

QUOTIENT 6

3	6	9
12	15	18
21	24	27

21	6	18
9	3	15
24	12	27

6	15	24
18	24	27
3	9	12

27	3	6
9	12	15
18	21	24

21	24	6
18	9	15
3	27	12

9	18	3
12	27	6
24	15	21

15	18	3
21	6	24
9	27	12

12	21	15
6	18	8
27	24	9

12	9	3
15	21	24
27	18	6

9	15	12
24	18	27
6	21	3

18	15	21
3	27	6
24	9	12

6	24	21
27	3	18
9	12	15

APPLICATIONS

LEVEL 7

UNIT II

WHOLE NUMBERS

BIBLIOGRAPHY FOR APPLICATIONS KIT

Adler, Irving, Readings in Mathematics Book 1, Ginn and Co., Toronto, 1972

Adler, Irving, Readings in Mathematics Book 2, Ginn and Co., Toronto, 1972

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Friebel & Gingrich, Math Applications Kit, SRA, Toronto, 1971

Horne, Sylvia, Patterns and Puzzles in Mathematics, Franklin Publications, Chicago, 1968

Jacobs, Harold R., Mathematics a Human Endeavor, W. H. Freeman and Co., San Francisco, 1970

Johnson, et al, Applications in Mathematics course A Scotts Foresman, Glenview, Illinois, 1972

Johnson, et al, Applications in Mathematics course B Scotts Foresman, Glenview, Illinois, 1974

Lyng, Meconi, Lwick, Career Mathematics: Industry and the Trades, Houghton Mifflin, Boston, 1974

Schor, Meng, Insights and Skills Parts 1, 2 and 3, Globe Book Co., New York 1973

Stein, Practical Applications in Mathematics, Allyn and Bacon Inc., Boston, 1972

Witherding, Margaret F., From Fingers to Computers, Franklin Publications Inc., Chicago, 1970

VIDEO TAPES

ETV Math Series produced in Ontario Tape #1 Part D

Approximating & Estimations
Good for Grade 8 Measurement Applications

Tape #3 Part B

So You Want to Buy a Car
(Application in credit buying Grade 9 level)

Tape #5 Part A

Art from Computers
Useful as maxirational unit for applying Math to Art any grade level.

OBJECTIVES — UNIT II	Reference Section
WHOLE NUMBERS	
Students should be able to:	
1. Compare the set of whole numbers and the set of natural numbers by plotting both sets on a number line.	
2. Demonstrate the need for whole numbers (e.g. closure property) .	A
3. Write any whole number in expanded form.	F
4. Write any whole number in standard form given the expanded form.	
5. Perform the operations of addition and subtraction with whole numbers using the following terminology: addends, sum, difference, subtrahend, minuend.	B,F
6. Perform the operations of multiplication and division with whole numbers using the following terminology: multiplicand, multiplier, product, divisor, dividend, quotient.	C
7. Use symbols and notation illustrated below. (a) Multiplication (i) $3 \times 4 = (3)(4) = 3(4)$ (ii) $3x = (3)(x) = 3(x)$ (b) Division (i) $\frac{9}{3} = 9 \div 3 = 3\overline{)9}$ (ii) $\frac{x}{9} = x \div 9 = 9\overline{)x}$	
8. Demonstrate the use of identity elements and the zero property. (De-emphasize terminology).	
9. Demonstrate that division by zero is undefined.	
10. Evaluate an expression by using the properties to produce short cuts in computation. (De-emphasize terminology). e.g. $4 \times 23 \times 25 = n$ $100 \times 23 = n$	
11. Evaluate expressions involving the order of operations. (Limit: expressions of the type $7 + 5 - 4 - 3 \times 1 =$).	
12. Evaluate expressions by substituting for the variable. (Limit: whole numbers and one variable.)	G

Reference Section

13. Solve conditions for equality using an informal approach for the following types. (Limit: whole numbers).

    (a) $A + x = B$ (c) $Ax + B = C$
    (b) $Ax = B$ (d) $Ax + Bx = C$

    Note: Make sure you check the level of difficulty before having students use an informal approach.

14. Verify solutions of conditions by substitution.

15. Use some formalized approach to solving conditions, (e.g. flow charts, related number sentences).

    (a) $A + x = B$ (c) $Ax + B = C$
    (b) $Ax = B$ (d) $Ax + Bx = C$

16. Write English sentences for mathematical sentences.

17. Write mathematical sentences for English sentences.

18. Solve problems using any organized approach. G

UNIT II - WHOLE NUMBER APPLICATIONS

A.

1. Get a dictionary. Find out how many words are defined in it. (This often is listed on the title page or dust cover).

2. Guess how many of these words begin with 'A', how many with 'B', and so on, through 'Z'.

3. Find out how good your guesses were. Which letters were surprising to you?

4. Make a graph showing how the 26 numbers compare.

5. If you were starting a print shop and were about to order 50,000 letters for type setting, how many of each letter would you buy?

6. Design a typewriter keyboard which you feel would have the best placement of letters. How does your design compare with the standard keyboard?

B. Ten-Pin Bowling

1. Find the dimensions of a bowling alley. How far does the bowling ball have to travel from the foul line to the first pin?

2. The pin placement is arranged in what pattern? Is the ball big enough to hit all the pins? What has to happen so that all the pins get knocked down?

3. Learn how a game is scored. What is a strike? What is a spare? What score do you get for each of these rolls?

4.

Name	1	2	3	4	. . .
John	5/3	8 ◢	9/1	■	
	8				. . .
Mary	■	7/3	7 ◢	1/9	
					. . .

Fill in the scores of the first four frames of a game between John and Mary. Can you fill in John's fourth frame? Who is the better bowler after the firs two frames have been scored?

5. If a bowler's score is 30 in the first frame, what did he bowl in the first three frames?

6. What would a bowler's score be if he got a spare in each frame, first getting nine pins down, then getting one pin down? What would the score be if he fir got one pin down and then the other nine in each frame?

7. Compute the score frame by frame of a perfect game.

8. Make a scale drawing of a bowling alley. Show the position of the pins and the diamond spots on the alley. Show the best paths for a bowling ball to take for both right and left-handed players.

9. Arrange to go to a bowling alley and score your own game(s).

C.

1. A man who was down to his last dollar joined a "Double or Nothing" gambling game. His luck changed and he won 20 times in a row. How much did he win?

Hand Played	Dollars Won
0	1
1	2
2	___
3	___
4	___
5	___
6	___
7	___
8	___
9	___
10	___
11	___
12	___
13	___
14	___
15	___
16	___
17	___
18	___
19	___
20	___

2. The Amoeba is a one-celled animal. Find out how it reproduces.

D. How Many Beans Fill A Big Bottle?

1. Fill a pint jar with dry beans.

Fill a 1/4 cup measure with beans from the jar. Now count the beans. How many beans would you guess are in the measure? Check your guess by counting. Keep a table.

	The Count	The Estimate
First 1/4 Cup		
Second 1/4 Cup		

Fill the 1/4 cup measure for the third time. How many beans do you think are in the measure? Count them. How close is the estimate to the count? How many beans would you expect to find in a full 1/4 cup measure? How many more, or fewer, beans might there be than you expect?

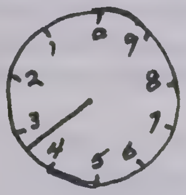
E.

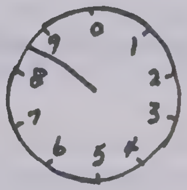
1. Find the number of 1/4 cup measures of beans in the jar filled with beans. Can you estimate how many beans there are in the full jar? How many more beans might there be in the jar? How accurate do you think the estimate is?

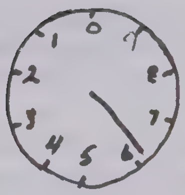
2. Fill a small bottle with rice. Use a tablespoon to estimate the number of grains of rice in the bottle. How accurate do you think your estimate is?

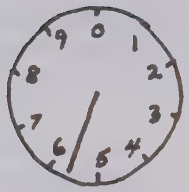
3. Can you estimate the number of tiles on your room floor?

F. Cost of Electricity









3000 + 800 + 60 + 5 = 3865 k

To read an electric meter, start at the left. Write down the last figure that the pointer on the dial has passed. Next to this figure write the last figure that the pointer on the next dial has passed. Do the same for the third dial, and finally for the fourth.

1. What unit of measure does an electric meter use?

2. Find the meter for your home and read it every day for a week at the same time.

3. How much electricity was used between the first and second readings?

4. How much was used each day after that?

5. Estimate how much electricity would be used at your house in a month.

6. Compare your estimate with the actual amount of your electric bill for a particular month.

G.

1. Find the wattage of appliances and light bulbs in your home. How long must each be turned on to use one kilowatt hour?

2. Are there other utilities in your home that are measured?

3. Why do the hands on an electric meter turn in alternate directions?

4. If the reading on the meter in June was 7621 kwh and one month later the reading was 7979 kwh, calculate the cost if the cost for the first 300 kwh is _____ per kwh and for all additional kwh the cost is _____ per kwh.

5. Visit your Electric Company's Local Office. Ask how an electric bill is computed. Does the company charge a fixed rate or does the rate change?

H. Fibonacci Sequence

1. Some ancient mathematicians were very interested in counting objects such as the number of kernels in an ear of corn, the number of seeds in a sun flower, the number of branches on a bush, or the number of rabbits in each generation. As strange as it may seem, number patterns resulted from counting these things. Fibonacci developed the following sequence to describ the results. He started with the number '1' followed by another '1' and then each number is the sum of the previous two, i.e., 1, 1, 2, 3, 5, 8.

(a) What are the next ten numbers in the Fibonacci sequence?

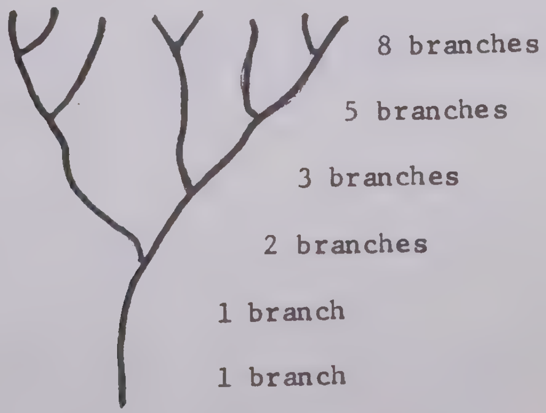
(b) One branch of biology called genetics, is concerned with the reproduction among succeeding generations of plants and animals. To find the number of pairs of rabbits living at the end of each month, if they reproduce two months after birth with one pair of offspring each month, consider the following:

Let **B** represent a rabbit at birth
Let **A** represent a rabbit at adolescence
Let **M** represent a mature rabbit (parent)

Birth	**B B**	1 pair
1 month	**A A**	1 pair
2 months	**MM BB**	2 pairs
3 months	**MM AA BB**	3 pairs
4 months	**MM MM AA BB BB**	5 pairs
5 months		
6 months		

Complete the chart for the fifth and sixth months. How many pairs of rabbits would there be after 10 months? 15 months?

(c)



The number of branches on a bush approximates numbers in the Fibonacci series. Try counting the number of branches on a bush and see if this hold true for your example.

(d) Obtain an ear of corn which has kernels divided into two sections. (Frequently two gaps run lenthwise along an ear of corn dividing the kernels into two sets). Fibonacci predicted the number of kernels in each group would be two consecutive numbers in the Fibonacci sequence. Was he right for your ear of corn? Was he close?

2. Many other examples of applications of this sequence can be found. Read about them in some of the following books.

"Mathematics: Man's Key to Progress" Book B, published by Franklin Publications, Inc. (1968)

"Mathematics: A Human Endeavour", published by W. H. Freeman and Company (1970)

"Story of Mathematics For Young People", published by Random House of Canada Ltd.

"The Giant Golden Book of Mathematics", published by Golden Press, New York

"Mathematics", life Science Library

"Readings in Mathematics", Ginn, pp. 59-61

HISTORY

LEVEL 7

UNIT II

WHOLE NUMBERS

REFERENCES

The Committee recommends the following references as primary sources of information for Junior High School teachers and students. We suggest that those books labelled (T) be available as teacher references and those labelled (S) be available in quantities of 3 - 5 for class use. Many of these books may be in your library now and extra copies may be borrowed from the Library Service Centre.

These references are numbered (1 - 14) for referral in the following outline:

1. Adler, Irving. *The Giant Golden Book of Mathematics*. Golden Press, New York, 1966
   510 Ad 59g (S)

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Black, Gerald J. *Canada Goes Metric*. Doubleday Canada Ltd., Toronto, 1974.

Posters

1. Walch, J.W. (Publisher) "Posters on Famous Mathematics". Available on loan from the Library Service Centre.
2. I.B.M., Timeline "Men of Mathematics", available from I.B.M. Library, Calgary. Ask for Item #5050003. (Free)

Busts

"Mathematicians of the Century", available from Moyer. Available on loan from the Library Service Centre. (Price $48.00)

Movies

CK 10591 "Possibly So Pythagoras". Available on loan from Instructional Aids Department.

CK 538 "Donald Duck in Math Magic Land". Instructional Aids.

Games

1. Euclid. Western Educational Activities. For advanced students.

The resource list on Posters, Busts, Movies and Games was taken from Men of Mathematics - A Resource Unit developed by J. Barnes.

E. T. V. Math Series (Produced in Ontario). Available from Central Office.

Tape #3 part (a) Square Root: Newton's Method. (Time 20 min., 275 ft.)
Useful for introducing square roots in grades 8 or 9.

Tape #5 part (b) History of Computers.
Useful as a motivational unit.

Tape #5 part (f) Number Systems.
Useful for introducing number theory, grade 7.

Tape #6 part (a) History of Numerals
Useful in grade 7 whole numbers.

Tape #6 part (b) History of π.
Grade 9 geometry.

Tape #6 part (c) From Time to Time
Development of calendar.

Tape #6 part (f) History of India(n) Mathematics
Laid the basis for our present number system and useful in History of Math in an option.

Tape #7 part (a) Inverse Variation
Grade 9 functions.

Tape #7 part (b) Graphs
Grade 8 coordinate system (Descarte).

Tape #9 part (a) Fibonacci Sequence
Grade 8 real numbers.

Tape #9 part (b) The Divine Proportion: Golden Section
Grade 9 geometry.

Tape #9 part (c) Map Making
Useful for upper ability students in grade 9 solid geometry.

Tape #10 part (c) What are Numbers
History of development of number systems. Useful as an introduction to grade 7 number systems.

LEVEL 7 WHOLE NUMBERS APPLICATIONS

UNIT II

OBJECTIVES	Reference Section
WHOLE NUMBERS	
Students should be able to:	
1. Compare the set of whole numbers and the set of natural numbers by plotting both sets on a number line.	
2. Demonstrate the need for whole numbers (e.g. closure property) .	A
3. Write any whole number in expanded form.	F
4. Write any whole number in standard form given the expanded form.	
5. Perform the operations of addition and subtraction with whole numbers using the following terminology: addends, sum, difference, subtrahend, minuend.	B,F
6. Perform the operations of multiplication and division with whole numbers using the following terminology: multiplicand, multiplier, product, divisor, dividend, quotient.	C
7. Use symbols and notation illustrated below. (a) Multiplication (i) $3 \times 4 = (3)(4) = 3(4)$ (ii) $3x = (3)(x) = 3(x)$ (b) Division (i) $\frac{9}{3} = 9 \div 3 = 3\overline{)9}$ (ii) $\frac{x}{9} = x \div 9 = 9\overline{)x}$	
8. Demonstrate the use of identity elements and the zero property. (De-emphasize terminology).	
9. Demonstrate that division by zero is undefined.	
10. Evaluate an expression by using the properties to produce short cuts in computation. (De-emphasize terminology). e.g. $4 \times 23 \times 25 = n$ $100 \times 23 = n$	
11. Evaluate expressions involving the order of operations. (Limit: expressions of the type $7 + 5 - 4 - 3 \times 1 =$).	
12. Evaluate expressions by substituting for the variable. (Limit: whole numbers and one variable.)	G

Reference Section

13. Solve conditions for equality using an informal approach for the following types. (Limit: whole numbers).

(a) $A + x = B$ (c) $Ax + B = C$
(b) $Ax = B$ (d) $Ax + Bx = C$

Note: Make sure you check the level of difficulty before having students use an informal approach.

14. Verify solutions of conditions by substitution.

15. Use some formalized approach to solving conditions, (e.g. flow charts, related number sentences).

(a) $A + x = B$ (c) $Ax + B = C$
(b) $Ax = B$ (d) $Ax + Bx = C$

16. Write English sentences for mathematical sentences.

17. Write mathematical sentences for English sentences.

18. Solve problems using any organized approach. G

UNIT II: WHOLE NUMBERS

RESOURCES

Napier (1550 - 1616) - Discovered a system for easy multiplication of whole numbers.

Reference #5, pages 22-23
Reference #12, pages 77-78

Archimedes (287 - 212 B.C.) - Worked with large numbers.

Reference #12, pages 52-55
Reference #11, page 3

The Story of the Calendar

Reference #13, pages 59-61

The Writing of Numbers

English Tally Stick
Chinese
Egyptians
Romans

Reference #13, pages 7-11

Karl Friedrick Gauss (1777 - 1855) - A system for adding arithmetic sequences.

Reference #5, pages 86-87
Reference #11, page 10
Reference #12, pages 96-99
Reference #5, pages 150-153
Reference #4, pages 218-269
Reference #2, pages 93-94

UNIT II: WHOLE NUMBERS

ACTIVITIES

1. Make a set of Napier's bones and learn how to use them.

   (a) Who invented Napier's bones?

   (b) Why were they useful at that time?

   (c) Who was the close associate of Napier, who continued his work?

   References:

   Mathematics (reference #5) page 22
   The Story of Math For Young People (reference #12) pages 77-79

2. In "Story of Math for Young People" pages 50-55, reference #4

   (a) What is the "Sand Reckoner"

   (b) How did Archimedes solve the "Cattle Problem"?

3. (a) Read pages 93 and 94 in Readings in Math: Book One (reference #2) and do the exercise at the end.

   (b) Use Gauss' method to find the sum of 51+52+53+...+135.

   References:

   Readings in Math: Book One (reference #2) pages 93-94.
   or Mathematics Enrichment (reference #11) page 10.

4. "The Story of Mathematics"

   (a) On pages 59, 60, 61 in "The Story of the Calendar (reference #13) describe the:
      a) Egyptian Calendar
      b) Roman Calendar or Julian Calendar
      c) Gregorian Calendar.

   (b) Which of the above calendars forms the basis for our present day calendar, and who developed it?

   (c) Which calendar led to the development of the rhyme "Thirty days has September, April, June and November" etc.?

(d) Why did Europeanians in 1752 feel they might lose 11 days in their lives?

5. (a) Write the numerals from 1 to 50 in at least one of these systems.

Egyptian
Chinese
Roman
Babylonian
Mayan

Refer to:

The Wonderful World of Math (reference #9) pages 10, 11
Mathematics: Man's Key to Progress (reference #6) pages 45, 46
The Story of Mathematics (reference #13) pages 7-10

(b) Complete the investigation on page 46 of Mathematics: Man's Key to Progress (reference #6) page 46.

6. Give an application of Sexagesimal. (reference #5) pages 15-18, Life Science Mathematics.

7. (a) The magic! Magic square in the painting on the following page was invented by Albrech Durer (1471 - 1528). The numbers do not show clearly so it is reproduced below. Find as many ways as you can of getting the magic number (34) in this square.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

(b) Durer also managed to slip in the year he completed the painting in the magic square. What was the year?

Refer to: Mathematics: A Human Endeavor (reference # 15) p. 35

LEARNING
PACKAGE

LEVEL 7

UNIT II

WHOLE NUMBERS

UNIT II - WHOLE NUMBERS

PERFORMANCE OBJECTIVES

Students should be able to:

1. Compare the set of whole numbers and the set of natural numbers by plotting both sets on a number line.

2. Demonstrate the need for whole numbers (e.g. closure property).

3. Write any whole number in expanded form.

4. Write any whole number in standard form given the expanded form.

5. Perform the operations of addition and subtraction with whole numbers using the following terminology: addends, sum, difference, subtrahend, minuend.

6. Perform the operations of multiplication and division with whole numbers using the following terminology: multiplicand, multiplier, product, divisor, dividend, quotient.

7. Use symbols and notation illustrated below.

   (a) Multiplication
   (i) $3 \times 4 = (3)(4) = 3(4)$
   (ii) $3x = (3)(x) = 3(x)$

   (b) Division
   (i) $\frac{9}{3} = 9 \div 3 = 3\overline{)\,9}$
   (ii) $\frac{x}{9} = x \div 9 = 9\overline{)\,x}$

8. Demonstrate the use of identity elements and the zero property. (De-emphasize terminology).

9. Demonstrate that division by zero is undefined.

10. Evaluate an expression by using the properties to produce short cuts in computation. (De-emphasize terminology).

    e.g. $4 \times 23 \times 25 = n$
    $100 \times 23 = n$

11. Evaluate expressions involving the order of operations. (Limit: expressions of the type $7 + 5 - 4 - 3 \times 1 =$).

12. Evaluate expressions by substituting for the variable. (Limit: whole numbers and one variable).

13. Solve conditions for equality using an informal approach for the followin types. (Limit: whole numbers).

    (a) $A + x = B$
    (b) $Ax = B$
    (c) $Ax + B = C$
    (d) $Ax + Bx = C$

    Note: Make sure you check the level of difficulty before having students use an informal approach.

14. Verify solutions of conditions by substitution.

15. Use some formalized approach to solving conditions, (e.g. flow charts, related number sentences).

    (a) $A + x = B$ (c) $Ax + B = C$
    (b) $Ax = B$ (d) $Ax + Bx = C$

16. Write English sentences for mathematical sentences.

17. Write mathematical sentences for English sentences.

18. Solve problems using any organized approach.

DEVELOPMENT AND EXERCISES

STRAND: *Whole Numbers* LEVEL: 7

UNIT: II OBJECTIVE NUMBER: 1 & 2

OBJECTIVE: 1. Compare the set of whole numbers and the set of natural numbers by plotting both sets on a number line.

2. Demonstrate the need for whole numbers. (e.g. closure property)

SUGGESTED DEVELOPMENT:

1) Define the Set of Whole Numbers as:

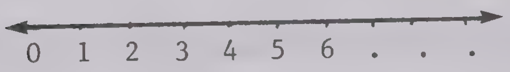
$W = \{0, 1, 2, 3, . . .\}$

2) Define the Set of Natural Numbers as:

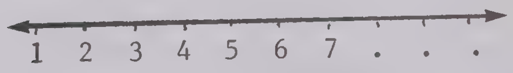
$N = \{1, 2, 3, 4, . . .\}$

3) Draw two number lines. Divide both into equal units.

4) Begin the numbering of one number line with a zero; begin the numbering of the other number line with a one.



Number line for whole numbers



Number line for natural numbers.

5) a. Show that the difference between consecutive numbers in both sets is one.

b. Show that the next greater number and the next lesser number can be determined. Point out the exception for lesser in W and in N. This idea shows order of whole numbers.

c. Discuss betweeness; not always possible to find whole numbers between two other whole numbers; similar with natural numbers.

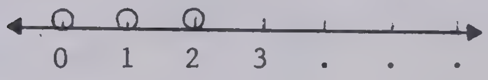
  d. Each natural number and whole number is associated with one point in the line.

6) Discuss the lack of zero in the set of natural numbers. This forms the whole numbers.

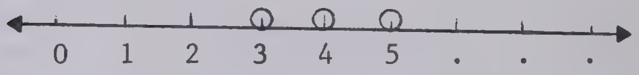
7) Discuss the closure property for (5 - 5), (7 - 7) and the need for zero.

8) Graph sets of whole or natural numbers by Drawing a circle around the corresponding points on the number line. Infinite sets are indicated by drawing a circle around the three dots under the number line.

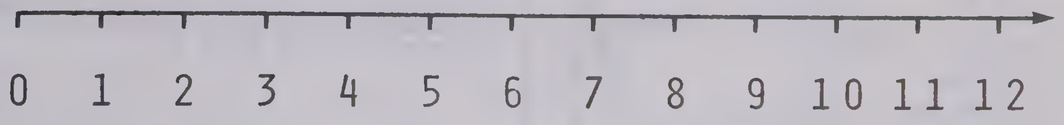
$\{0, 1, 2\}$



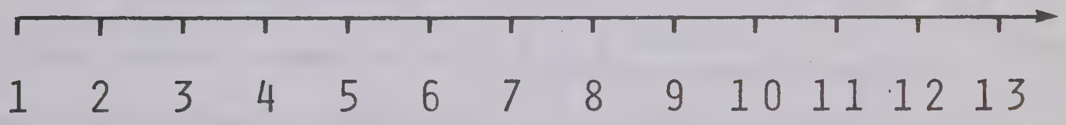
$\{3, 4, 5\}$



WHOLE NUMBERS



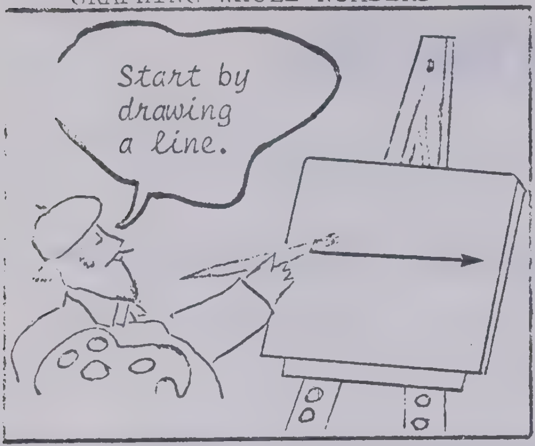
NATURAL NUMBERS

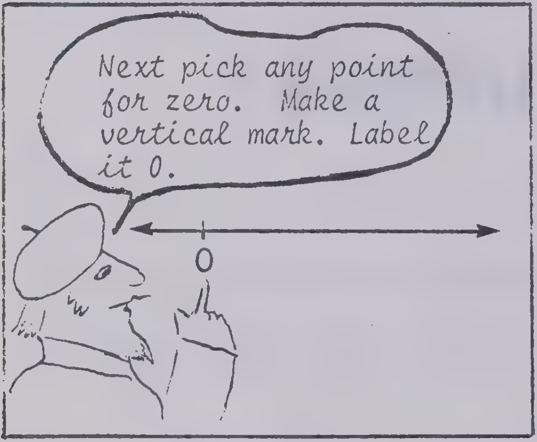


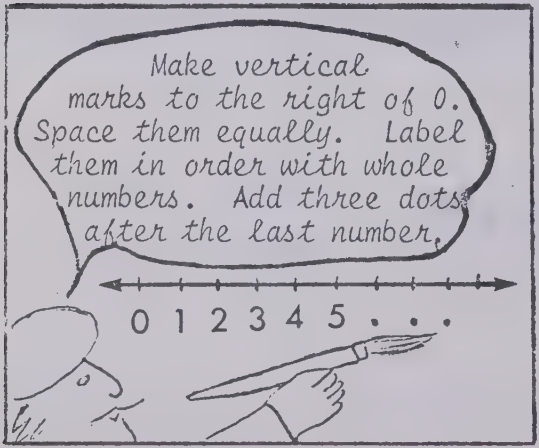
$U = N$ $5 - 5 = ?$ No Answer

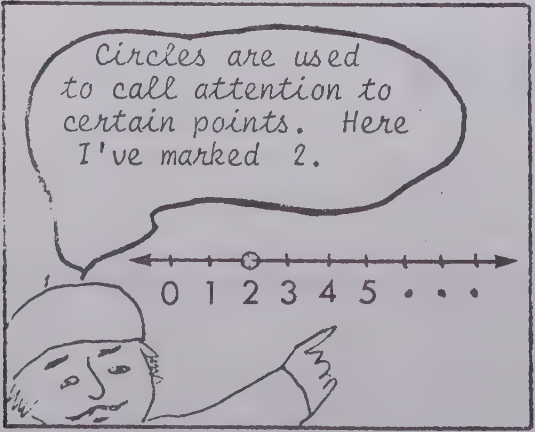
$U = W$ $5 - 5 = 0$

GRAPHING WHOLE NUMBERS


Start by drawing a line.

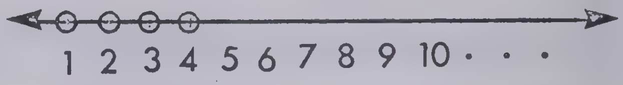

Next pick any point for zero. Make a vertical mark. Label it 0.
0


Make vertical marks to the right of 0. Space them equally. Label them in order with whole numbers. Add three dots after the last number.
0 1 2 3 4 5 . . .


Circles are used to call attention to certain points. Here I've marked 2.
0 1 2 3 4 5 . . .

The Line Up

Graph the solution set on the NATURAL NUMBER LINE of all the numbers less than 5.

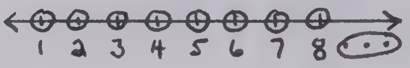


Solution Set: $\{1, 2, 3, 4\}$

Graph the following:

1. The natural numbers.
2. Whole numbers greater than 3.
3. The natural numbers less than 8.
4. The whole numbers greater than 4 and less than 5.
5. Whole numbers.
6. Natural numbers less than 1.
7. Whole numbers less than 1.

1.



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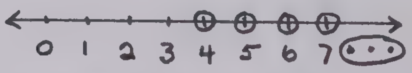

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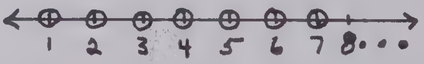

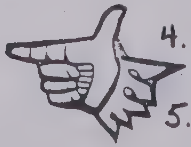

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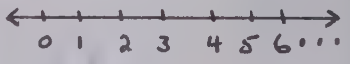

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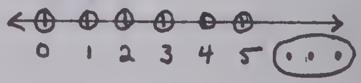

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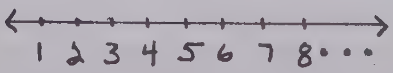

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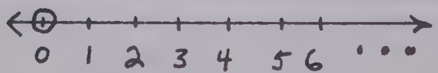


DEVELOPMENT AND EXERCISES

STRAND: *Whole Numbers* LEVEL: *7*

UNIT: *II* OBJECTIVE NO: *3 & 4*

OBJECTIVE: *(3) Write any whole number in expanded form.*

(4) Write any whole number in standard form given the expanded form.

SUGGESTED DEVELOPMENT:

1. (a) Construct a place value chart to illustrate the property of place value.

6	HUNDRED TRILLIONS
5	TEN TRILLIONS
3	TRILLIONS
9	HUNDRED BILLIONS
7	TEN BILLIONS
2	BILLIONS
0	HUNDRED MILLIONS
9	TEN MILLIONS
1	MILLIONS
7	HUNDRED THOUSANDS
6	TEN THOUSANDS
3	THOUSANDS
8	HUNDREDS
5	TENS
4	ONES

(b) The digits in a number are separated by a space. Read the digits in groups. This is a good place for exercises in reading and writing large numbers.

2. (a) From a list of numbers consisting of any number of digits, write the name showing the number of units, tens, hundreds, etc.
(*preferred method*)
8347 = (8 x 1000) + (3 x 100) + (4 x 10) + (7 x 1)

(b) Rewrite the hundreds, thousands and so on to show each by using 10 as a factor as many times as necessary.
8347 = (8 x 10 x 10 x 10) + (3 x 10 x 10)
+ (4 x 10) + (7 x 1)

3. (a) Convert several expanded numbers into standard form.
(5 x 10) + (2 x 1) = 52
(6 x 100) + (3 x 10) + (4 x 1) = 634
(7 x 1000) + (5 x 100) + (0 x 10) + (2 x 1) = 7502
(*preferred method*)

(b) Convert any expanded number factored by tens into standard form.
(7 x 10 x 10) + (3 x 10) + (0 x 1) = 730
(7 x 10 x 10 x 10) + (5 x 10 x 10) + (4 x 10) +
(2 x 1) = 7542

4. *Property of Place Value*

The number represented by each digit is determined by the place the digit occupies.
9234 - What does the 9 mean?
- What does the 2 mean?
- What does the 3 mean?
- What does the 4 mean?

What digit in 9234 represents the largest number? The smallest number?

5. Optional for those students with sufficient background.
(a) Review powers of 10 so that the following can be recognized very quickly.
$10^0 = 1$; $10^1 = 10$; $10^2 = 100$; $10^3 = 1000$

(b) Rewrite in expanded form using exponential notation.
$8347 = (8 \times 10^3) + (3 \times 10^2) + (4 \times 10^1) + (7 \times 10^0)$

(c) Review powers of 10 so that they can be quickly recognized.
10 x 10 is 10^2 or 100

(d) Convert any expanded numeral into standard form by using exponential notation.
$(7 \times 10^3) + (5 \times 10^2) + (4 \times 10^1) + (2 \times 10^0)$
$= 7542$

EXPANDED NOTATION

32 415

= (3 × 10 000) + (2 × 1000)

+ (4 × 100) + (1 × 10)

+ (5 × 1)

STANDARD NOTATION

(4 × 10 000) + (6 × 100)

+ (3 × 10) + (4 × 1)

= 40 634

EXERCISES:

Expanded Form

OBJECTIVE NO. 3 & 4

NOTE: The SI system does not use commas but leaves a space.

Examples:

(a) Write in expanded form:

27 638 = (2 x 10 000) + (7 x 1000) + (6 x 100) + (3 x 10)
+ (8 x 1)

(b) Write in standard form:

(7 x 100 000) + (3 x 10 000) + (0 x 1000) + (6 x 100) +
(0 x 10) + (5 x 1) = 730 605

(c) Write in word form:

1396 = One thousand three hundred ninty-six

1. Write in expanded form:

(a) 39 (3 x 10) + (9 x 1)
(b) 74 (7 x 10) + (4 x 1)
(c) 312 (3 x 100) + (1 x 10) + (2 x 1)
(d) 4000 (4 x 1000) + (0 x 100) + (0 x 10) + (0 x 1)
(e) 357 (3 x 100) + (5 x 10) + (7 x 1)
(f) 6009 (6 x 1000) + (0 x 100) + (0 x 10) + (9 x 1)
(g) 42 734 (4 x 10 000) + (2 x 1000) + (7 x 100) + (3 x 10) + (4 x 1)
(h) 625 416 (6 x 100 000) + (2 x 10 000) + (5 x 100 + (4 x 100) + (1 x 10) + (6 x 1)
(i) 433 (4 x 100) + (3 x 10) + (3 x 1)
(j) 392 (3 x 100) + (9 x 10) + (2 x 1)

2. Write in expanded form:

(a) 5632 (5 x 1000) + (6 x 100) + (3 x 10) + (2 x 1)
(b) 321 (3 x 100) + (2 x 10) + (1 x 1)
(c) 4134 (4 x 1000) + (1 x 100) + (3 x 10) + (4 x 1)
(d) 34 435 (3 x 10 000) + (4 x 1000) + (4 x 100) + (3 x 10) + (5 x 1)
(e) 4387 (4 x 1000) + (3 x 100) + (8 x 10) + (7 x 1)
(f) 4 021 304 (4 x 1 000 000) + (0 x 100 000) + (2 x 10 000) + (1 x 1000) + (3 x 100) + (0 x 10) + (4 x 1)
(g) 400 003 (4 x 100 000) + (0 x 10 000) + (0 x 1000) + (0 x 100) + (0 x 10) + (3 x 1)
(h) 234 007 (2 x 100 000) + (3 x 10 000) + (4 x 1000) + (0 x 100) + (0 x 10) + (7 x 1)
(i) 42 132 (4 x 10 000) + (2 x 1000) + (1 x 100) + (3 x 10) + (2 x 1)
(j) 764 452 160 (7 x 100 000 000) + (6 x 10 000 + (4 x 1000 000) + (4 x 100 000) + (5 x 10 000) + (2 x 1000) + (1 x 100) + (6 x 10) + (0 x 1)

3. Write in word form:

(a) 4395 Four thousand three hundred and ninty five
(b) 2 000 416 Two million one hundred and sixteen
(c) 7600 Seven thousand Six hundred
(d) 5 900 000 Five million nine hundred thousand
(e) 538 738 Five hundred and thirtyeight thousand Seven hundred and thirtyeight
(f) 4657 Four thousand six hundred and fiftyseven

4. In the number 4 526 197 what is the place value of each digit?
4 million 5 hundred thousand 2 ten thousand 6 one thousand 1 hundred 9 tens 7 ones

5. Which digit in 1 375 426 represents the largest number? The smallest number? 1 millions , 6 ones

6. Write in standard form:

(a) (7 x 10 000) + (3 x 1000) + (0 x 100) + (5 x 10) + (6 x 1)
73 056

(b) (8 x 1 000 000) + (5 x 100 000) + (3 x 10 000) + (2 x 1000) + (6 x 100) + (1 x 10) + (9 x 1)
8 532 619

(c) (5 x 10 000) + (3 x 1000) + (9 x 100) + (6 x 10) + (0 x 1)
53 960

(d) (4 x 1) + (3 x 10) + (6 x 100) + (5 x 1000) + (7 x 10 000) + (2 x 100 000) 275 634

(e) (0 x 1) + (0 x 10) + (0 x 100) + (0 x 1000) + (4 x 10 000)
40 000

7. Write in standard form:

(a) (3 x 10 x 10 x 10) + (5 x 10 x 1 0) + (0 x 10) + (3 x 1)
3503

(b) (6 x 10 x 10 x 10) + (2 x 10 x 10) + (8 x 10) + (0 x 1)
6280

(c) (4 x 10 x 10 x 10 x 10 x 10) + (7 x 10 x 10 x 10 x 10) + (8 x 10 x 10 x 10) 478 000

(d) (4 x 1) + (4 x 10) + (6 x 10 x 10) + (2 x 10 x 10 x 10) +(8 x 10 x 10 x 10 x 10) 82 644

(e) (1 x 1) + (0 x 10) + (2 x 10 x 10) + (0 x 10 x 10 x 10) + (4 x 10 x 10 x 10 x 10) + (3 x 10 x 10 x 10 x 10 x 10)
340 201

*8. Give the number in standard form:

(a) 10^4 10 000

(b) 10^5 100 000

(c) 10^0 1

(d) 10^2 100

(e) 10^1 10

(f) 10^9 1 000 000 000

(g) 10^3 1000

(h) 10^7 10 000 000

(i) 10^8 100 000 000

(j) 10^6 1 000 000

*9. Write in exponential form:

(a) $10 \times 10 \times 10$ 10^3

(b) 10×10 10^2

(c) $10 \times 10 \times 10 \times 10 \times 10$ 10^5

(d) $10 \times 10 \times 10 \times 10$ 10^4

(e) $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ 10^7

*10. Write in standard form:

(a) $(8 \times 10^4) + (5 \times 10^3) + (0 \times 10^2) + (4 \times 10^1) + (6 \times 10^0)$

85 046

(b) $(8 \times 10^6) + (6 \times 10^5) + (7 \times 10^4) + (4 \times 10^3) + (6 \times 10^2)$
$+ (1 \times 10^1) + (3 \times 10^0)$ 8 674 613

(c) $(5 \times 10^4) + (4 \times 10^3) + (8 \times 10^2) + (8 \times 10^1) + (0 \times 10^0)$

54 880

(d) $(3 \times 10^0) + (5 \times 10^1) + (6 \times 10^2) + (8 \times 10^3) + (6 \times 10^4)$
$+ (7 \times 10^5)$ 768 653

(e) $(7 \times 10^0) + (3 \times 10^1) + (2 \times 10^2) + (6 \times 10^3) + (8 \times 10^4)$

86 237

DEVELOPMENT AND EXERCISES

STRAND: *Whole numbers* LEVEL: 7

UNIT: II OBJECTIVE NO: 5

OBJECTIVE: *Perform the operations of addition and subtraction with whole numbers using the following terminology: addend, sum, difference, subtrahend, minuend.*

SUGGESTED DEVELOPMENT:

I. *ADDITION*

Certain skills must be mastered if one is to be able to add correctly and efficiently, and it is these skills that must be reviewed and practised. One basic approach would be to present the more troublesome skills using examples. We should also keep in mind the terminology connected with addition and try to make frequent use of these terms during this review.

1. Adding zero:

0	addend
+42	addend
42	sum

$$\underset{\text{addend}}{\underset{\downarrow}{951}} + \underset{\text{addend}}{\underset{\downarrow}{0}} = \underset{\text{sum}}{\underset{\downarrow}{951}}$$

2. Simple regrouping:

①

65	addend
+26	addend
91	sum

$$\underset{\text{addend}}{\underset{\downarrow}{43}} + \underset{\text{addend}}{\underset{\downarrow}{39}} = \underset{\text{sum}}{\underset{\downarrow}{82}}$$

3. Repeat regrouping:

①①

333	addend
+579	addend
912	sum

$$\underset{\text{addend}}{\underset{\downarrow}{457}} + \underset{\text{addend}}{\underset{\downarrow}{275}} = \underset{\text{sum}}{\underset{\downarrow}{732}}$$

NOTE:

Computation does not make use of SI convention. Only use SI for reading purposes.

II. *SUBTRACTION*

The same approach that was used for addition can be used here but care should be taken in describing some of the skills. Once again an introduction of the terminology associated with subtraction would be in order.

1. Subtracting zero:

$$\begin{array}{rl} 49 & \text{minuend} \\ \underline{-0} & \text{subtrahend} \\ 49 & \text{difference} \end{array}$$

$$\begin{tikzcd}[column sep=small] 75 \arrow[d] & - & 0 \arrow[d] & = & 75 \arrow[d] \\ \text{minuend} & & \text{subtrahend} & & \text{difference} \end{tikzcd}$$

2. Simple regrouping:

$$\begin{array}{rl} 6 & \\ 6\cancel{7}2 & \text{minuend} \\ \underline{-257} & \text{subtrahend} \\ 415 & \text{difference} \end{array}$$

3. Regrouping across zero:

$$\begin{array}{r} 69 \\ \cancel{7}\cancel{0}4 \\ \underline{-335} \\ 369 \end{array} \qquad\qquad \begin{array}{r} 799 \\ \cancel{2}\cancel{0}\cancel{0}4 \\ \underline{-2589} \\ 5415 \end{array}$$

4. Repeated regrouping:

$$\begin{array}{r} 815 \\ \cancel{9}\cancel{6}5 \\ \underline{-689} \\ 276 \end{array}$$

5. Unequal length:

$$\begin{array}{r} 6 \\ 128\cancel{7}3 \\ \underline{-49} \\ 12824 \end{array}$$

EXERCISES:

Addition

OBJECTIVE NO. 5

1. 34
25
59

2. 67
32
99

3. 20
58
78

4. 63
26
89

5. 42
27
19
88

6. 25
50
83
158

7. 86
92
58
236

8. 65
7
94
166

9. 34
79
88
201

10. 7
28
72
107

11. 582
143
214
939

12. 861
495
827
2183

13. 779
698
579
2056

14. 926
15
658
1599

15. 639
852
543
2034

16. 34832
11913
22741
10420
79906

17. 56834
67509
31915
47628
203886

18. 42763
96833
47485
92739
279820

19. 6445
95214
87
1756
103502

20. 76542
81276
998
5823
164639

21. 698523
34769
178
9874
968452
789347
59791
2560934

22. 487944
921593
678105
637050
986967
369725
917862
4 999 246

23. 2459630
25400
580575
243125
8549
9783484
15338
13 116 101

24. 4369565
7436859
3050762
4598083
6452136
7587327
3967009
37461741

25. 97
2
421
0
33
553

26. 427
95
522

27. 39
72
111

28. 348
83
592
203
1226

29. 84953
48769
34822
168544

30. 46321
34
502485
2981
551821

31. 30
733
49
4892
5704

32. 3 + 36 + 15 + 294 = 348

33. 45 + 327 = 372

34. 10 + 291 + 0 + 84 = 385

35. x + y = z

x, y and z represent an addition problem.

x and y are called addends

and z is called sum

EXERCISES:

Subtraction

OBJECTIVE NO. 5

1. 85 -61 24	2. 66 -35 31	3. 98 -26 72
4. 49 -15 34	5. 57 -23 34	6. 475 -132 343
7. 978 -642 336	8. 362 -150 212	9. 549 -325 224
10. 853 -721 132	11. 8654 -2431 6223	12. 4296 -3003 1293
13. 6582 -4172 2410	14. 9728 -6215 3513	15. 7849 -3812 4037
16. 38762 -26521 12241	17. 93596 -61593 32003	18. 55706 -52304 3402
19. 45383 -32162 13227	20. 79648 -36136 43512	21. 645987 -314265 331722
22. 705961 -536809 169152	23. 658235 -621791 36444	24. 450000 -328542 121458
25. 532645 - 2867 529778	26. 2963475 -1724133 1239342	27. 7694251 - 86206 7608045
28. 2500000 -1758625 741375	29. 4000000 -2950966 1049034	30. 349 - 24 325
31. 483 - 74 409	32. 98491 - 8993 89498	33. 62461 - 5 62456
34. 903 - 75 828	35. 10031 - 34 9997	36. 27604 - 8875 18729
37. 94 - 0 = 94	38. 4241 - 52 = 4189	39. 342 - 125 = 217

40. x - y = z

x, y and z each represent a certain part of a subtraction problem. We have names for these parts.

x is called minuend
y is called subtrahend
z is called difference

EXERCISES:

OBJECTIVE NO. 5

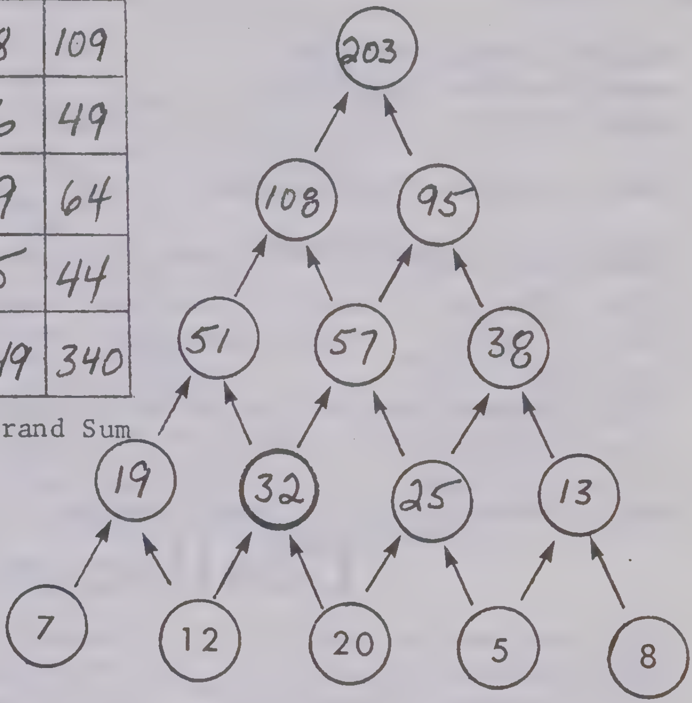
Please do the following pages in your note book! ★ ★ ★ ★

1. Complete the Chart

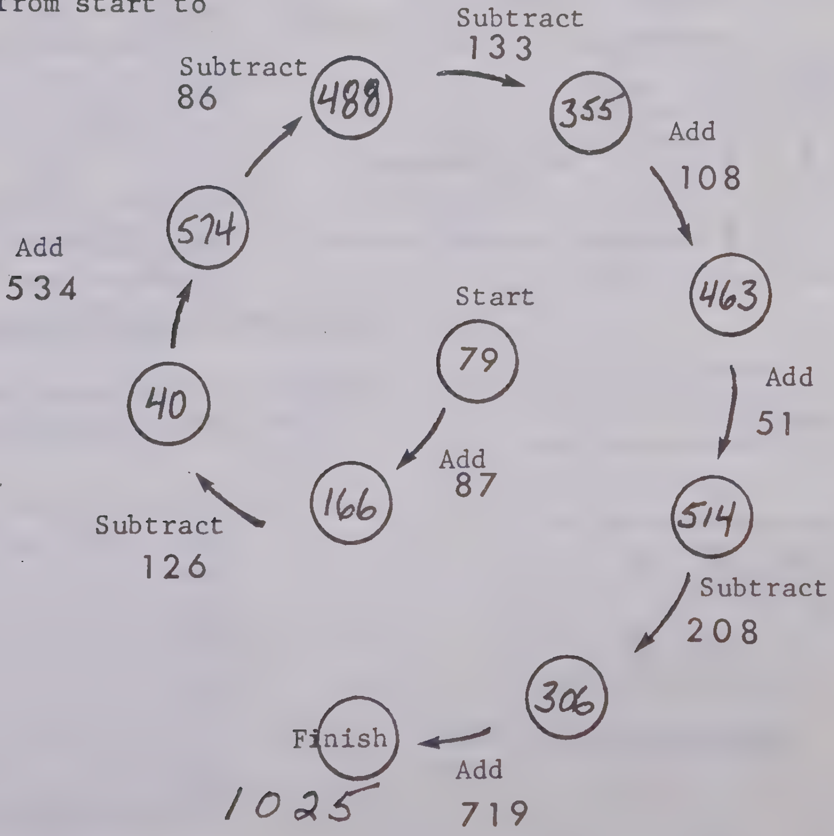
+	2	9	18	10	5	sum
6	8	15	24	16	11	74
13	15	22	31	23	18	109
1	3	10	19	11	6	49
4	6	13	22	14	9	64
0	2	9	18	10	5	44
sum	34	69	114	74	49	340

Grand Sum

2. Add each pair of numbers to get the number above. Work your way to the top.



3. Follow the path from start to finish.



EXERCISES: (CONT'D)

Magic Squares

OBJECTIVE NO. 5

In a *magic square*, the sums of the numbers in each row, each column, and each diagonal are equal. This sum is called the *magic constant*.

			27
3	13	11	27
17	9	1	27
7	5	15	27
27	27	27	27

1. Show by adding that the example at the right is a magic square.

   Magic constant = 27

2. Complete the magic squares.

4	19	16
25	13	1
10	7	22

Magic constant = 39.

4	14	12
18	10	2
8	6	16

Magic constant = 30.

Challenge

3. It's your turn to make a magic square. Use each of the numbers 1 through 9.

8	1	6
3	5	7
4	9	2

Magic constant = 15

4. Complete the magic squares:

(a)

19	3	2	16
6	12	13	9
11	7	8	14
4	18	17	1

CHALLENGE Magic constant = 40.

(b)

22	4	3	19
7	15	16	10
14	8	9	17
5	21	20	2

Magic constant = 48.

5. Fill in the magic square. Use each of the numbers 1 through 16.

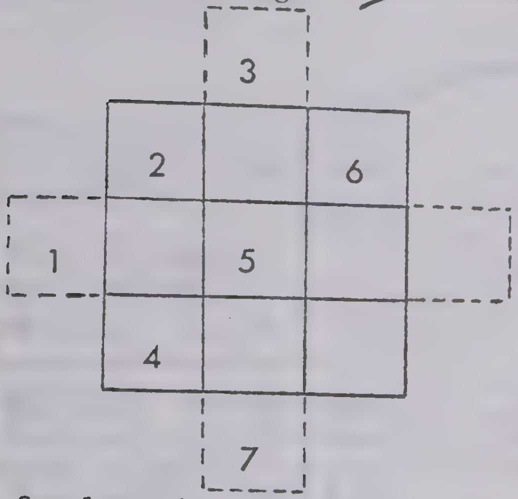
4	15	14	1
9	6	7	12
5	10	11	8
16	3	2	13

Magic constant = 34.

The problems below show a method for making magic squares. Can you figure it out? Use the pattern on the left to make a magic square on the right.

1. a) Complete the pattern. Use the consecutive whole numbers 1 through 9.

b) Complete the magic square.



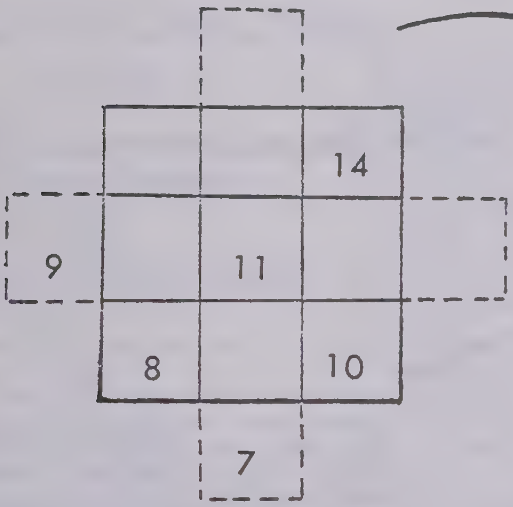
2	7	6
9	5	1
4	3	8

Magic constant = 15

Do you see where the numbers in the broken-line boxes went?

2. a) Complete the pattern. Use the consecutive whole numbers 7 through 15.

b) Complete the magic square.



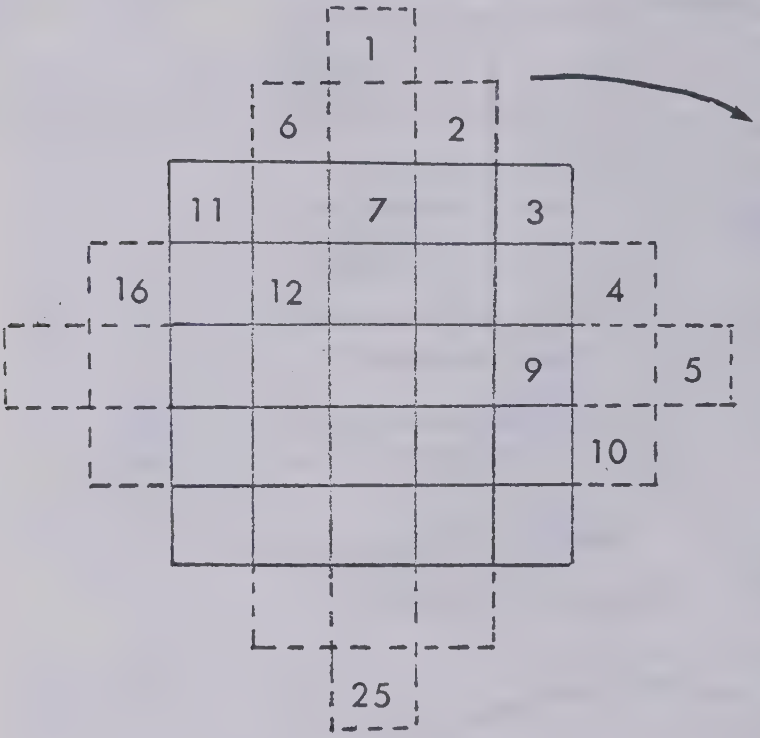
12	7	14
13	11	9
8	15	10

Magic constant = 33

Use the pattern on the left to make a magic square on the right.

1. a) Complete the pattern. Use the consecutive whole numbers 1 through 25.

   b) Now complete the magic square. (Hint: Where did the 4, 10 and 5 go?)

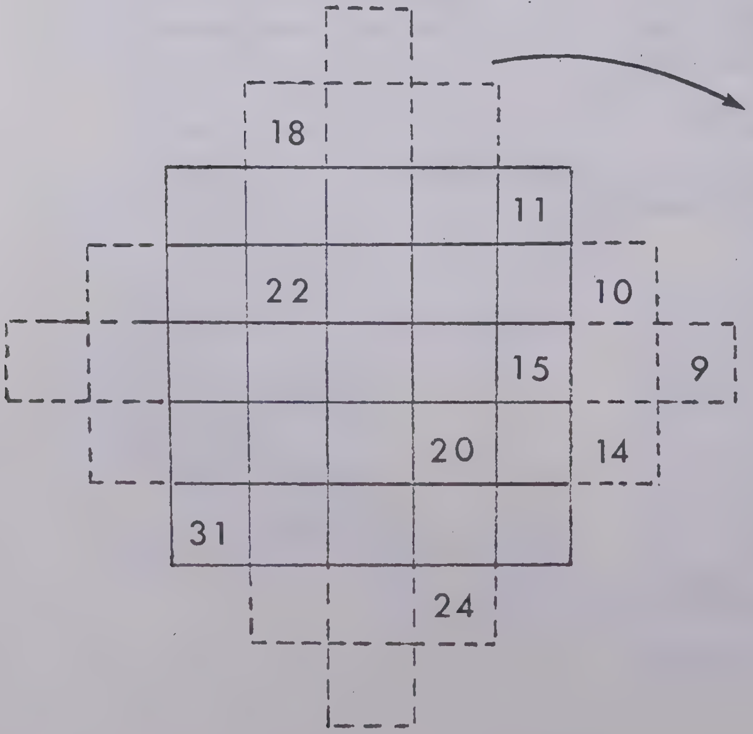


11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

Magic constant = 65

2. a) Use the consecutive whole numbers 9 through 33 to complete the pattern.

   b) Complete the magic square.



23	30	17	24	11
10	22	29	16	28
27	9	21	33	15
14	26	13	20	32
31	18	25	12	19

Magic constant = ________

Palindromes

Palindromes read the same way front or back.

Napoleon was born in 1769.

	1769	
Reverse 1769.	+ 9671	
Add.	11440	Not a palindrome
Reverse 11440.	+ 04411	
Add.	15851	A palindrome

- It took two reversals to get a palindrome.

1. In what year were you born? ________

   How many reversals are needed to get a palindrome? ____________

2. Try this with the year of birth of other members of your family.

   Who has the most reversals? ____________

3. Now try some famous people.

NAME	YEAR BORN	PALINDROME	NUMBER OF REVERSALS
Napoleon	1769	15851	2
Christopher Columbus	1446	7887	1
Sitting Bull	1834	25 652	3
George Washington Carver	1864	22 022	3
Babe Ruth	1895	91 619	4
Helen Keller	1880	7 777	3
Billie Holiday	1915	45 254	3
Dwight Eisenhower	1890	69 696	5
Albert Einstein	1879	233 332	6
Elizabeth I	1533	4 884	1
Abraham Lincoln	1809	79 497	4
Shakespeare	1564	25 652	3

Here comes the troublemaker!

Jesse James	1847	88 555 588	9

4. There are 22 two-digit dates in a 31-day month. Which dates require more than one reversal to give you a palindrome? 19, 28

DEVELOPMENT AND EXERCISES

STRAND: *Whole numbers*　　LEVEL: 7

UNIT: II　　OBJECTIVE NO: 6

OBJECTIVE: *Perform the operations of multiplication and division with whole numbers, using the following terminology: multiplicand, multiplier, product, divisor, dividend, quotient.*

SUGGESTED DEVELOPMENT:

This objective provides an opportunity for review and practise of the fundamentals of these operations. An examination of these basics using examples would be one method of development.

I. *MULTIPLICATION:*

1. One times a number:

```
  45  multiplicand     1
x  1  multiplier    x 45
  45  product         45
```

2. Zero times a number;

```
  92          0
x  0        x92
   0          0
```

3. One digit multiplier:

```
 435        937
 x 3        x 9
1305       8433
```

4. Multiplying by 10 and multiples of 10:

```
 31      49      321
x10     x30     x100
310    1470    32100
```

5. Two digit multiples:

```
  34       7893
 x53        x84
 102      31572
170      631440
1802     663012
```

6. Three digit multiples:

```
   378        529
  x412       x508
   756       4232
  378       26450
1512       268732
155736
```

NOTE:

Computation does not make use of SI convention. Only use SI for reading purposes.

II. *DIVISION*

1. Zero divided by any number (except zero) :

$$\begin{array}{rll} & 0 & \longleftarrow \text{quotient} \\ \text{divisor} \longrightarrow 6 & \overline{)\,0} & \longleftarrow \text{dividend} \end{array}$$

$$\underset{\text{dividend}}{\underset{\downarrow}{0}} \quad \div \quad \underset{\text{divisor}}{\underset{\downarrow}{9}} \quad = \quad \underset{\text{quotient}}{\underset{\downarrow}{0}}$$

2. (a) One digit divisor:

$$\begin{array}{rr} & 4 \\ 9 & \overline{)\,36} \\ & \underline{36} \\ & 0 \end{array} \qquad\qquad \begin{array}{rr} & 55 \\ 6 & \overline{)\,330} \\ & \underline{30\downarrow} \\ & 30 \\ & \underline{30} \\ & 0 \end{array}$$

(b) One digit divisor:

$$\begin{array}{rr} & 70 \\ 7 & \overline{)\,491} \\ & \underline{49\downarrow} \\ & 01 \\ & \underline{00} \\ & 1 \end{array} \qquad\qquad \begin{array}{rr} & 8 \\ 5 & \overline{)\,42} \\ & \underline{40} \\ & 2 \end{array}$$

3. (a) Two digit divisor:

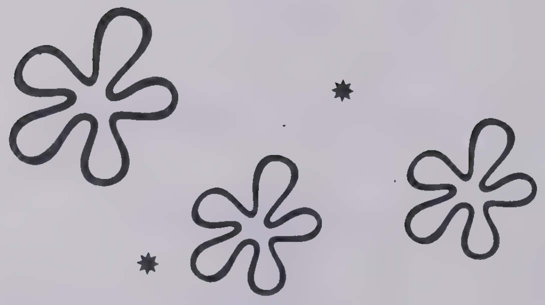
$$\begin{array}{rr} & 201 \\ 45 & \overline{)\,9045} \\ & \underline{90\downarrow} \\ & 04 \\ & \underline{00\downarrow} \\ & 45 \\ & \underline{45} \\ & 0 \end{array} \qquad\qquad \begin{array}{rr} & 362 \\ 29 & \overline{)\,10498} \\ & \underline{87\downarrow} \\ & 179 \\ & \underline{174\downarrow} \\ & 58 \\ & \underline{58} \\ & 0 \end{array}$$

3. (b) Two digit divisor:

$$\begin{array}{r}
68\\
48\;\overline{)3282}\\
288\downarrow\\
402\\
\underline{384}\\
18
\end{array}
\qquad\qquad
\begin{array}{r}
129\\
93\;\overline{)\,12043}\\
\underline{93\downarrow\;}\\
274\\
\underline{186\downarrow}\\
883\\
\underline{837}\\
46
\end{array}$$

4. Three digit divisor:

$$\begin{array}{r}
1533\\
321\;\overline{)\,492369}\\
\underline{321\downarrow\;\;\;}\\
1713\;\;\\
\underline{1605\downarrow\;}\\
1086\;\\
\underline{963\downarrow}\\
1239\\
\underline{963}\\
276
\end{array}$$



EXERCISES: **Multiplication** 21.

OBJECTIVE NO. 6

1. 2304
x 200
460 800

2. 900
x 4
3600

3. 289
x 74
21 386

4. 3492
x 8
27 936

5. 95
x 87
8265

6. 1432
x 207
296 424

7. 2093
x 51
106 743

8. 5831
x 0
0

9. 562
x 1
562

10. 3089 x 47 =
145 183

11. 43
x 72
3096

12. 57
x 38
2166

13. 813
x 74
60 162

14. 1638
x 43
70 434

15. 8536
x 91
776 776

16. 283
x 164
46 412

17. 342
x 193
66 006

18. 8204
x 672
5 513 088

19. 91625
x 405
37 108 125

20. 49872
x 5003
249 509 616

21. 38976
x 1039
40 496 064

22. 2826
x 32405
91 576 530

23. 38391
x 46145
1 771 552 695

24. 14638
x 247
3 615 586

25. a
x b
c
a is called multiplicand
b is called multiplier
c is called product

26. 76
x 36
2736

27. 2876
x 7
20 132

28. 6478
x 24
155 472

29. 8632
x 73
630 136

30. 6389
x 49
313 061

31. 52890
x 58
3 067 620

32. 8249
x 407
3 357 343

33. 1960
x 260
509 600

34. 3769
x 93
350 517

35. 6475
x 25
161 875

36. 47036
x 46
2 163 656

37. 5700
x 809
4 611 300

38. 30087
x 36
1 083 132

39. 51608
x 29
1 496 632

40. Complete the following chart:

×	9	18	27	36	45	54	Sums
9	81	162	243	324	405	486	1701
18	162	324	486	648	810	972	3402
27	243	486	729	972	1215	1458	5103
36	324	648	972	1296	1620	1944	6804
45	405	810	1215	1620	2025	2430	8505
54	486	972	1458	1944	2430	2916	10206
Sums	1701	3402	5103	6804	8505	10206	

Grand Sum
35 721

Add the products in each column and place the sum in the box below the column. Find the grand sum.

EXERCISES: Quaint Quotients OBJECTIVE NO. 6

1. $\begin{array}{r} 812 \\ 8\overline{)6496} \end{array}$

2. $\begin{array}{r} \text{30 r 1} \\ 3\overline{)91} \end{array}$

3. $\begin{array}{r} \text{20 r 1} \\ 45\overline{)901} \end{array}$

4. $\begin{array}{r} \text{1199 r 79} \\ 781\overline{)936498} \end{array}$

5. $\begin{array}{r} \text{2334 r 34} \\ 42\overline{)98062} \end{array}$

6. $\begin{array}{r} 61 \\ 1\overline{)61} \end{array}$

7. $\begin{array}{r} 6 \\ 14\overline{)84} \end{array}$

8. $\begin{array}{r} \text{1189 r 76} \\ 83\overline{)98763} \end{array}$

9. $\begin{array}{r} 0 \\ 41\overline{)0} \end{array}$

10. $\begin{array}{r} \text{504 r 357} \\ 978\overline{)493269} \end{array}$

11. $\begin{array}{r} z \\ x\overline{)y} \end{array}$

x is called the divisor

y is called the dividend

z is called the quotient

12. $\begin{array}{r} 647 \\ 76\overline{)49172} \end{array}$

13. $\begin{array}{r} 901 \\ 22\overline{)19822} \end{array}$

14. $\begin{array}{r} \text{20 r 23} \\ 26\overline{)543} \end{array}$

15. $\begin{array}{r} \text{606 r 1} \\ 73\overline{)44239} \end{array}$

16. $\begin{array}{r} 14002 \\ 6\overline{)84012} \end{array}$

17. $\begin{array}{r} \text{10842 r 7} \\ 8\overline{)86743} \end{array}$

18. $\begin{array}{r} \text{839 r 3} \\ 7\overline{)5876} \end{array}$

19. 60 309 ÷ 9 = 6701

20. 18 639 ÷ 19 = 981

21. 7864 ÷ 32 = 245 r 24

22. 6785 ÷ 83 = 81 r 62

23. 71 825 ÷ 72 = 997 r 41

24. 867 166 ÷ 604 = 1435 r 4

25. 84 864 ÷ 416 = 204

26. 86 526 ÷ 506 = 171

27. 85 617 ÷ 652 = 131 r 205

Choose any one of the problems below. Estimate the product. Then find the exact product. Cross out the product on the *MATHO* card.

You should try to cross out five products in a row or five in a column or five on a diagonal. The first player to do so wins. Do as few problems as possible.

1. 157 x 276 = 43 332
2. 37 x 52 = 1924
3. 216 x 941 = 203 256
4. 113 x 19 = 2147
5. 87 x 26 = 2262
6. 356 x 21 = 7476
7. 134 x 28 = 3752
8. 351 x 474 = 166 374
9. 27 x 91 = 2457
10. 225 x 92 = 20 700
11. 128 x 39 = 4992
12. 425 x 68 = 28 900
13. 64 x 98 = 6272
14. 719 x 532 = 382 508
15. 31 x 206 = 6386
16. 631 x 48 = 30 288
17. 75 x 54 = 4050
18. 83 x 47 = 3901
19. 408 x 701 = 286 008
20. 782 x 52 = 40 664
21. 627 x 811 = 508 497
22. 17 x 32 = 544
23. 832 x 54 = 44928
24. 572 x 44 = 25 168
25. 12 x 72 = 864

MATHO

3752	4050	44 928	1924	286 008
3901	203 256	20 700	2147	544
166 374	7476	43 332	28 900	40 664
2262	25 168	4992	508 497	6272
30 288	2457	382 508	864	6386

The Square Zero

YOU WILL NEED: scissors, a ruler

There are eight polygons below. You can see only four of them right now. The other four are incomplete.

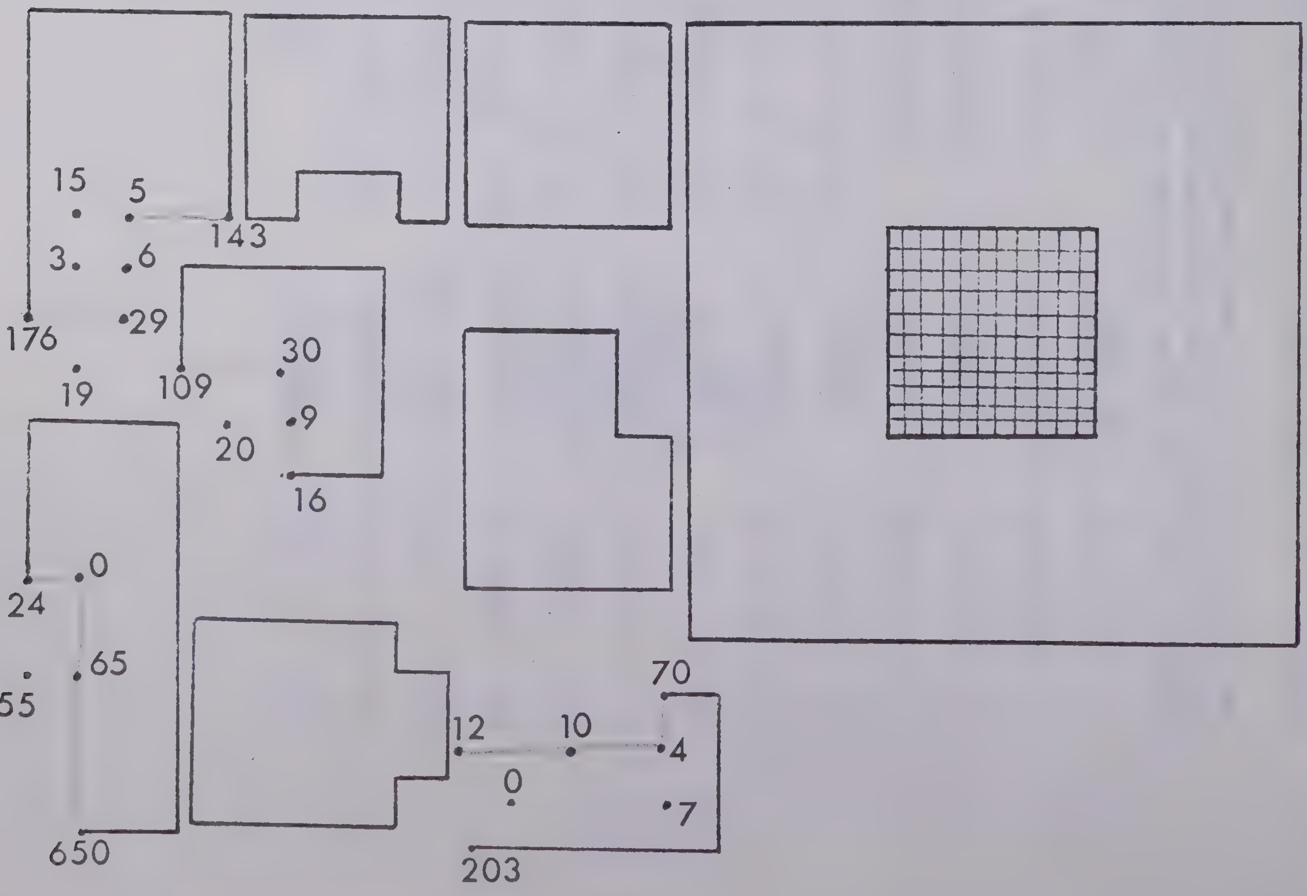
YOUR JOB:

To complete the polygons.
Then fit all eight polygons together to make a square zero.

HERE'S WHAT YOU DO:

First do the division problems below.
The quotients and remainders are shown next to some of the dots below.
Connect each quotient with the remainder.
When all eight polygons are complete, cut them out and fit them together. They should cover the square zero at the right.

1. 209 ÷ 36 = 5 r 29
2. 576 ÷ 24 = 24 r 0
3. 814 ÷ 49 = 16 r 30
4. 784 ÷ 65 = 12 r 4
5. 8871 ÷ 62 = 143 r 5
6. 2524 ÷ 36 = 70 r 4
7. 17 453 ÷ 99 = 176 r 29
8. 7150 ÷ 11 = 650 r 0
9. 6352 ÷ 58 = 109 r 30
10. 15 643 ÷ 77 = 203 r 12

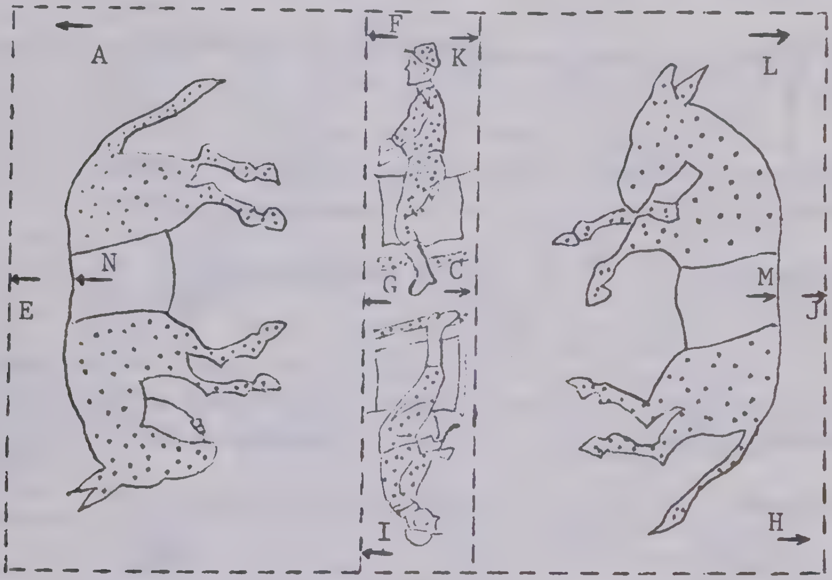


Leaping Donkeys

1. Sam Loyd was a famous American chess player who lived in the nineteenth century. He is the one who invented the game of Parcheesi.

   Sam loved to make up puzzles like the one below. It looks like a simple puzzle. See if you can solve it.

   Cut out the three rectangles along the broken lines. Without doing any more cutting or folding, arrange the three rectangles so that the jockeys are riding the two donkeys.



2. You don't think it can be done? Here's a little help.

   First do all the division problems in column 1.

COLUMN 1	COLUMN 2
A. 476 ÷ 27 = K	I. 475 R18
B. 598 ÷ 32 = O	J. 46 R21
C. 1980 ÷ 86 = N	K. 17 R17
D. 2201 ÷ 12 = P	L. 453
E. 2735 ÷ 59 = J	M. 237 R4
F. 10 419 ÷ 23 = L	N. 23 R2
G. 2137 ÷ 9 = M	O. 18 R22
H. 30 893 ÷ 65 = I	P. 183 R5

The answers to the problems are in column 2. The letters for six of the problems and their answers are on the rectangles. Arrange the rectangles so that the tips of the arrows of matching letters touch.

Challenge: Do You Know Your Terms

1. Given: addend = 535
sum = 912

Find the other addend.

377

2. Given: subtrahend = 906
difference = 409

Find the minuend.

1315

3. Given: multiplicand = 894
product = 63 474

Find the multiplier.

71

4. Given: divisor = 1736
quotient = 28

Find the dividend.

48608

5. Given: product = 40 152
multiplier = 84

Find the multiplicand.

478

6. Given: sum = 751
addend = 245

Find the other addend.

5[illegible]

7. Given: dividend = 2738
quotient = 37

Find the divisor.

74

8. Given: difference = 777
minuend = 838

Find the subtrahend.

61

Review

This is a crossnumber puzzle. Do all the *ACROSS* problems and all the *DOWN* problems. Your *ACROSS* answers and your *DOWN* answers will fit together if they are correct. If they don't agree, look for your mistake.

^{1}1	^{2}3	^{3}9	^{4}1	^{5}4	^{6}5	▨	^{7}1	^{8}7	▨	^{9}8
^{10}7	8	4	7	▨	^{11}4	^{12}6	4	▨	^{13}2	3
^{14}5	7	2	▨	^{15}2	2	3	▨	^{16}4	5	6
^{17}8	5	▨	^{18}1	4	8	▨	^{19}9	4	2	▨
^{20}9	▨	^{21}5	7	3	▨	^{22}2	7	0	▨	^{23}3
▨	^{24}2	6	1	▨	^{25}2	9	4	▨	^{26}6	1
^{27}5	0	0	▨	^{28}2	5	7	▨	^{29}1	8	7
^{30}3	0	▨	^{31}6	8	5	▨	^{32}3	4	9	▨
^{33}7	▨	^{34}3	0	7	▨	^{35}2	9	4	▨	^{36}1
▨	^{37}2	4	0	▨	^{38}5	1	6	▨	^{39}3	7
^{40}6	8	9	▨	^{41}7	4	1	▨	^{42}1	1	4

DOWN

1. 100 006 - 82 417 17589
2. 155 x 25 3875
3. 8478 ÷ 9 942
4. 1006 - 989 17
5. 308 ÷ 77 4
6. 1357 x 4 5428
7. 2 x 7 14
8. 2 + 5 7
9. 40 500 - 39 664 836
12. 10 000 - 9937 63
13. 504 ÷ 2 252
15. 78 + 165 243
16. 440 ÷ 1 440
18. 9 x 19 171
19. 2 x 487 974
21. 1120 ÷ 2 560
22. 653 − 356 297
23. 74 256 - 73 939 317
24. 15 400 ÷ 77 200
25. 765 ÷ 3 255
26. 9160 - 8471 689
27. 540 - 3 537
28. 92 + 195 287
29. 87 696 ÷ 609 144
31. 526 + 74 600
32. 2772 ÷ 7 396
34. 349 x 1 349
35. 2 + 190 + 19 211
36. 174 x 1 174
37. 784 ÷ 28 28
38. 9 x 6 54
39. 124 ÷ 4 31
40. 1 000 000 - 999 994 6
41. 973 ÷ 139 7
42. 1 x 1 1

ACROSS

1. 74 316 + 64 829 139 145
7. 5 + 12 17
9. 3 + 5 8
10. 413 x 19 7847
11. 95 + 369 464
13. 8 + 9 + 6 23
14. 20 020 ÷ 35 572
15. 8474 ÷ 38 223
16. 8 x 57 456
17. 7310 ÷ 86 85
18. 4 x 37 148
19. 53 + 889 942
20. 106 - 97 9
21. 1000 - 427 573
22. 6 x 45 270
23. 3 x 1 3
24. 290 - 29 261
25. 1 + 291 + 2 294
26. 1 + 55 + 5 61
27. 1500 ÷ 3 500
28. 100 000 - 99 743 257
29. 9724 ÷ 52 187
30. 7 + 23 30
31. 5 x 137 685
32. 1139 - 790 349
34. 188 + 119 307
35. 147 x 2 294
36. 1 x 1 1
37. 273 - 33 240
38. 6 + 510 516
39. 2516 ÷ 68 37
40. 900 - 211 689
41. 222 + 519 741
42. 10 + 95 + 9 114

DEVELOPMENT AND EXERCISES

STRAND: Whole numbers

LEVEL: 7

UNIT: II

OBJECTIVE NO: 7

OBJECTIVE: Use symbols and notation illustrated below.

SUGGESTED DEVELOPMENT:

1. Multiplication can be shown in several different ways.

   eg. $5 \times 6 = 5\ (6) = (5 \times 6)$

   $3x = 3\ (x)$

2. Division can be shown in several different ways.

   eg. $14 \div 7 = 7\overline{)14} = \frac{14}{7}$

   $x \div 3 = \frac{x}{3} = x\overline{)3}$

NOTE: SI does not use · to represent the operation of multiplication.

3. Place the following column on the board. Have students indicate different symbols for the operation.

a) $28\ (17) =$ 28 x 17

b) $13 \div 8 =$ ______

c) $12 \times M =$ ______

d) $(5)\ (2n) =$ ______

e) $13\ (11r) =$ ______

f) $7\overline{)8} =$ ______

g) $\frac{4r}{5} =$ ______

h) $\frac{s}{3} =$ ______

i) $4 \times 13\ (y) =$ ______

j) $6y \div 3 =$ ______

k) $\frac{(24 \times 3)}{6} =$ ______

l) $18\overline{)360} =$ ______

m) $\frac{6y}{7} =$ ______

SYMBOLS & NOTATION

MULTIPLICATION

$$3 \times 5 = 3(5) = (3)(5)$$

$$5 \times a = 5(a) = 5a$$

DIVISION

$$14 \div 7 = 7\overline{)14} = \frac{14}{7}$$

$$a \div 3 = \frac{a}{3} = \frac{1}{3}a$$

WHOLE NOS. 7 OBJ. 7

DEVELOPMENT AND EXERCISES

STRAND: Whole numbers LEVEL: 7

UNIT: II OBJECTIVE NO: 8 & 9

OBJECTIVE: (8) Demonstrate the use of identity elements and the zero property. (De-emphasize terminology)

(9) Demonstrate that division by zero is undefined.

SUGGESTED DEVELOPMENT:

(8) 1. Select a one digit number i.e. 7 and show the addition of 7 + (in sequence 5, 4, 3, 2, 1)

i.e.
7 + 5 = 12
7 + 4 = 11
7 + 3 = 10
7 + 2 = 9
7 + 1 = 8
7 + 0 = __

From this descending series of answers it should be clear to the students that 7 + 0 = 7 (the original, or identical number).

2. Similar to #1 show the products 7 x 5, 7 x 4, etc. to 7 x 1 by studying the set of products they should see 7 x 1 = 7.

3. Name 0 - the identity element for addition.
1 - the identity element for multiplication.

4. Indicate to students that
0 x 0 = 0
1 x 0 = 0
2 x 0 = 0
3 x 0 = 0

This effect of zero on multiplication is called the property, i.e. a x 0 = 0

(9)

I. 1. Show with several examples that in other division problems the product of quotient x divisor = dividend

i.e. $\begin{array}{r} 2 \\ 4\overline{)\,8} \end{array}$ s x 4 = 8

2. Assume that division by zero is possible:

then $0\,\overset{n}{\overline{)\,9}}$ n = (some whole number) and n x 0 = 9

but - no true replacement for n can be found

- we know 0 x *any number* = 0

Therefore - Division by zero is not defined

II. Show that for any division:

$a\,\overset{n}{\overline{)\,b}}$ for any b; as a becomes smaller, n becomes larger

eg.

$100\,\overset{1}{\overline{)\,100}}$ $50\,\overset{2}{\overline{)\,100}}$

$25\,\overset{4}{\overline{)\,100}}$ $10\,\overset{10}{\overline{)\,100}}$

$5\,\overset{20}{\overline{)\,100}}$ $1\,\overset{100}{\overline{)\,100}}$

Therefore - as a approaches 0, n approaches infinity, so

$0\,\overline{)\,\text{any number}}$ is undefined

* * * * *

IDENTITY ELEMENTS

ADDITION

WHEN ZERO IS ADDED TO
A NUMBER IT DOES NOT
CHANGE THE IDENTITY (VALUE)
OF THAT NUMBER

$$\left.\begin{aligned} 4 + 0 &= 4 \\ 342 + 0 &= 342 \\ a + 0 &= a \end{aligned}\right\}$$

ZERO IS THE
IDENTITY ELEMENT
FOR ADDITION

MULTIPLICATION

WHEN ONE IS MULTIPLIED
BY A NUMBER IT DOES NOT
CHANGE THE IDENTITY (VALUE)
OF THAT NUMBER

$$\left.\begin{aligned} 9 \times 1 &= 9 \\ 36 \times 1 &= 36 \\ a \times 1 &= a \end{aligned}\right\}$$

ONE IS THE
IDENTITY ELEMENT
FOR MULTIPLICATION

ZERO

MULTIPLICATION BY ZERO GIVES A PRODUCT OF ZERO

$$4 \times 0 = 0$$

$$1\,425\,631 \times 0 = 0$$

$$a \times 0 = 0$$

DIVISION BY ZERO IS UNDEFINED

1. $\begin{array}{r} 9 \\ 4\overline{)36} \end{array}$ SO $4 \times 9 = 36$

2. $\begin{array}{r} n \\ 0\overline{)42} \end{array}$ SO $n \times 0 = 42$

WHAT VALUES

OF "n" MAKE

STATEMENT 2 TRUE?

DEVELOPMENT AND EXERCISES

STRAND: Whole numbers LEVEL: 7

UNIT: II OBJECTIVE NO: 10

OBJECTIVE: Evaluate an expression by using the properties to produce short cuts in the computation. (De-emphasize terminology)

SUGGESTED DEVELOPMENT:

1. When you combine two numbers (x or +), there is only one product or sum.

2. Illustrate commutative property of multiplication with examples.

   eg. 3 x 8 = 8 x 3

   15 x 2 = 2 x 15

   So if it makes it easier to change the order in multiplication, then do so.

3. Show that if we have a sum or product with more than two numbers, it becomes necessary to add or multiply numbers in two's. Group the numbers in such a way as to make computation easier.

   eg. 17 + 15 + 13 = (17 + 13) + 15 = 45

   8 x 5 x 7 x 2 = (8 x 7) x (5 x 2) = 560

4. Use distributive property in evaluating expressions.

   eg. 14 x 6 = 10 x 6 + 4 x 6 = 60 + 24 = 84

   17 x 22 = 17 x 20 + 17 x 2 = 340 + 34 = 374

In Math we say: $a \times 1 = 1 \times a = a$ The product of any whole number and 1 equals the whole number
i.e. $7 \times 1 = 1 \times 7 = 7$

We also say: $a + 0 = 0 + a = a$ The sum of any whole number and 0 equals the whole number
i.e. $5 + 0 = 0 + 5 = 5$

These numbers have a special name

REMEMBER

0 - the *IDENTITY* element for *ADDITION*
1 - the *IDENTITY* element for *MULTIPLICATION*

NOTE:

$0 \times 0 = 0$
$0 \times 1 = 0$
$0 \times 2 = 0$
$0 \times n = 0$

The product of zero and any whole number is zero. This is called the *ZERO PROPERTY*.

1. *ADD*

a) $1 + 0 = ?$
b) $0 + 5 = ?$
c) $14 - 0 = ?$
d) $14 - 0 = ?$
e) $247 + 0 = ?$
f) $0 + 0 = ?$
g) $70 + 0 = ?$
h) $83 - 0 = ?$

2. *MULTIPLY*

a) $3 \times 1 = ?$
b) $17 \times 1 = ?$
c) $1 \times 24 = ?$
d) $238 \times 1 = ?$
e) $1 \times 10\ 004 = ?$
f) $1 \times 56 = ?$

DIVISION BY ZERO

Recall: The *CHECK* for division is *QUOTIENT X DIVISOR = DIVIDEND*

eg. $\begin{array}{r} 3 \\ 4\overline{)12} \end{array}$ Check: $3 \times 4 = 12$

NOW: Assume that 0 goes into 8, n times - i.e. $\begin{array}{r} n \\ 0\overline{)8} \end{array}$

CHECK: $n \times 0 = 8$
$\therefore \quad 0 = 8$

BUT the product of any whole number and zero = 0
$\therefore$ $n \times 0 = 0$ is false
$\therefore$ Division by zero is *UNDEFINED*!

3. *DIVIDE* ALL are UNDEFINED

a) $\frac{14}{0 \times 14} = ?$
b) $8 \div 0 = ?$
c) $\frac{12}{(4 - 4)} = ?$
d) $\frac{5}{2\ (3 - 3)} = ?$
e) $24 \div \frac{(0 \times 16)}{8} = ?$
f) $\frac{27}{0} = ?$

EXERCISES:

Properties

OBJECTIVE NO. 10

COMMUTATIVE PROPERTY

a + b = b + a
3 + 4 = 4 + 3
a x b = b x a
3 x 4 = 4 x 3

The *commutative property* allows us to change the order of the numbers when we are ADDING OR MULTIPLYING.

ASSOCIATIVE PROPERTY

(a + b) + c = a + (b + c)
(5 + 4) + 3 = 5 + (4 + 3)
(a x b) x c = a x (b x c)
(5 x 4) x 3 = 5 x (4 x 3)

The *associative property* allows us to regroup the numbers differently when ADDING OR MULTIPLYING.

DISTRIBUTIVE PROPERTY

a (b + c) = ab + ac
5 (2 + 3) = 5 x 2 + 5 x 3
a (b - c) = ab - ac

The *distributive property* tells us to multiply each number in the brackets by the number outside.

---

1. Use the properties to evaluate the following mentally:
Change the order and regroup if necessary.
eg. 4 + 12 + 6 = 10 + 12
= 22

a) 7 + 9 + 3 = ? 19
b) 4 + 13 + 16 = ? 33
c) 15 + 23 + 25 = ? 63
d) 140 + 113 + 260 = ? 513
e) 12 + 25 + 75 = ? 112
f) 2 x 3 x 4 x 5 x 7 = ? 840
g) 3 + 6 + 7 + 4 = ? 20
h) 4 x 21 x 25 = ? 2 100
i) 35 + 12 + 5 + 8 =? 60
j) 50 x 12 x 2 = ? 1 200

2. Using the properties, solve the following:
eg. 6 + 8 + 7 + 9 = 6 + 9 + 8 + 7
= 15 + 15
= 30

5 x 8 x 2 = 2 x 5 x 8
= 10 x 8
= 80

a) 2 x 9 x 5 = ? 90
b) 7 x 8 x 10 = ? 560
c) 7 + 6 + 3 + 4 = ? 20
d) 13 + 15 + 5 + 7 = ? 40
e) 24 + 31 + 16 + 9 = ? 80
f) 42 + 14 + 6 + 18 = ? 80
g) 14 + 15 + 15 + 16 = ? 60
h) 13 x 10 x 2 = ? 260

3. Using the distributive property, solve the following problems for n, p, y, z. Study the following examples:

eg.

$$\begin{array}{c}\underbrace{35 \text{ x } 7}\\ \underbrace{30\ (7)} + \underbrace{5\ (7)}\\ 210 + 35\\ \searrow\ \swarrow\\ 245\end{array}$$

$$\begin{array}{c}\underbrace{48 \text{ x } 5}\\ \underbrace{n\ (5)} + \underbrace{8\ (5)}\\ p + y\\ \searrow\ \swarrow\\ z\end{array}$$

$$\begin{array}{c}\underbrace{48 \text{ x } 5}\\ \underbrace{40\ (5)} + \underbrace{8\ (5)}\\ 200 + 40\\ \searrow\ \swarrow\\ 240\end{array}$$

n = 40, p = 200, y = 40, z = 240

a) 15×5
$\underbrace{5\ (5)} + \underbrace{n\ (5)}$
$p + y$
z
n = 10
p = 25
y = 50
z = 75

b) 17×8
$\underbrace{10\ (8)} + \underbrace{n\ (8)}$
$p + y$
z
n = 7
p = 80
y = 56
z = 136

c) 26×3
$\underbrace{n\ (3)} + \underbrace{20\ (3)}$
$p + y$
z
n = 6
p = 18
y = 60
z = 78

d) 9×25
$\underbrace{4\ (25)} + \underbrace{n\ (25)}$
$p + y$
z
n = 5
p = 100
y = 125
z = 225

4. Evaluate the following expressions. Use the distributive property to assist you.
eg. 7 x 13 = 7 (10 + 3)
= (7 x 10) + (7 x 3)
= 70 + 21
= 91

a) 43 x 9 = ? 387
b) 59 x 8 = ? 472
c) 69 x 5 = ? 345
d) 48 x 4 = ? 192
e) 56 x 12 = ? 672
f) 42 x 13 = ? 546
g) 66 x 22 = ? 1 452
h) 402 x 13 = ? 5 226
i) 526 x 14 = ? 7 364

5. Without calculating, state whether the following are true or false.

a) (17 x 395) + (14 x 395) = (17 + 14) x 395 T
b) (14 x 963) + (6 x 963) = 20 x 963 T
c) (5 x 7) + (3 x 3) = 8 x 10 F
d) (11 x 53) + (1 x 53) = 12 x 53 T
e) (17 x 97) + 97 = 18 x 97 T
f) (27 x 351) + 351 = 28 x 351 T
g) (16 x 5) + (4 x 5) = 20 x 10 F
h) (14 x 7) + 14 = 28 x 7 F

6. Using the properties you have learned, calculate the following as quickley as possible.

a) 19 + 8 + 12 + 1 = ? 40
b) 5 x 7 x 9 x 2 = ? 630
c) 4 x 7 x 25 = ? 700
d) 5 x 8 x 2 x 2 x 5 = ? 800
e) 13 + 8 + 6 + 2 + 4 = ? 33
f) 16 x 4 x 5 = ? 320
g) 9 + 6 + 2 + 4 + 8 = ? 29
h) 58 + 41 + 19 + 12 = ? 130
i) 5 x 7 x 4 x 9 = ? 1 260
j) 5 + 23 + 17 + 41 = ? 86
k) 5 x 4 x 7 x 2 x 5 = ? 1400
l) 14 + 23 + 16 + 7 = ? 60
m) 251 + 64 + 9 + 36 = ? 360
n) 9 x 2 x 3 x 5 = ? 270
o) 6 x 3 x 5 x 4 = ? 360
p) 4 x 5 x 89 x 5 = ? 890
q) 21 + 6 + 9 + 11 = ? 47
r) 12 + 8 + 4 + 2 + 3 = ? 29
s) 9 x 2 x 5 x 4 = ? 360
t) 54 + 9 + 6 + 21 = ? 90
u) 6 x 2 x 3 x 5 x 2 = ? 360
v) 25 x 5 x 8 = ? 1 000
w) 51 + 64 + 16 + 9 + 35 = ? 175

IF YOU TOOK: 2 minutes- genius
3 minutes- wizard
4 minutes- super
5 minutes- bright
6 minutes- more practice.

COMPUTATIONAL SHORTCUTS

ADDITION

$4 + 3 + 6 + 7$

ASSOCIATIVE

$= (4 + 6) + (3 + 7) \leftarrow$ AND COMMUTATIVE

PROPERTIES

$= 10 + 10$

$= 20$

MULTIPLICATION

$8 \times 5 \times 7 \times 2$

ASSOCIATIVE

$= (8 \times 7) \times (5 \times 2) \leftarrow$ AND COMMUTATIVE

PROPERTIES

$= 56 \times 10$

$= 560$

DEVELOPMENT AND EXERCISES

STRAND: Whole numbers
LEVEL: 7
UNIT: II
OBJECTIVE NO: 11
OBJECTIVE: Evaluate expressions involving the order of operations
(Limit: expression of the type 7 + 5 - 4 - 3 x 1 =)

SUGGESTED DEVELOPMENT:

1. Illustrate the confusion that occurs when evaluating an expression such as 7 + 5 x 2.

2. Explain carefully agreements regarding use of parentheses.

3. Rules for order of operations:

   a) First, perform operations within parentheses.
   b) Second, perform all multiplications and divisions in left-to-right order.
   c) Third, perform all additions and subtractions in left-to-right order.

   Examples:

   i) $4 \underset{\text{2nd}}{+} 6 \underset{\text{1st}}{\times} 5$

   ii) $3 \underset{\text{1st}}{\times} 2 \underset{\text{2nd}}{+} 8$

   iii) $9 \underset{\text{1st}}{\div} 3 \underset{\text{2nd}}{\times} 7$

   iv) $12 \underset{\text{3rd}}{-} 9 \underset{\text{4th}}{+} 8 \underset{\text{1st}}{\div} 4 \underset{\text{2nd}}{\times} 3$

ORDER OF OPERATIONS

1. PERFORM OPERATIONS WITHIN PARENTHESES
2. PERFORM MULTIPLICATIONS AND DIVISIONS
3. PERFORM ADDITIONS AND SUBTRACTIONS

$$
\begin{aligned}
& (12 - 9) + 8 \div 4 \times 3 \\
= \; & 3 + 8 \div 4 \times 3 \\
= \; & 3 + 2 \times 3 \\
= \; & 3 + 6 \\
= \; & 9
\end{aligned}
$$

EXERCISES:

Evaluation

OBJECTIVE NO. 11

When more than one operation occurs in an expression follow the following rule:

B - Brackets

O - Of (means multiply when used with fractions)

D } Division } in the order that they
M } Multiplication } appear from left to right

A } Addition } in the order that they
S } Subtraction } appear from left to right

eg. 1.
4 + 5 x 6
= 4 + 30
= 34

eg. 2.
132 - (22 + 18)
= 132 - 40
= 92

eg. 3.
(24 + 6) ÷ 5 + 8 x 3
= 30 ÷ 5 + 8 x 3
= 6 + 8 x 3
= 6 + 24
= 30

I. Evaluate each of the following number sentences by following the rules for order of operations. Complete the sentence below using the number letter key provided.

1) 4 - 2 x 2 = ? 0
2) 6 + 3 ÷ 3 = ? 7
3) 7 x 6 - 3 = ? 39
4) 8 ÷ 4 - 1 = ? 1
5) 20 - 5 x 2 = ? 10
6) 18 - 8 ÷ 2 = ? 14
7) 3 + 5 x 4 = ? 23
8) 9 - 9 ÷ 3 = ? 6
9) 14 + 4 ÷ 2 = ? 16
10) 9 - 8 ÷ 4 = ? 7
11) 5 x 4 + 19 = ? 39

WHAT DID ONE MATH STUDENT SAY TO ANOTHER?

M	A	T	H	I	S	G	R	E	A	T
1	2	3	4	5	6	7	8	9	10	11

Number-Letter Key

A = 7
E = 16
G = 23
H = 1
I = 10
M = 0
R = 6
S = 14
T = 39

II. Evaluate each of the number sentences below, following the rules for order of operation. Use the number-letter key to complete the sentence on the following page.

1) 7 + 9 - 4 x 2 = ? 8
2) 18 ÷ 3 + 7 - 8 = ? 5
3) 6 x 4 ÷ 8 + 9 = ? 12
4) 24 - 3 x 4 ÷ 12 = ? 23
5) 27 ÷ 9 + 3 x 7 = ? 24
6) 13 + 9 - 24 ÷ 3 = ? 14
7) 7 x 6 + 27 ÷ 3 = ? 51
8) 3 + 8 x 6 ÷ 12 = ? 7
9) 25 + 15 ÷ 5 x 6 = ? 43
10) 12 ÷ 4 x 2 - 6 = ? 0
11) 17 - 9 + 7 x 4 = ? 36
12) 4 x 6 - 21 ÷ 3 = ? 17
13) 5 + 4 x 2 = ? 13

Number-Letter Key

A = 7
B = 23
E = 24 or 0
F = 36
M = 12
N = 8 or 13
R = 14 or 43
S = 51
U = 5 or 17

WHAT DID ONE BOOKKEEPER SAY TO ANOTHER BOOKKEEPER?

N	U	m	B	E	R	S	A	R	E	F	U	N
1	2	3	4	5	6	7	8	9	10	11	12	13

III. Evaluate each of the following number sentences below. Discover the sentences below by using the number letter key provided.

1) 7 + 11 - 9 x 6 ÷ 3 = ? 0
2) 14 - 6 x 3 ÷ 9 + 5 = ? 17
3) 7 x 4 ÷ 2 + 7 - 9 = ? 12
4) 36 ÷ 4 + 6 - 3 x 4 = ? 3
5) 5 x 7 + 21 ÷ 7 - 9 = ? 29
6) 5 + 4 x 8 - 18 ÷ 3 = ? 31
7) 40 - 9 ÷ 3 x 7 + 5 = ? 24
8) 48 ÷ 16 x 7 + 13 - 18 = ? 16
9) 18 + 14 ÷ 2 - 3 x 4 = ? 13
10) 23 - 51 ÷ 3 + 7 x 8 = ? 62
11) 6 x 9 - 63 ÷ 9 + 5 = ? 52
12) 72 ÷ 9 - 8 + 7 x 7 = ? 49
13) 4 + 8 ÷ 2 x 11 = ? 48
14) 9 + 18 ÷ 9 - 3 = ? 8
15) 12 - 4 ÷ 2 + 3 = ? 11
16) 10 - 8 x 3 ÷ 1 = ? 2
17) 13 + 2 x 10 + 5 = ? 38

WHAT DID ONE DOCTOR SAY TO ANOTHER DOCTOR?

O	P	E	R	A	T	I	O	N	S	A	R	E	E	A	S	Y
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

Number-Letter Key

A = 29, 52, or 11
E = 12, 48, or 8
I = 24
N = 13

O = 16 or 0
P = 17
R = 3 or 49
S = 62 or 2

T = 31
Y = 38

IV. Evaluate each of the number sentences below, following rules pertaining to grouping symbols and order of operations. Discover the sentence that follows using the number key provided.

1) 15 ÷ (12 ÷ 2 - 1) = ? 3
2) (8 + 4) ÷ (24 ÷ 8) = ? 4
3) (16 - 8) x 4 - 20 = ? 12
4) (5 + 9) ÷ 7 + 5 = ? 7
5) (2 + 10) ÷ (6 - 4) = ? 6
6) [21 ÷ (10 - 3)] ÷ 3 = ? 1
7) (7 x 4 + 4) ÷ 4 = ? 8
8) [(19 - 4) x 2] ÷ 6 = ? 5
9) (24 ÷ 4 + 6) + 3 = ? 15
10) [(27 - 6) ÷ 3] x 7 = ? 49
11) [(12 + 3) ÷ 5] x 9 = ? 27
12) [(67 - 4) ÷ 9] x 6 = ? 42

Number-Letter Key

D = 8
I = 7, 5, 49
S = 42

E = 4, 1
K = 6
T = 27

G = 15
L = 12
W = 3

WHAT DID THE FINGERS SAY TO THE TOES?

W	E	L	I	K	E	D	I	G	I	T	S
1	2	3	4	5	6	7	8	9	10	11	12

Review

REVIEW EXERCISES: OBJECTIVE NO'S 1-11

1. Write the following whole numbers in expanded form:

a) 231 (2x100)+(3x10)+(1x1)
b) 4 295 (4x1000)+(2x100)+(9x10)+(5x1)
c) 9 001 (9x1000)+(0x100)+(0x10)+(1x1)
d) 1 040 (1x1000)+(0x100)+(4x10)+(0x1)
e) 99 840 049 (9x10 000 000)+(9x1 000 000)+(8x100 000)+(4x10 000)+(0x1000)+(0x100)+(4x10)+(9x1)
f) 80 000 (8x10000)+(0x1000)+(0x100)+(0x10)+(0x1)
g) 421 (4x100)+(2x10)+(1x1)
h) 98 (9x10)+(8x1)
i) 3 421 (3x1000)+(4x100)+(2x10)+(1x1)
j) 90 042 (9x10 000)+(0x1000)+(0x100)*(4x10)+(2x1)

2. Write the following in standard form:

a) (9 x 10 x 10 x 10 x 10)+(8 x 10 x 10 x 10)+(0 x 10 x 10)+(1 x 10)+(0 x 1) 98 010
b) (1 x 1 000 000)+(0 x 100 000)+(0 x 10 000)+(2 x 1 000)+(1 x 100)+(0 x 10)+(1 x 1) 1 002 101
c) (1 x 10)+(2 x 1) 12
d) (4 x 1) 4
e) (5 x 10 x 10 x 10)+(0 x 10)+(2 x 1) 5 002
f) (9 x 10)+(8 x 1) 98
g) (3 x 10 000 000)+(2 x 1 000 000)+(9 x 100 000)+(0 x 10 000)+(0 x 1 000)+(2 x 100)+(5 x 10)+(5 x 1) 32 900 255
h) (4 x 10 x 10)+(5 x 10)+(4 x 1) 454
i) (3 x 10 x 10 x 10)+(0 x 10 x 10)+(1 x 10)+(6 x 1) 3016
j) (4 x 10 x 10 x 10 x 10 x 10 x 10)+(3 x 10 x 10 x 10 x 10 x 10)+(9 x 10 x 10 x 10 x 10)+(7 x 10 x 10 x 10)+(0 x 10 x 10)+(1 x 10)+(2 x 1) 4 397 012

3. Perform the following subtractions:

a) 984
-29
955

b) 243
-81
162

c) 49
-0
49

d) 489
-21
468

e) 4001
-984
3017

f) 99812
-9994
89 818

g) 34
-31
3

h) 49001
-9885
39 116

i) 56241
-8492
47749

j) 981
-92
889

4. Add the following:

a) 432
92
1284
1808

b) 891
0
891

c) 9813
946
361
98192
109 312

d) 45
29
131
1349
1554

e) 99
881
36
984
2000

f) 1498431
936494
2 434925

g) 9832
761
10593

h) 45 + 91 + 321 + 983 + 18 = ? 1158
i) 94 + 89 + 453 + 9812 + 2 = ? 10 450
j) 45 + 911 + 3391 + 482 = ? 4 829

5. Multiply the following whole numbers:

a) 983 x13 = 12779
b) 43 x2 = 86
c) 19 x21 = 399
d) 92 x421 = 38 732
e) 4012 x981 = 3 935 772
f) 81 x399 = 32 319
g) 4841 x119 = 576 079
h) 42 x894 = 37 548

6. Divide the following whole numbers:

a) 810 ÷ 9 = 90
b) 408 ÷ 8 = 51
c) 79)67229 851
d) 43)31261 727
e) 981)30411 31
f) 88)41086 466 r 80
g) 912)4396752 4821
h) 93)372186 4002

7. Perform the following operations using short cuts to make computation easier:

a) 4 x 9 x 25 900
b) 49 + 13 + 1 + 7 70
c) 20 x 19 x 5 1900
d) 5 x 14 x 2 140
e) 17 + 21 + 9 + 13 60
f) 15 x 17 x 2 510
g) 5 + 19 + 15 + 21 60
h) 49 + 57 + 13 + 11 130
i) 25 x 94 x 4 x 10 94 000
j) 491 + 18 + 9 + 2 520

8. Label the parts of the following operations:

49
+3
52

a) addends
b) sum

982
-41
941

c) minvend
d) subtrahend
e) difference

```
     163
  3 )491
     3
     19
     18
      11
       9
       2
```

f) divison
g) quotient
h) dividend
i) remainder

45
x2
90

j) multiplicand
k) multiplier
l) product

9. Evaluate the following pairs of expressions, then put the correct symbol between them (<, > , =)

a) 45 ÷ 3 + 6 > 45 ÷ (3 + 6)
b) 15 - 4 x 3 + 2 = 15 - (4 x 3) + 2
c) 5 + 6 - 3 ÷ 3 > 60 ÷ 6 - 5
d) 18 ÷ 3 - 2 x 2 > 21 ÷ 7 - 4 ÷ 2

Review

^{1}1	2	^{2}8	▨	^{3}1	9	^{4}9	9	^{5}2	0	▨
▨	▨	^{6}5	5	6	▨	^{7}9	▨	8	▨	▨
▨	▨	^{8}8	▨	^{9}3	9	1	1	6	▨	^{10}9
^{11}1	6	9	▨	▨	▨	▨	▨	8	▨	3
7	▨	▨	^{12}2	7	^{13}0	0	^{14}4	▨	^{15}5	6
^{16}1	2	^{17}1	3	▨	▨	▨	3	▨	6	▨
2	▨	^{18}4	1	^{19}7	▨	▨	^{20}1	^{21}1	▨	^{22}2
▨	▨	^{23}4	6	4	4	▨	^{24}5	3	5	5

ACROSS		DOWN	
1.	16 x 8	2.	60 123 ÷ 7
3.	714 x 280	3.	489 ÷ 3
6.	140 + 131 + 56 + 137 + 92	4.	855 + 136
7.	2 700 ÷ 300	5.	478 x 6
8.	36 ÷ (3 x 3) + 4	10.	(45 + 5 - 11) x (9 x 3 - 3)
9.	889 x 44	11.	214 x 8
11.	8 x 9 - 15 ÷ 5 + 100	12.	873 + 528 + 915
12.	628 x 43	13.	39 ÷ (10 + 3) - 3
15.	10 + 3 x 14 - 6 + 10	14.	863 x 5
16.	945 + 268	15.	6 + 43 + 7
18.	3 + 20 x 20 + 14	17.	12 x 12
20.	23 + 15 - 9 x 3	19.	49 + 8 ÷ 8 + 49 - 25
23.	516 x 9	21.	91 ÷ 7
24.	765 x 7	22.	28 ÷ 4 + 3 x 8 - 6

STRAND: *Whole numbers* LEVEL: *7*

UNIT: *II* OBJECTIVE NO: *12*

OBJECTIVE: *Evaluate expressions by substituting for the variable. (Limit: Whole numbers and one variable)*

SUGGESTED DEVELOPMENT:

Discuss with the class:

1. Only when you choose a value for the number n can you determine the value of n + 4.

2. Because n holds the place for a member of a specified set (in this case, W), n is a variable.

3. Evaluate expressions for values given to a variable;

   Example: n x 5; the replacement set is $\{1,2,3,4\}$

   1 x 5 = 5 2 x 5 = 10

   3 x 5 = 15 4 x 5 = 20

4. Complete with class as an example.

   Replacement set $\{1,2,3,4\}$

n	2 n + 1
1	2 x 1 + 1 = 2 + 1 = 3
2	5
3	7
4	9

EVALUATING EXPRESSIONS

THE VALUE OF EXPRESSIONS CONTAINING A VARIABLE CHANGES IF THE VALUE OF THE VARIABLE CHANGES

$4a + 5$ IF $a = 3$

$= 4(3) + 5$

$= 12 + 5$

$= 17$

$4a + 5$ IF $a = 7$

$= 4(7) + 5$

$= 28 + 5$

$= 33$

y	$3y + 3$
0	3
1	____
2	9
3	____
4	15

WHOLE NOS. 7 OBJ. 12

EXERCISES: OBJECTIVE NO. 12

1. Copy and complete these tables of values.

Example: Replacement set $\{1,2,3,4\}$

n	2 n + 1		
	SUBSTITUTION	COMPUTATION	VALUE
1	2 (1) + 1	2 + 1	3
2			
3			
4			

REPLACEMENTS

a)

n	3 n + 1
1	4
2	7
3	10
4	13
5	16

b)

n	2 n
2	4
3	6
4	8
5	10
7	14

c)

n	5 n - 2
5	23
8	38
9	43
6	28
7	33

d)

n	5 x 2 n
7	70
5	50
6	60
3	30
2	20

e)

n	11 - n
1	10
2	9
5	6
4	7
9	2

f)

n	n x 7
2	14
6	42
3	21
9	63
7	49

2. Evaluate the following for the values of the variable given.

a) n + n + n when the replacement set is $\{0, 1, 2, 3, 4\}$ $\{0, 3, 6, 9, 12\}$

b) 4 x a when the replacement set is $\{0, 1, 2, 3, 4\}$. $\{0, 4, 8, 12, 16\}$

c) 12 ÷ n when the replacement set is $\{2, 4, 6\}$. $\{6, 3, 2\}$

d) c + c + c + c + c when the replacement set is $\{1, 3, 5, 7\}$. $\{5, 15, 25, 35\}$

e) 5 x n when the replacement set is $\{1, 2, 3,\}$. $\{5, 10, 15\}$

f) 3a when the replacement set is $\{2, 4, 6, 8\}$. $\{6, 12, 18, 24\}$

g) 6f when the replacement set is $\{2, 4, 6, 8\}$. $\{12, 24, 36, 48\}$

h) a + 2a when the replacement set is $\{1, 2, 3, 4\}$. $\{3, 6, 9, 12\}$

i) 2a + 3a when the replacement set is $\{0, 1, 2, 3,\}$. $\{0, 5, 10, 15\}$

j) 2a + 2a + 2a when the replacement set is $\{2, 4, 6\}$. $\{12, 24, 36\}$

DEVELOPMENT AND EXERCISES

STRAND: Whole numbers 7

UNIT: II 13

OBJECTIVE: Solve conditions for equality using an informal approach (Limits: whole numbers.)

SUGGESTED DEVELOPMENT:

1. Use conditions only of the form:

   (a) $x + A = B$
   (b) $Ax = B$
   (c) $Ax + B = C$
   (d) $Ax + Bx = C$

2. Sentences such as $a + 7 = 11$, and $x - 8 = 13$ are conditions (equations)

3. Symbols, such as x, n or d are variables.

4. Give the students practice in intuitively finding the correct replacement for the variable in a condition. (Exercise 2). The students must also understand that after finding the replacement the quantities on each side of the equal sign must be the same.

SUGGESTION:

Look at the condition $x + 3 = 10$

Cover x with your thumb. $+ 3 = 10$

What number must be under my thumb in order that the statement be true?

1. Solve these conditions:

Cover the variable with your thumb. Ask yourself, "What number must be under my thumb in order that the statement be true?"

a) $n + 7 = 17$
10

b) $n - 9 = 8$
17

c) $n - 5 = 6$
11

d) $n + 3 = 8$
5

e) $n + 12 = 19$
7

f) $n - 5 = 9$
14

g) $x - 3 = 16$
19

h) $x + 3 = 37$
34

i) $x - 1 = 12$
13

j) $x + 1 = 11$
10

k) $n - 6 = 17$
23

l) $n + 3 = 15$
12

m) $n - 8 = 15$
23

n) $n - 6 = 7$
13

o) $n + 20 = 39$
19

p) $a + 7 = 9$
2

q) $x + 9 = 17$
8

r) $x - 7 = 11$
18

s) $x + 23 = 67$
44

t) $x - 59 = 93$
152

2. Solve these conditions.

a) $6n = 30$
5

b) $2n = 8$
4

c) $4n = 28$
7

d) $\frac{n}{5} = 11$
55

e) $8n = 56$
7

f) $\frac{x}{3} = 8$
24

g) $15x = 60$
4

h) $24x = 96$
4

i) $6x = 96$
16

j) $3x = 51$
17

k) $n \div 6 = 12$
72

l) $n \div 2 = 9$
18

m) $11n = 77$
7

n) $\frac{n}{9} = 7$
63

o) $\frac{n}{7} = 7$
49

p) $27x = 81$
3

q) $x \div 7 = 13$
91

r) $\frac{x}{8} = 12$
96

s) $\frac{x}{9} = 8$
72

t) $\frac{x}{5} = 24$
120

To answer the riddles which follow, solve each condition (equation) for the variable. Then place the variable in the space which corresponds to its value. (The first one has been done for you.)

1. *WHAT DO THEY CALL A SMART DUCK IN SCHOOL?*

A	w	i	s	e		q	u	a	c	k	e	r
7	2	4	20	6		8	3	7	13	14	6	15

eg. $w + 3 = 5$ $w = 2$

$a + 5 = 12$
$9 + q = 17$
$e + 9 = 15$

$s - 17 = 3$
$9 + u = 12$
$i - o = 4$

$k - 5 = 9$
$c - 12 = 1$
$r - 9 = 6$

2. *WHY DO DRAGONS SLEEP IN THE DAYTIME?*

B	e	c	a	u	s	e		t	h	e	y
18	2	3	27	1	8	2		4	7	2	13

c	a	n		h	u	n	t		k	n	i	g	h	t	s
3	27	6		7	1	6	4		11	6	10	5	7	4	8

$3g = 15$
$h \times 6 = 42$
$9t = 36$
$15c = 45$

$2y = 26$
$a \div 9 = 3$
$n \div 6 = 1$
$19 \times u = 19$

$s \div 1 = 8$
$16 = e \times 8$
$3k = 33$
$i \div 5 = 2$
$b \div 2 = 9$

3. *WHY IS YOUR NOSE IN THE MIDDLE OF YOUR FACE?*

B	e	c	a	u	s	e		i	t		i	s		a
5	3	6	2	7	4	3		21	15		21	4		2

s	c	e	n	t	e	r
4	6	3	9	15	3	13

$s + 17 = 21$
$12e = 36$
$5a + 6 = 16$

$t \div 5 = 3$
$18 + c = 24$
$3b + 1 = 16$

$27 - u = 20$
$2i = 42$
$23 + n = 32$
$3r = 39$

DEVELOPMENT AND EXERCISES

STRAND: Whole numbers
LEVEL: 7
UNIT: II
OBJECTIVE NO: 14
OBJECTIVE: Verify solutions of conditions by substitution.

SUGGESTED DEVELOPMENT:

1. Variables in conditions can be replaced by numbers in order to evaluate the expression.

2. Replacement means to substitute one thing for another.

   Examples:

   a) A basketball player being substituted for another player.

   b) A wet football being replaced by a dry one.

   c) Winter tires replacing summer tires.

3. In mathematics we make replacements within mathematical sentences. We replace variables with numerals which will make the mathematical sentence true.

   Examples:

   a) $r + 8 = 11$
      i) variable is r, replacement is $\{3\}$.

   b) $x + 3 = 17$
      i) variable is x, replacement is $\{14\}$.

   c) $y - 8 = 13$
      i) variable is y, replacement is $\{21\}$.

   d) $n - 5 = 16$
      i) replacement is $\{21\}$.

   e) $9 - q = 4$
      i) variable is q, replacement is $\{5\}$.

4. The condition becomes true when a replacement for a variable causes both sides of the condition to name the same number.

   Example:

Solution	Verification
$y - 8 = 13$	$21 - 8 = 13$
$y = 21$	$13 = 13$
$9 - q = 4$	$9 - 5 = 4$
$q = 5$	$4 = 4$

CHECKING SOLUTIONS

SUBSTITUTING THE SOLUTION FOR THE VARIABLE IN A CONDITION WILL GIVE A TRUE STATEMENT IF THE SOLUTION IS CORRECT

LEFT HAND SIDE = RIGHT HAND SIDE

$$4x + 3 = 11$$

$$x = 2$$

LEFT HAND SIDE = RIGHT HAND SIDE

$$4(2) + 3 = 11$$

$$8 + 3 = 11$$

$$11 = 11$$

EXERCISES:

Verification

OBJECTIVE NO. 14

Find a replacement for the variable. Verify by substituting the solution for the variable. The first one has been done for you.

CONDITION	SOLUTION	VERIFICATION
1. $2 + x = 5$	$x = 3$	$2 + (3) = 5$ $5 = 5$
2. $y + 3 = 7$	$y = 4$	$(4)+3 = 7 \quad 7 = 7$
3. $z - 2 = 5$	$z = 7$	$(7)-2 = 5 \quad 5 = 5$
4. $r - 6 = 2$	$r = 8$	$(8)-6 = 2 \quad 2 = 2$
5. $3r = 12$	$r = 4$	$3(4) = 12 \quad 12 = 12$
6. $5x = 20$	$x = 4$	$5(4) = 20 \quad 20 = 20$
7. $4 \times z = 28$	$z = 7$	$4 \times (7) = 28 \quad 28 = 28$
8. $2y + 4 = 10$	$y = 3$	$2(3)+4 = 10 \quad 10 = 10$
9. $2x + x = 9$	$x = 3$	$2(3)+(3) = 9 \quad 9 = 9$
10. $4x + 5 = 17$	$x = 3$	$4(3)+5 = 17 \quad 17 = 17$
11. $3p + 7 = 16$	$p = 3$	$3(3)+7 = 16 \quad 16 = 16$
12. $6x = 12$	$x = 2$	$6(2) = 12 \quad 12 = 12$
13. $2y = 60$	$y = 30$	$2(30) = 60 \quad 60 = 60$
14. $z \div 7 = 12$	$z = 84$	$(84) \div 7 = 12 \quad 12 = 12$
15. $2n + 3n = 15$	$n = 3$	$2(3)+3(3) = 15 \quad 15 = 15$

DEVELOPMENT AND EXERCISES

STRAND: Whole numbers LEVEL: 7

UNIT: II OBJECTIVE NO: 15

OBJECTIVE: Use some formalized approach to solving conditions. (eg. flow charts, related numbers.)

SUGGESTED DEVELOPMENT: 1. Use conditions only of the form:

a) $x + A = B$
b) $Ax = B$
c) $Ax + B = C$
d) $Ax + Bx = C$

2. The balance approach:

a)
$$\begin{aligned} x + A &= B \\ x + A - A &= B - A \end{aligned}$$

$$\begin{aligned} x + 3 &= 7 \\ x + 3 - 3 &= 7 - 3 \\ x &= 4 \end{aligned}$$

b)
$$\begin{aligned} x - A &= B \\ x - A + A &= B + A \\ x &= B + A \end{aligned}$$

$$\begin{aligned} x - 4 &= 11 \\ x - 4 + 4 &= 11 + 4 \\ x &= 15 \end{aligned}$$

c)
$$\begin{aligned} 3x + 4 &= 16 \\ 3x + 4 - 4 &= 16 - 4 \\ 3x &= 12 \\ \frac{3x}{3} &= \frac{12}{3} \\ x &= 4 \end{aligned}$$

3. Related number sentence

a)
$$\begin{aligned} x + 3 &= 7 \\ x &= 7 - 3 \\ x &= 4 \end{aligned}$$

b)
$$\begin{aligned} x - 4 &= 11 \\ x &= 11 + 4 \\ x &= 15 \end{aligned}$$

c)
$$\begin{aligned} Ax &= B \\ x &= B \div A \end{aligned}$$

$$\begin{aligned} 3x &= 12 \\ x &= 12 \div 3 \\ x &= 4 \end{aligned}$$

d)
$$\begin{aligned} x \div A &= B \\ x &= BA \end{aligned}$$

$$\begin{aligned} x \div 3 &= 4 \\ x &= 4 \times 3 \\ x &= 12 \end{aligned}$$

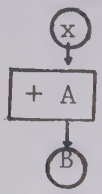
e)
$$\begin{aligned} 3x + 4 &= 16 \\ 3x &= 16 - 4 \\ 3x &= 12 \\ x &= \frac{12}{3} \\ x &= 4 \end{aligned}$$

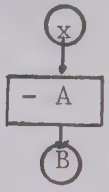
f)
$$\begin{aligned} 3x + 4x &= 14 \\ (3 + 4)x &= 14 \\ 7x &= \frac{14}{7} \\ x &= 2 \end{aligned}$$

4. The flow chart approach:

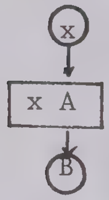
a) State the condition in a flow chart.

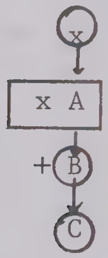
eg. x + A = B as   x - A = B as





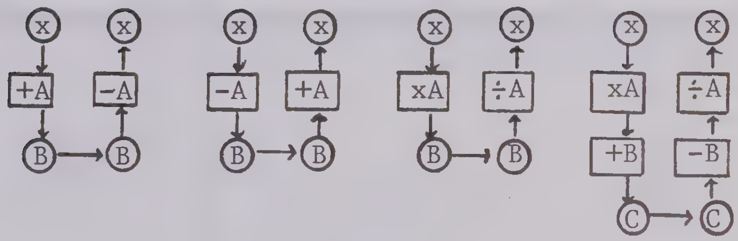
Ax = B as   Ax + B = C as



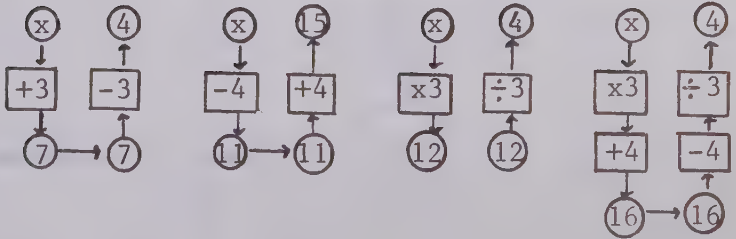


b) Reverse the flow chart on the right using inverse operations to solve the condition.

eg.



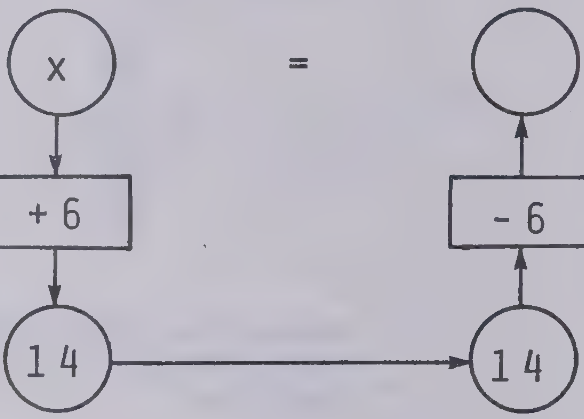
x + 3 = 7   x - 4 = 11   3x = 12   3x + 4 = 16



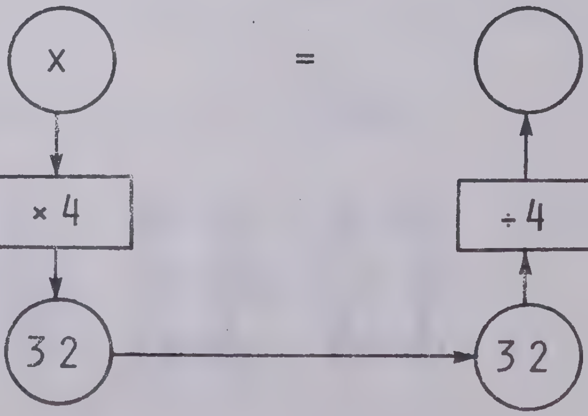
SOLVING CONDITIONS

FLOW CHARTS

$x + 6 = 14$



$4x = 32$



WHOLE NOS. 7 OBJ. 15

SOLVING CONDITIONS

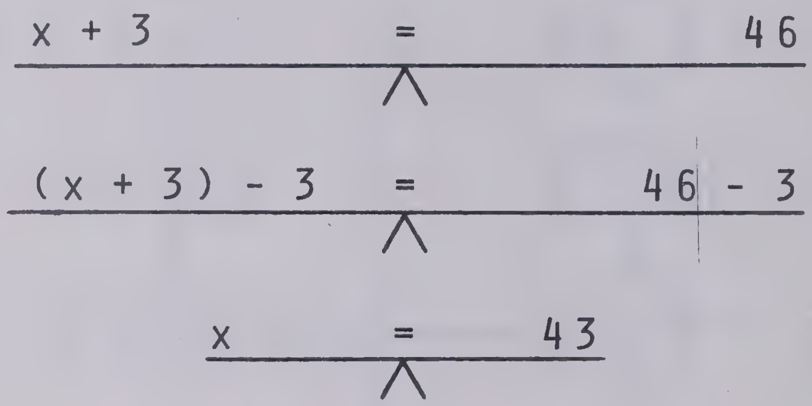
RELATED NUMBER SENTENCES

$3 + 5 = 8$ $5 + 3 = 8$ $3 = 8 - 5$ $5 = 8 - 3$	$4 + x = 9$ $x + 4 = 9$ $4 = 9 - x$ $x = 9 - 4$
$4 \times 7 = 28$ $7 \times 4 = 28$ $4 = 28 \div 7$ $7 = 28 \div 4$	$5a = 30$ $a \times 5 = 30$ $5 = 30 \div a$ $a = 30 \div 5$

SOLVING CONDITIONS

BALANCE

$$x + 3 = 46$$

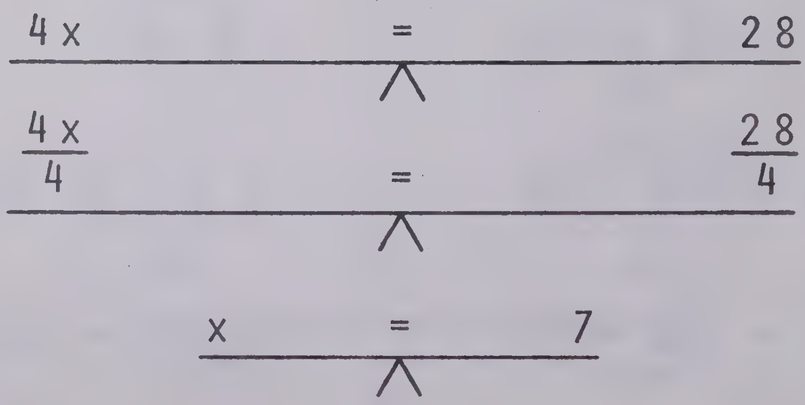


$$x + 3 = 46$$

$$(x + 3) - 3 = 46 - 3$$

$$x = 43$$

$$4x = 28$$



$$4x = 28$$

$$\frac{4x}{4} = \frac{28}{4}$$

$$x = 7$$

EXERCISES: Fun With Conditions

OBJECTIVE NO. 15

1. Solve these conditions.

a) $n + 17 = 173$
$n = 173 - 17$
$n = 156$

b) $n - 23 = 48$
$n = 48 + 23$
$n = 71$

c) $n - 15 = 37$ 52
d) $n + 26 = 54$ 28
e) $n + 12 = 91$ 79
f) $n - 9 = 28$ 37
g) $x - 47 = 83$ 130
h) $n - 166 = 24$ 190
i) $13 + n = 59$ 46
j) $35 + n = 223$ 188
k) $x - 341 = 29$ 370
l) $a + 48 = 97$ 49
m) $23 + n = 108$ 85
n) $x - 52 = 17$ 69
o) $b + 18 = 61$ 43
p) $a - 30 = 24$ 54
q) $x - 92 = 11$ 103

2. Solve these conditions.

a) $6n = 78$
$n = 78 \div 6$
$n = 13$

b) $n \div 14 = 9$
$n = 9 \times 14$
$n = 126$

c) $13n = 169$ 13
d) $21n = 126$ 6
e) $17x = 204$ 12
f) $7x = 91$ 13
g) $a \div 17 = 6$ 102
h) $x \div 8 = 22$ 176
i) $47a = 188$ 4
j) $n \div 37 = 18$ 666
k) $53x = 583$ 11
l) $a \times 19 = 133$ 7
m) $8n = 120$ 15
n) $x \times 27 = 216$ 8

3. Solve the following conditions.

a) $2x + 4 = 20$
$x = (20 - 4) \div 2$
$x = 8$

b) $5a - 4 = 21$
$a = (21 + 4) \div 5$
$a = 5$

c) $3n + 2 = 11$ 3
d) $4x - 3 = 17$ 5
e) $7a + 13 = 76$ 9
f) $14x - 38 = 116$ 11
g) $15x + 20 = 95$ 5
h) $7z - 22 = 27$ 7
i) $9x + 14 = 95$ 9
j) $12n - 11 = 133$ 12
k) $4n + 8 = 36$ 7
l) $25x + 14 = 139$ 5
m) $20a - 7 = 73$ 4
n) $6x - 25 = 29$ 9

4. Solve these conditions.

a) $3a + a = 24$
$4a = 24$
$a = 24 \div 4$
$a = 6$

b) $4x + 3x = 56$
$7x = 56$
$x = 56 \div 7$
$x = 8$

c) $9x + 7x = 128$ 8
d) $15a - 9a = 90$ 15
e) $38x - 7x = 93$ 3
f) $5n + 17n = 242$ 11
g) $22a - 15a = 98$ 14
h) $13x + 29x = 336$ 8
i) $8n + 17n = 200$ 8
j) $105x - 75x = 120$ 4

DEVELOPMENT AND EXERCISES

STRAND: Whole numbers LEVEL: 7

UNIT: II OBJECTIVE NO: 16

OBJECTIVE: Write English sentences for mathematical sentences.

SUGGESTED DEVELOPMENT: Discuss with the class.

1. Commutative property is used in types such as $3x + 7 = 23$. Interpretation can be either of the following ways:
   a) Three times a number is added to 7 results in 23.
   b) Seven added to three times a number equals 23.
2. For mathematical sentences such as $28 - x = 19$ there is no commutativity for subtraction. However, the sentence can be interpreted in the following way:
   a) If a number is subtracted from 28, the result would be 19.
3. Some mathematical sentences involve two basic operations such as $\frac{29 - 5}{6} = x$.

   Interpretations can be:
   a) The difference of 29 and 5 divided by 6 equals what number?
   b) The quotient of the difference of 29 and 5 divided by 6 will be what number?
4. Some mathematical sentences involve all of the four basic operations such as $63 - (x \div 7) = 2(x) + 3$
   a) The difference of 63 and a number divided by 7 is equal to the sum of twice the number and 3. Find the number.
   b) Twice a number added to 3 equals the difference between 63 and the quotient of the number and 7. Find the number.
5. Words similar to add such as increase, greater larger, and subtract (diminish, smaller, less) need illustration at this time. Examples:

a)	$x + 6 = 9$	A number increased by six is nine.
b)	$x - 2 = 11$	A number diminished by 2 is eleven.
c)	$2x = 8$	Twice some number is 8.
d)	$x \div 3 = 10$	A number divided by three equals ten.
e)	$12 - n = 4 + n$	Twelve minus a number is four plus the same number.

EXERCISES:

Translations

OBJECTIVE NO. 16

1. Write English phrases for the following mathematical expressions:

Example: 1. $x + 7$ A number increased by seven.
2. $7 - x$ Seven diminished by a number.

a) $x + 2$ a number plus 2
b) $3x$ 3 times a number
c) $x - 5$ a number diminished by 5
d) $6 - x$ 6 diminished by a number
e) $2x + 1$ twice a number increased by 1
f) $3x - 2$ three times a number subtract 2
g) $4x + 6$ 6 added to 4 times a number
h) $6 + 3x$ 3 times a number added to 6
i) $2 + x + 2x$ 2 plus a number plus twice a num
j) $17 - 2x$ 17 diminished by tw a number

2. Write English sentences for the following mathematical sentences:

Example: $x + 3 = 8$ A number increased by 3 is equal to 8.

a) $x - 2 = 7$ a number diminished by 2 equals 7
b) $5x = 15$ 5 times a number equals 15
c) $3x = 18$ 3 times a number is 18
d) $3x - 5 = 13$ 3 times a number diminished by 5 is 13
e) $4x = x + 15$ 4 times a number equals a number plus 15
f) $23 = x - 10$ 23 equals a number diminished by 10
g) $5 + x = 18$ 5 increased by a number is eighteen
h) $17 - x = 9$ 17 diminished by a number is
i) $10 + x = 12$ 10 plus a is 12
j) $14 = 20 - 2x$ 14 equals 20 diminis by twice a num

3. Make up a problem to match each of these conditions:

Example: $x + 7 = 15$ Paul had some hockey cards. He collected 7 more and then he had 15 cards. How many cards did he start with?

a) $2x = 10$
b) $17 - x = 12$
c) $x + 13 = 25$
d) $2x + 4 = 16$
e) $3x - 5 = 22$
f) $4x = 24$

a) Twice a number is ten. Find the number

b) Mary had $17. After she spent some money she had $12 left. How much did she spend?

c) A number increased by 13 is 25. Find the number

d) John has some coins. If we double the number of coins Jo has and add 4, we get 16. Ho many coins does John have

4. *WHERE DOES A 400 kg CANARY SIT?*

To solve the riddle match the English phrases with the correct mathematical expressions; transfer the letter to the numbers below.

ENGLISH PHRASE

1. A number decreased by six
2. A certain number doubled
3. Eight is increased by a number
4. Twelve more than a number
5. An unknown number is added to 17
6. An unknown number is subtracted from 5
7. A number increased by 3
8. 6 added to a number
9. The product of seven and a number
10. Eight is increased by twice a number
11. A number tripled
12. Five is subtracted from an unknown number
13. The sum of a certain number and fourteen
14. A certain number multiplied by five
15. Three times a number diminished by two

EXPRESSION

A $x - 6$
E $5 - x$
E $5x$
E $8 + 2x$
E $x + 6$
H $17 + x$
H $7x$
I $x - 5$
K $x + 14$
L $3x$
N $2x$
R $x + 3$
S $3x - 2$
W $x + 12$
Y $8 + x$

A	N	Y	W	H	E	R	E	H	E
1	2	3	4	5	6	7	8	9	10

L	I	K	E	S
11	12	13	14	15

3. e) Three times a number diminished by five is twenty-two. What is the number?

f) Four times Jason's age is 24. How old is Jason?

DEVELOPMENT AND EXERCISES

STRAND: Whole numbers LEVEL: 7

UNIT: II OBJECTIVE NO: 17

OBJECTIVE: Write mathematical sentences for English sentences.

SUGGESTED DEVELOPMENT:

1. Generally, difficulty lies in determining the basic mathematical operations involved in mathematical sentences. Terminology such as increase, greater, more, larger, diminish, smaller, and less need emphasis.

2. The many terms which can describe the verb such as: is, results, will be, etc. also needs emphasis.

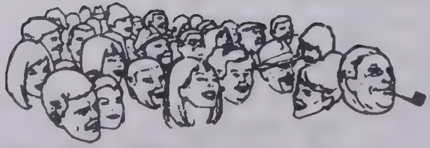
3. It should also be emphasized that an assigned variable should be replaced by the same number if it occurs more than once in a sentence.

4. Addition and multiplication are commutative so mathematical sentences involving addition and multiplication can be written as:

   a) $x + 3 = 12$ or $3 + x = 12$
   b) $x \times 4 = 20$ or $4 \times x = 20$

5. Sentences involving subtraction and division need more attention since commutativity does not exist for subtraction and division.

   a) $x - 5 = 12$ can not be $5 - x = 12$
   b) $x \div 6 = 5$ can not be $6 \div x = 5$



EXERCISES:

Translations

OBJECTIVE NO. 17

1. Write the mathematical sentences for the following:
DO NOT SOLVE.
Example: A number diminished by seven is equal to five.
$x - 7 = 5$

   a) The sum of five and nine is fourteen. $5+9=14$
   b) The product of five and four is twenty. $5 \times 4 = 20$
   c) The quotient of nine and three is three. $9 \div 3 = 3$
   d) If you multiply a number by five, the result is 35. $5x = 35$
   e) If you add 6 to a number, the result is 13. $x+6=13$
   f) If you subtract 7 from a number, the result is 9. $x-7=9$
   g) If you multiply a number by 11, you get 121. $11x = 121$
   h) A number divided by 9 is equal to 21. $x \div 9 = 21$
   i) If you add 17 to a number, the sum is 35. $x+17=35$
   j) If you add 15 to a number, the result is 34. $x+15=34$
   k) A number divided by 7 is equal to 12. $\frac{x}{7} = 12$
   l) If you subtract 307 from a number, the result is 814. $x-307=814$
   m) The product of a number and 7 is the same as 21. $7x = 21$
   n) If you decrease a number by 5, the result is 6. $x-5=6$
   o) A number divided by 6 equals 9. $\frac{x}{6} = 9$

2. Write mathematical sentences for the following:
DO NOT SOLVE.
Example: The sum of an unknown and 6 is 24.

   $x + 6 = 24$

   a) A number less 12 is 15. $x-12=15$
   b) 14 increased by a number results in 24. $14+x=24$
   c) A number diminished by 13 is 21. $x-13=21$
   d) 22 greater than a number will be 33. $x+22=33$
   e) Twice a number is 12. $2x=12$
   f) A number is 6 larger than 10. $x=10+6$
   g) The sum of 14 and 3 is a number. $14+3=x$
   h) A number tripled yields 27. $3x=27$
   i) The difference between 24 and 11 is a number. $24-11=x$
   j) 6 less than a number is 4. $x-6=4$
   k) 14 is 6 smaller than a certain number. $14=x-6$
   l) 4 times a number less 6 is 26. $4x-6=26$
   m) 6 more than a number is 29. $x+6=29$
   n) 5 more than 3 times a number results in 20. $3x+5=20$
   o) 4 more than a number results in 6 less than 3 times the number. $x+4=3x-6$

3. Write conditions for each of the following problems. Let "x" represent the unknown. DO NOT SOLVE.

Example: Jason had 33 pieces of gum. After he gave some away, he had 19 left. How many did he give away?

$33 - x = 19$

a) 232 grade 7 students at Balwin Junior High were divided up into 8 equal classes. How many students would be in each class?

$232 \div 8 = x$

b) Wendy bought a skirt for 19 dollars. She then had 17 dollars left. How many dollars did she have before she bought the skirt?

$x - 19 = 17$

c) Mr. Auto's car averages 8 km/ℓ of gasoline. If the tank holds 72 litres, how far can he drive on one tankful of fuel?

$8(72) = x$

d) Mr. Chips has a board 50 dm long. If he cuts 4 pieces each 9 dm long, how much board is left? $50 - 4(9) = x$

e) A number plus three times the number is 28. What is the number?

$x + 3x = 28$

f) The sum of two numbers is 4963. One number is 1947. Find the other number. $1947 + x = 4963$

g) The quotient of two numbers is 19. The smaller number is 25. What is the larger number? $\frac{x}{25} = 19$

h) Jim gets an allowance of 75¢ per week. If he saves his allowance for six weeks and then purchases a pocket knife worth $3.98, how much does he have left?

$6 \times 75 - 398 = x$

30 Days Has September

4. *WHICH MONTH HAS 28 DAYS?*

To answer the puzzle, match the English sentence with the condition; transfer the letter to the numbers below.

1. A number increased by seven is equal to 13.	A. $20 = 3x + 5$
2. Five more than a number is 24.	A. $x - 9 = 12$
3. A number plus 8 is equal to 19.	E. $x + 8 = 19$
4. If a number is doubled, the result is 20.	E. $2x + 3 = 19$
5. Twenty is 5 more than three times a number.	H. $x + 5 = 24$
6. When a certain number is subtracted from 11, the result is 8.	H. $x + 2x = 15$
7. Four times an unknown number is 28.	L. $4x = 28$
8. A number plus twice the number is 15.	L. $11 - x = 8$
9. A number diminished by 9 is 12.	T. $x + 7 = 13$
10. Three times a number increased by 2 is 14.	V. $3x + 2 = 14$
11. A number doubled, plus three is equal to 19.	Y. $2x = 20$

T	H	E	Y	A	L	L	H	A	V	E
1	2	3	4	5	6	7	8	9	10	11

DEVELOPMENT AND EXERCISES

STRAND: *Whole numbers* LEVEL: *7*

UNIT: *II* OBJECTIVE NO: *18*

OBJECTIVE: *Solve problems using any organized approach.*

SUGGESTED DEVELOPMENT: Discuss problem solving procedure with students:

1. State the variable.
2. Write the condition.
3. Solve the condition; flow charts can be used.
4. Write a statement answer to the question.

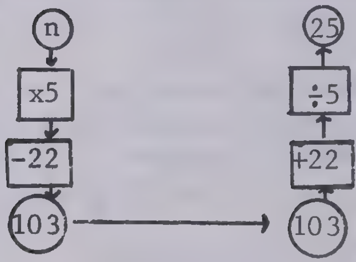
Example:

Bill has a stamp collection. If we multiply the number of stamps in his collection by 5 and subtract 22, we get 103. How many stamps does he have?

Let the number of stamps be "n"

Method 1

$(5 \times x) - 22 = 103$



Method 2

$$\begin{array}{l} 5n - 22 = 103 \\ \underline{\quad + 22 + 22} \\ \frac{5n}{5} \qquad \frac{125}{5} \\ n \qquad 25 \end{array}$$

Bill has 25 stamps.

Method 3

$$\begin{aligned} 5n - 22 &= 103 \\ 5n - 22 + 22 &= 103 + 22 \\ 5n &= 125 \\ n &= \frac{125}{5} \\ n &= 25 \end{aligned}$$

Bill has 25 stamps.

Method 4

$$\begin{aligned} 5n - 22 &= 103 \\ 5n &= 103 + 22 \\ 5n &= 125 \\ n &= \frac{125}{5} \\ n &= 25 \end{aligned}$$

Bill has 25 stamps.

EXERCISES: **We Got Problems!** OBJECTIVE NO. 18

A. Solve the following problems:

SHOW ALL THE STEPS (1) *VARIABLES:* Let x = _________
(2) *CONDITION:*
(3) *SOLUTION:* x = _________
(4) *ANSWER:* A Statement

Example: Mr. James bought 16 sheets of plywood. He paid \$8.45 a sheet. How much did the plywood cost him?

(1) Let x = the total cost of the plywood
(2) x = 16 (8.45)
(3) x = 135.20
(4) The plywood cost him \$135.20

1. Mark had 31 chocolate bars. After he gave some away, he had 18 left. How many did he give away? 13 bars

2. Mike had 15 Batman comic books. He had 24 after he bought some more. How many more did he buy? 9 comic books

3. John bought a bicycle from his savings. He paid \$45.00 for the bicycle and had \$9.00 left. How much money had John saved?
\$54.00

4. Mr. Olson has 115 tomato plants to set out. After one hour, he had set out 76. How many plants did he have left to set out?
39 plants

5. Sue had 20 quarters. After she spent some of them she had 12 quarters left. How many quarters did Sue spend?
8 quarters

6. Joe has 14 more dimes than nickels. He has 45 nickels. How many dimes does he have? 59 dimes

7. Mutt drinks 3 times as much milk as his brother Jeff. Together they drink 24 glasses in a week. How many glasses do they each drink per week? 6 glasses and 18 glasses

8. A school wants to buy some baseball bats. If they decided to buy 3 bats for \$ 8.85, how much did they pay for each bat?
\$2.95

9. Roast beef costs \$2.87 a kg, minced steak costs \$1.49 a kg and turkey costs \$.99 a kg. How much would you spend if you bought a 3 kg roast of beef, 2 kg of minced steak and a 11 kg turkey? \$22.48

10. The population of a city was 67 209. The population was made up of 22 914 men, 9 016 boys, 23 785 women and a number of girls. How many girls were there in the city?
11 494 girls

11. The Montrose basketball team bought 8 basketballs at $14.85 each. How much did they pay for the basketballs? $118.80

12. Sirloin roasts cost $3.85 a kg and pork chops $2.78 a kg. Mrs. Brown bought a 3 kg roast and 2 kg of chops. How much change did she get back from a $20. bill? $2.89

13. Jim bought a bicycle out of his savings. He paid $79.95 for the bicycle and had $133.42 left in his savings. How much did he have in savings before his purchase? $213.37

14. Janet had 876 stamps in her collection. After saving for another six months she had 1 073 stamps in her collection. How many more did Janet save during that time? 197 stamps

15. Mrs. Allen bought 6 cans of soup at 47¢ a can, 2 kg of oranges at $1.13 a kg, 5 grapefruit at 19¢ each, and 3 kg of hamburger at $1.19 a kg. What did she pay for her purchases? $9.60

16. During his Olympic training program, George ran a certain number of laps every day for a week. He ran 12 laps on Monday, 16 on Tuesday, 13 on Wednesday, 15 on Friday, 18 on Saturday, and 17 on Sunday. He ran a total of 108 laps during the week. How many laps did he run on Thursday? 17 laps

17. Mr. Jones bought 38 litres of gasoline at 14¢ per litre. What was the cost of the gasoline? $5.32

18. On a recent trip Mr. Brown averaged 6 km to a litre of gasoline. If he used 36 litres of gasoline, how far did he drive? 216 km

19. Mrs. Dorsey bought 3 kg of apples at 39¢ a kg and some bananas at 33¢ a kg. If she spent $1.83 in all how many kilograms of bananas did she buy? 2 kg

20. On Tuesday Jack delivered 17 more papers than he had on Monday. His total delivery for the two days was 103. How many papers did he deliver on Tuesday? 60 papers

B. ABSTRACT PROBLEMS:

Example: If you multiply a number by 7, then add 17 to the product, the result is 115. What is the number?

(1) Let x = the number
(2) 7 x x + 17 = 115
(3) x = 14
(4) The number is 14.

1. A number less 23 is equal to 85. What is the number?
108
2. Fourteen times a certain number is equal to 896. What is the number? 64
3. Three times a number diminished by 5 is equal to 88. What is the number? 31
4. Twice a certain number diminished by 47 is equal to 23. What is the number? 35
5. The product of a certain number and 12 is increased by 27. The result is 63. What is the number?
3
6. If you multiply a number by 6, then add 20 to the product, you will get 110. What is the number?
15
7. If you divide a number by 4, then subtract 5 from the quotient, you will get 8. What is the number?
52
8. The product of 12 and 13 increased by 17 equals a number. Find the number. 173
9. A number divided by 3 is 4 less than 12.
24
10. Twice a number decreased by 21 is 15.
18
11. Three times a number diminished by 7 equals 26.
11
12. If you increase 4 times a number by 76, the result is 664. What is the number? 147

REVIEW EXERCISES: Review OBJECTIVE NO'S 11 - 18

1. Solve for x:

(a) $3x + 1 = 4$ $x=1$
(b) $5x + 2x = 28$ $x=4$

2. Make these sentences true:

(a) $9 - 5 = 6 - ?$ 2
(b) $3 \times 7 = 20 +$ 1

3. Find the value of the following:

(a) $3x + 2x + 1$ if $x = 4$ 21
(b) $\frac{(w \times 3) + 30}{w\ (w - 4)}$ if $w = 6$ 4

4. (a) $532 - 329 + 27 = ?$ 230

5. Insert <, > or = to make a true statement
(a) $4 + 7 - (2 \times 3) < (10 - 5)\ 5 - 5$
(b) $(6 + 8)\ \frac{(9)}{3} + 30 > (5 + 7) + 2 \times 3$
(c) $3\ \frac{(15)}{15} \times 3 \times 3 = \frac{(60 - 6)}{2}$

6. Evaluate the following:

(a) $4 + 3x - 2x + 4 + x$ if $x = 3$ 14
(b) $m + (n + n) - m + (n + n)$ if $m = 2$ and $n = 4$ 16
(c) $2x - 3 + y$ if $x = 5$ and $y = 7$ 14

7. Solve for the variable in each case:

(a) $5x = 115$ 23
(b) $3a - 2 = 17 - 1$ 6
(c) $5a + a = 30$ 5

8. (a) $y + 8 = 14$ 6
(b) $x - 3 = 2$ 5

9. (a) $3x - 2 = 10$ 4

10. (a) $5x - 3 + x$ for $x = 4$ 21
(b) $\frac{3x + 14}{4}$ for $x = 2$ 5

11. (a) $7 + x = 12$ 5
(b) $18 + x = 25$ 7
(c) $7x = 42$ 6
(d) $6x = 18$ 3
(e) $4x + 7 = 19$ 3
(f) $3x + 11 = 38$ 9
(g) $5x + 3x = 64$ 8
(h) $9x + 4x = 78$ 6

12. Solve:

(a) $x + 5 = 8$ 3
(b) $2x - 3 = 11$ 7

13. Solve:

(a) $11 + x = 24$ 13
(b) $x - 8 = 3$ 11

REVIEW EXERCISES: OBJECTIVE NO'S 16 - 18

14. Write mathematical expressions for the following:

- (a) Six more than a number $x+6$
- (b) Four more than twice a number $2x+4$
- (c) Five times a number less eight $5x-8$
- (d) Nine times twice a number $9(2x)$
- (e) Eight less than twice a number $2x-8$
- (f) Nine times a number less eight $9x-8$

15. Write mathematical sentences for the following:

- (a) Three times a number is equal to 27. $3x=27$
- (b) Six more than a number is equal to 31. $x+6=31$
- (c) Five more than twice a number is equal to 120. $2x+5=120$
- (d) Four times a number less six is equal to 130. $4x-6=130$
- (e) Seven less than three times a number is equal to 32. $3x-7=32$
- (f) Fourteen less than three times a number is equal to 8. $3x-14=8$

16. Write conditions for these problems: DO NOT SOLVE.

- (a) If 16 is subtracted from three times a number, the result is 80. $3x-16=80$
- (b) The product of eight and twice a number is 144. $8(2x)=144$
- (c) Sixty diminished by twice a number is 30. $60-2x=30$
- (d) The difference between twice a number and 16 is 42. $2x-16=42$
- (e) Jack is collecting hockey cards. In January he collected 52 cards, and twice as many in February. How many cards did he collect during the two months? $52+2(52)=x$
- (f) On his paper route Tom collects \$19.50 during one week. If he collected \$7.00 on Monday and \$6.75 on Tuesday, how much more does he have to collect? $7.00+6.75+x=19.50$
- (g) The population of Edmonton is approximately 450 000. How much more would the population have to increase by to reach 525 000? $450\,000+x=525\,000$
- (h) During a football season the Calgary Stampeders scored a total of 384 points in 16 games. What was their average score per game? $384\div 16=x$

17. Solve the problems:

- (a) Nancy has four dollars. She gets fifteen dollars as a birthday present. She then spends three dollars on each of her two brothers. How much money does she now have? \$13.00
- (b) Tony purchases five articles at the same price and then spends four dollars on a record. Altogether he spends nineteen dollars. Find the cost of each article. \$3.00
- (c) Betty has some records. When she buys five more she will have 3 times the amount that Susan has. Susan has 8 records. How many records does Betty have at present? 19 records
- (d) There are three times as many students in the Drama Club as there are in the Science Club. There are 92 students altogether in the two clubs. How many students are there in the Science Club? ★ ★ 23 students

LABS

LEVEL 7

UNIT III

RATIONAL NUMBERS

OBJECTIVE: *To discover the relationship between equivalent fractions when reducing fractions to their lowest form.*

SUGGESTED DEVELOPMENT FOR THE LAB APPROACH:

TIME: minimum of 60 minutes

MATERIALS: centimetre grid paper, pencil

ORGANIZATION: Students should work in pairs with each student having their own materials.

DIRECTION TO STUDENTS:

READ CAREFULLY steps 1 and 2 of the method before using the FLOW CHART.

After columns 2, 3 and 4 of the table have been completed, compare your findings with those of your partner. When both you and your partner have completed your labs bring them to me for checking. After I have checked your lab you may proceed with the exercise sheet that I will give you. When all labs are completed and corrected a group discussion will follow.

DIRECTIONS TO TEACHERS:

You should work through this lab at least once on your own before having your class carry it out. Be sure to read through all instructions carefully with the class before allowing any of the students to go ahead on their own. It may be necessary for you to go through one example with a class of weaker students. It is necessary that the students have a previous knowledge of factors and greatest common factors as well as the use of flow charts!

PURPOSE: To discover the relationship between equivalent fractions when reducing fractions to their lowest terms.

MATERIALS: 3 - cm grid papers
1 - pencil

METHOD:

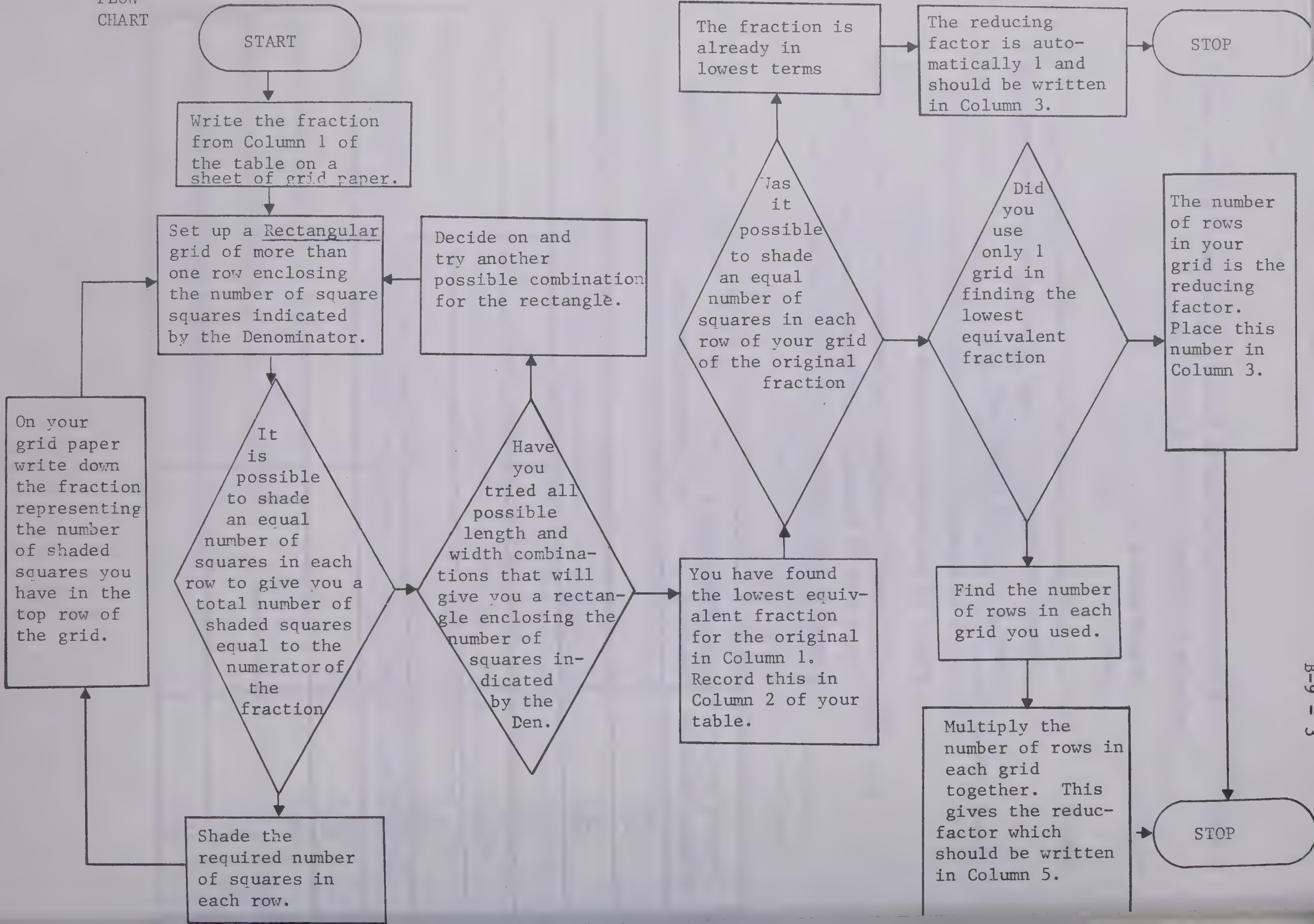
(1) On the following table you will find a column labelled ORIGINAL FRACTIONS. For each fraction in this column, follow through the given flow chart.

(2) As you go through the flow chart, fill in the information gained in the appropriate columns of the table.

(3) After completing columns two and three of the table, complete column four by finding the greatest common factor for the numerator and denominator for each original fraction.

1 ORIGINAL FRACTION	2 LOWEST EQUIVALENT FRACTION	3 REDUCING FRACTION	4 GREATEST COMMON FACTOR
$\frac{8}{28}$			
$\frac{5}{16}$			
$\frac{9}{12}$			
$\frac{13}{18}$			
$\frac{14}{35}$			
$\frac{20}{60}$			
$\frac{20}{44}$			
$\frac{36}{64}$			
$\frac{24}{56}$			
$\frac{48}{75}$			

FLOW
CHART


START
Write the fraction from Column 1 of the table on a sheet of grid paper.
Set up a Rectangular grid of more than one row enclosing the number of square squares indicated by the Denominator.
It is possible to shade an equal number of squares in each row to give you a total number of shaded squares equal to the numerator of the fraction
Shade the required number of squares in each row.
On your grid paper write down the fraction representing the number of shaded squares you have in the top row of the grid.
Have you tried all possible length and width combinations that will give you a rectangle enclosing the number of squares indicated by the Den.
Decide on and try another possible combination for the rectangle.
You have found the lowest equivalent fraction for the original in Column 1. Record this in Column 2 of your table.
Was it possible to shade an equal number of squares in each row of your grid of the original fraction
The fraction is already in lowest terms
The reducing factor is automatically 1 and should be written in Column 3.
STOP
Did you use only 1 grid in finding the lowest equivalent fraction
The number of rows in your grid is the reducing factor. Place this number in Column 3.
Find the number of rows in each grid you used.
Multiply the number of rows in each grid together. This gives the reducing factor which should be written in Column 5.
STOP

CONCLUSION: Answer each of the following questions based on the information gained during the LAB:

(1) What relationship exists between the numbers in column 3 (reducing factor) and column 4 (greatest common factor) from the chart?

(2) How may the above relationship be used in determining a method of finding the lowest equivalent form of a fraction?

DISCUSSION: (1) How could you use the above method to find equivalent fractions other than the lowest equivalent fractions:

(2) Can you suggest a method for simplifying improper fractions or mixed numerals using the above information?

EXERCISES: Reduce each of the following fractions to their lowest form by using Greatest Common Factors.

(1) $\frac{4}{38} =$

(2) $\frac{26}{54} =$

(3) $7\,\frac{48}{56}$

(4) $3\,\frac{24}{42} =$

(5) $\frac{12}{66} =$

(6) $\frac{50}{96} =$

(7) $\frac{26}{34} =$

(8) $12\,\frac{18}{66} =$

(9) $\frac{4}{35} =$

(10) $\frac{21}{48} =$

(11) $1\,\frac{16}{20} =$

(12) $\frac{30}{54} =$

(13) $\frac{28}{6} =$

(14) $\frac{72}{104} =$

(15) $3\,\frac{21}{6} =$

ANSWER SHEET TO EXERCISES:

(1) $\frac{2}{19}$

(2) $\frac{13}{27}$

(3) $7\frac{6}{7}$

(4) $3\frac{4}{7}$

(5) $\frac{2}{11}$

(6) $\frac{25}{48}$

(7) $\frac{13}{17}$

(8) $12\frac{3}{11}$

(9) $\frac{4}{35}$

(10) $\frac{7}{16}$

(11) $1\frac{4}{5}$

(12) $\frac{5}{9}$

(13) $4\frac{2}{3}$

(14) $\frac{9}{13}$

(15) $6\frac{1}{2}$

OBJECTIVE: *Demonstrate the use of reciprocals to obtain the identity element.*

TIME: Two periods will be required. One for constructing and measuring models, the second to perform the lab.

MATERIALS: 8 milk cartons, scissors, ruler, sand, bucket, masking tape, pen, per group.

ORGANIZATION: A few days in advance of the lab ask the students to bring milk cartons to the class. Depending on the number of cartons received and the size of the group desired, assign students to "lab stations" throughout the room.

TEACHER NOTE: Once materials have been prepared by a class, store the graduated cartons for future use.

Students should have covered multiplication of fractions before attempting this lab.

LAB PREPARATION

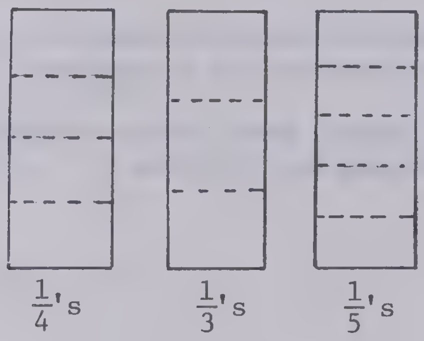
MATERIALS: 8 milk cartons, scissors, ruler, sand, bucket, masking tape, pen, per group

METHOD: Cut the top off 3 cartons and graduate as shown.

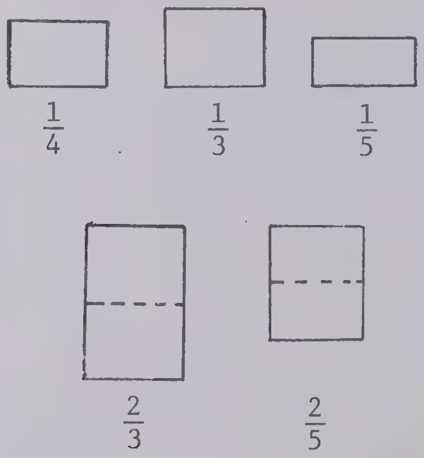
3 cartons - one graduated in quarters, one in thirds, and one in fifths.

Label the cartons. Write the graduations on masking tape and apply to carton.

These 3 cartons will be called "*containers*" in the lab.



Cut the remaining 5 cartons as indicated.



5 cartons: one cut to $\frac{1}{4}$
one cut to $\frac{1}{3}$
one cut to $\frac{1}{5}$
one cut to $\frac{2}{5}$
one cut to $\frac{2}{3}$

These 5 cartons will be called "*fillers*" in the lab.

Fill the indicated whole container with the appropriate filler and record your results in the table following:

TA

TABLE 1

A CONTAINER	B FILLER	C NO. OF FILLERS TO FILL CONTAINER	D PRODUCT OF B x C
$1 = \frac{4}{4}$	$\frac{1}{4}$		
$1 = \frac{3}{3}$	$\frac{1}{3}$		
$1 = \frac{5}{5}$	$\frac{1}{5}$		
$1 = \frac{5}{5}$	$\frac{2}{5}$		
$1 = \frac{3}{3}$	$\frac{2}{3}$		

What do you notice about Column D? ______________________

Using this conclusion about Column D complete the following table:

A CONTAINER	B FILLER	C NO. OF FILLERS TO FILL CONTAINERS	D PRODUCT OF B x C	E VALUE OF Y
$\frac{3}{3}$	$\frac{2}{3}$	Y	$\frac{2}{3} \times Y = 1$	______
$\frac{4}{4}$	$\frac{3}{4}$	Y	$\frac{3}{4} \times Y = 1$	______
$\frac{5}{5}$	$\frac{3}{5}$	Y	$\frac{3}{5} \times Y = 1$	______
$\frac{5}{5}$	Y	$\frac{5}{4}$	$Y \times \frac{5}{4} = 1$	______
$\frac{6}{6}$	Y	$\frac{6}{5}$	$Y \times \frac{6}{5} = 1$	______

PAIRS OF NUMBERS THAT HAVE 1 AS THEIR PRODUCT ARE CALLED RECIPROCALS

APPLICATION: List the reciprocals of the following:

1. $\frac{3}{4}$
2. $\frac{5}{6}$
3. $\frac{4}{5}$
4. $2\frac{1}{2}$
5. $4\frac{1}{3}$
6. $\frac{7}{2}$
7. $\frac{2}{7}$
8. $\frac{1}{8}$
9. $\frac{3}{8}$
10. 1

FURTHER DEVELOPMENT:

To find the number of groups in a number we divide.

e.g. There are 4 groups of 2 in 8. ($8 \div 2 = 4$)

To find the number of parts in a whole we can also divide.

e.g. There are 4 quarters in 1. ($1 \div \frac{1}{4} = 4$) This answer is also reached by $1 \times \frac{4}{1} = 4$.

How many thirds in 1?

$1 \div \frac{1}{3} =$ ___ or $1 \times \frac{3}{1} =$ ___

How many $\frac{2}{5}$ in 1?

$1 \div \frac{2}{5} =$ ___ or $1 \times \frac{5}{2} =$ ___

We find that we get the same answer if we divide by the part or if we multiply by the reciprocal of the part.

$1 \div \frac{1}{5} = 1 \times$ ___ = ___

$1 \div \frac{2}{3} = 1 \times$ ___ = ___

If you place a division sign between Columns A, B and an equal sign between column B and C in Table 1, do you get true statements? __________

Try this method to solve the following: (Fill your containers if necessary.)

1. $\frac{2}{3} \div \frac{1}{3} = \frac{2}{3} \times$ ___ = ___

2. $\frac{3}{4} \div \frac{1}{4} =$

3. $\frac{3}{5} \div \frac{2}{5} =$

4. $\frac{4}{5} \div \frac{2}{5} =$

5. $\frac{1}{5} \div \frac{1}{5} =$

Now you should be able to try the following division questions. Use the same method as above.

6. $\frac{2}{3} \div \frac{1}{4} =$

7. $\frac{4}{5} \div \frac{1}{3} =$

8. $\frac{3}{5} \div \frac{3}{4} =$

9. $\frac{1}{5} \div \frac{1}{4} =$

10. $\frac{3}{4} \div \frac{2}{5} =$

ACTIVITIES

LEVEL 7

UNIT III

RATIONAL NUMBERS

Deci Pairs

PURPOSE: To provide practice in decimal operations.

MATERIALS: Deck of 25 cards (illustrated below).

RULES:

1. Two, three, or four players.
2. Dealer deals all 25 cards.
3. Each player makes "tricks" from his hand by pairing up questions and correct solutions.
4. Play continues with players drawing a card from the previous player's hand. If the drawn card completes a trick, the player places the trick on the table.
5. The hand ends when one player goes out.
6. Scoring - 2 points per trick
   - 13 points against for "Deci" card with 2 players
   - 3 points against for "Deci" card with three or four players.
7. Game is three hands.

DECI PAIRS DECK:

0.7 ÷ .5 =	6.8 + 3.4 =	7.2 ÷ 4 =
4 ÷ .02 =	.035 + 0.60 =	.008 ÷ 100 =
.4 x 8 =	9.0 - .05 =	3.05 x 1.1 =
.027 x 1000 =	23.6 - 4 =	.006 x 0.01 =

1.4	10.2	1.8
200	0.635	0.00008
3.2	8.95	3.355
27	19.6	0.00006

"DECI" CARD

DECI PAIRS (Cont'd)

VARIATION: 4 players use the "Deci Pairs" deck without the "Deci" card to play Concentration.

NOTE: Similar decks may be developed to provide practice with fraction operations or more complex decimal operations.

APPLICATIONS

LEVEL 7

UNIT III

RATIONAL NUMBERS

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Schor, Meng, Insights and Skills Parts 1, 2 and 3, Globe Book Co., New York 1973

Stein, Practical Applications in Mathematics, Allyn and Bacon Inc., Boston, 1972

Witherding, Margaret F., From Fingers to Computers, Franklin Publications Inc. Chicago, 1970

VIDEO TAPES

ETV Math Series produced in Ontario Tape #1 Part D

Approximating & Estimations
Good for Grade 8 Measurement Applications

Tape #3 Part B

So You Want to Buy a Car
(Application in credit buying Grade 9 level)

Tape #5 Part A

Art from Computers
Useful as maxirational unit for applying Math to Art any grade level.

Reference Section

OBJECTIVES

UNIT III

RATIONAL NUMBERS

A. Factors and Multiples:

Students should be able to:

1. Maintain all previously developed skills and ideas: primes, composites, factors, multiples, and divisibility.

2. List the set of factors for whole numbers. (Limit: 100)

3. Write the prime factorization for whole numbers. (Limit: 100)

*4. Obtain the G.C.F. for a set of four whole numbers. (Limit: 100)

*5. Obtain the L.C.M. for a set of four whole numbers. (Limit: 100)

*6. Determine if a pair of whole numbers is relatively prime (e.g. prime factorization). (Limit: 100)

A

LEVEL 7

RATIONAL NUMBER APPLICATIONS

UNIT III

B. Fractional Numerals:

Students should be able to:

OBJECTIVES	Reference Section
1. Maintain previously developed skills and ideas: numerators, denominators, basic fraction, equivalent fractions, mixed numerals, improper fractions, and the order of fractions.	
2. Demonstrate need for fractional numbers (e.g. solve condition of the type 3x = 8)	
3. Recognize fractional numerals of the form $\frac{a}{b}$ as: (a) part of a whole (b) part of a group (c) a measure (d) a ratio (e) division	H
* 4. Demonstrate that the whole numbers are a subset of the fractional numbers using Venn diagrams.	
5. Demonstrate knowledge of the fractional numbers by plotting a given set on a number line.	J
6. Demonstrate the use of identity elements and zero property. (De-emphasize terminology)	
7. Generate sets of equivalent fractions. (Limit: 5).	
*8. Find the L.C.D. (e.g. using L.C.M.)	
9. Reduce any fraction to its basic form (e.g. using G.C.F.)	H
10. Perform the operations of addition and subtraction with fractional numbers. (Limit: 1 digit denominators)	D
11. Demonstrate the use of reciprocals to obtain the identity element.	
12. Perform the operations multiplication and division with fractional numbers. (Limit: 1 digit denominators)	D, H
13. Demonstrate that division by zero is undefined.	
14. Evaluate an expression by using the properties to produce short cuts in computation. (De-emphasize terminology) e.g. $\frac{1}{2} + \frac{1}{4} + \frac{1}{2} = 1 + \frac{1}{4} = 1\frac{1}{4}$	
15. Evaluate expressions involving order of operations. e.g. $\left(\frac{3}{4} \times \frac{4}{5} + \frac{6}{12} \div \frac{3}{4} = \right)$	
16. Order fractional numerals using the symbols $<$, $>$, $=$.	

UNIT III

C. **Decimals**:

Students should be able to:

OBJECTIVES	Reference Section
1. Maintain previously developed skills and ideas: change fractions to decimals by use of equivalent fractions, place value to 1 000ths.	
2. Write any decimal numeral in expanded form. (Use exponential notation only if previously taught).	
3. Write any decimal numeral in standard form given expanded form. (use exponential notation only if previously taught).	
4. Order decimal numerals using the symbols <, >, = .	
5. Demonstrate knowledge of decimals by plotting a given set on the number line. (Limit: 100th)	J
6. Perform operations of addition and subtraction with decimal numerals.	D, E, F, J
7. Round off decimal numerals. (Limit: 1 000ths)	
8. Multiply and divide decimal numerals by powers of 10.	
9. Perform operations of multiplication and division with decimal numerals.	B, C, D, E, F,
10. Convert fractions to decimals by dividing the numerator by the denominator. (Limit: denominators of 100)	D
11. Convert decimals to fractions. (Limit: repeating zero and 3rds)	
12. Use symbols and notation illustrated below. (Use fractions and decimals)	

(a) Multiplication
  (i) $0.25 \times 0.1 = (0.25)(0.1)$
  (ii) $0.3y = (0.3)(y) = 0.3(y)$

(b) Division
  (i) $\frac{0.5}{0.4} = 0.5 \div 0.4 = 0.4\overline{)\,.5} =$
  (ii) $\frac{0.2y}{0.5} = \frac{0.2y}{0.5} = 0.2y \div 0.5 = \frac{0.2}{0.5}(y)$

OBJECTIVES — UNIT III — Reference Section

Decimals Cont'd.

13. Evaluate expressions by substituting for the variable. (Use fractions, decimals and one variable).

14. Solve the following type of conditions. (Use fractions and decimals).

    (a) $A + x = B$ (c) $Ax + Bx = C$
    (b) $Ax = B$ (d) $Ax + B = C$

15. Verify solutions of conditions by substitution.

16. Write English sentences for mathematical sentences.

17. Write mathematical sentences for English sentences.

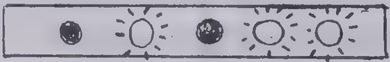
18. Solve problems using an organized approach. (Use decimals and fractions) — G, H (Unit IV)

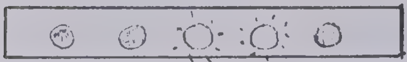
UNIT III - NUMBER THEORY APPLICATIONS

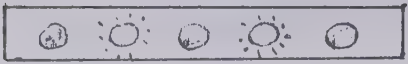
A. Base Two

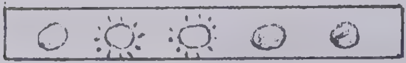
1. Computers use base two (binary arithmetic) to perform calculations electronically. Write the digits from 1 to 20 in base two.

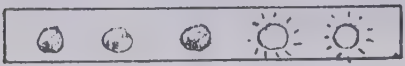
2. In computers, a light 'on' represents a one, and a light 'off', a zero. What base 10 number would each of the following represent:

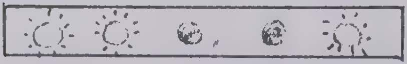
e.g.  represents 1011 base two or eleven.

 =

 =

 =

 =

 =

3. (a) Learn to add and multiply in base two, then complete the following tables and try the problems shown.

+	0	1
0		
1		

x	0	1
0		
1		

$$\begin{array}{r} 110 \\ \times\ 11 \\ \hline \end{array} \qquad \begin{array}{r} 1011 \\ 110 \\ +\ \ 11 \\ \hline \end{array} \qquad \begin{array}{r} 1101 \\ \times\ 101 \\ \hline \end{array}$$

$$\begin{array}{r} 11 \\ 110 \\ 1001 \\ +\ \ 11 \\ \hline \end{array} \qquad \begin{array}{r} 10110 \\ \times\ 11 \\ \hline \end{array} \qquad \begin{array}{r} 10101 \\ 1101 \\ +\ \ 11 \\ \hline \end{array}$$

(b) Check your results by converting the above problems to base 10.

4. Try to build binary adder or binary counter. See "Application Problems for Set Theory" for a plan for an electrical binary adder. Binary adders and counters can also be made mechanically without the use of electrical circuits. Could you design one? Can you find a plan for one in a book?

5. Additional applications can be found in:

SRA Math Applications Kit under

Appeteasers #10
Everyday Things #28

"From Fingers to Computers", Franklin Math Series

"The Roman Abacus", pp. 18 to 38

"Binary System", pp. 80 to 93

"Patterns and Puzzles in Mathematics", Franklin Math Series

"Secret Code", pp. 90, 91, 92

"Fraction Fun", pp. 64 to 67

B. Comparative Buying

1. For comparison shopping, find the unit cost for each of the following and determine the better buy:

   (a) A 3 kg bag of flour at 93¢ per bag or a 5 kg bag of flour at $1.35 per bag.

   (b) A 60 g jar of instant coffee at 77¢ per jar or a 150 g jar of instant coffee at $2.39 per jar.

   (c) A 1 ℓ bottle of orange juice at 63¢ per bottle or a 700 ml bottle of orange juice at 49¢ per bottle.

   (d) A 2 kg box of detergent for $1.89 or a 500 gm. box of detergent for 69¢.

   (e) Prune juice, 1 ℓ can at 48¢ or a 240 ml can at 18¢.

   (f) Fruit juice, 1 ℓ can at 41¢ or a 400 ml can at 24¢.

   (g) 360 gm glass of jelly at 34¢ or 1 kg jar at 89¢.

   (h) 864 gm of mayonnaise at 45¢ or 254 gm at 28¢.

2. Which is the better buy?

   Compare the prices of the different sizes of one brand of toothpaste at a local store. Which size is the most economical? By how much?

3. Try to determine which is the better size to buy for at least one other item in the store? Is the larger size always the most economical?

C. Find the Unit Price

1. Find the unit price (price per single item) in each of the following:

   (a) 4 tires sell for $140. ______

   (b) 10 pears sell for 89¢. ______

   (c) 6 oranges sell for 45¢. ______

   (d) 3 cans of corn sell for 79¢. ______

2. Find the unit price (price per kg) in each of the following:

   (a) 2 kg of onions sell for $1.49. ______

   (b) 1/2 kg of mushrooms sell for 98¢. ______

   (c) 5 kg of potatoes sell for $1.09. ______

   (d) 300 gm of tomatoes sell for 25¢. ______

3. Find the unit price (price per litre) in each of the following:

   (a) Milk, 4 ℓ carton sells for $1.23. ______

   (b) Paint, 5 ℓ can sells for $2.99. ______

PRICE LIST

D.

Groceries - Canned Goods

Corn 25¢ a can
Flour 2 kg for 67¢
Peaches 39¢ a can
Peas 23¢ a can
Pineapple 41¢ a can
Soap 14¢ a bar
Soap powder 81¢ a pkg.
Soups 2 cans for 41¢
String beans 25¢ a can
Sugar 450 g for 49¢
Tomatoes 2 cans for 43¢
Tomato juice 29¢ a can

Meats and Fish

Flounder $1.98/kg
Haddock $2.50/kg
Lamb $2.69/kg
Rib Roast $3.09/kg
Sirloin $3.72/kg
Smoked ham $1.80/kg
Chicken $1.75/kg
Veal $2.45/kg

Dairy

Butter $1.09/450 g
Eggs 95¢ a doz.
Milk 38¢/ℓ

Bakery

Assorted Cakes $1.20 a doz.
Bread 37¢ a loaf
Rolls 72¢ a doz.

Fruit and Produce

Potatoes 5 kg for 98¢
Carrots 16¢ a bunch
Peas 45¢/kg
Celery 29¢ a stalk
Beets 10¢ a bunch
Tomatoes 70¢/kg
Onions 2 kg for 69¢
Oranges 85¢ a doz.
Apples 1 kg for 47¢
Grapefruit 3 for 31¢
Pears 23¢/400 g
Grapes 35¢/500 g
Peaches 27¢/500 g
Lemons 80¢ a doz.

Using the above price list, find the cost of each of the following orders:

1. 1 doz. eggs
½ doz. rolls
2 cans peas
1 can pineapple
3 bars soap ________

2. 2 cans corn
900 g sirloin
2 ℓ milk
5 kg potatoes ________

3. 1 box soap flakes
800 g flounder
2 kg flour
700 g peas
6 cans soup ________

4. 3 cans string beans
22 kg sugar
2 kg lamb
1 doz. cakes
1 can peaches
2 bunches carrots ________

5. 4.6 kg chicken
2 stalks celery
500 g tomatoes
4 cans soup
1½ doz. eggs
1.5 kg apples ________

6. 3 cans tomato juice
4 grapefruit
2 loaves bread
2.4 kg rib roast
3 kg grapes
6 bars soap ________

7. 26 kg ham
900 g butter
1¼ doz. rolls
½ doz. cakes
1 kg peas
4 bunches beets
3 doz. oranges
5 kg flour __________

8. 3 ℓ milk
7.2 kg sugar
3.4 kg veal
750 g haddock
800 g pears
3 kg potatoes
3 kg onions
2½ doz. eggs __________

9. 2 cans tomatoes
900 g peas
9 lemons
1.6 kg sirloin
½ doz. cakes
1 can soup
2 cans pineapple
1350 g butter __________

10. What is the price of 100 g of chocolate if you buy a 20¢ bar or a 49¢ bar?

E. Finding the Total Cost

1. Andrew bought a new suit costing \$69.50; shoes, \$14,98; hat, \$7.75; shirt, \$6.98; and tie, \$2.98. How much did his new wardrobe cost?

2. Joan's father gave her \$90 to spend for new clothing. If she bought a dress for \$24.98, shoes for \$12.50, hat for \$4.95, bag for \$8.49, and a coat for \$31.75, how much money did she have left?

3. Find the cost of your wardrobe for the last 12 months.

F.

MAKING CHANGE

For each of the following sales write the number of each note and coin you would use in making the necessary change. Check your answers.

	Articles Sold	Amount Offered In Payment		CHANGE						
				COINS					NOTE	
				1¢	5¢	10¢	25¢	50¢	$1	$5
1.	4 ice cream cones @ 15¢	80¢	1.							
2.	700 g meat @ $1.10/kg	$2.00	2.							
3.	2 doz. oranges @ 79¢	$5.00	3.							
4.	3 pr. stockings @ $1.25	$10.00	4.							
5.	1 set dishes @ $19.49	$20.00	5.							
6.	8 l paint @ $1.09	$15.00	6.							
7.	2 l varnish @ $2.19	$9.00	7.							
8.	2½ doz. eggs @ 89¢	$5.00	8.							
9.	500 g butter @ 139/kg	75¢	9.							
10.	1 suit @ $74.50	$80.00	10.							
11.	1 pr. shoes @ $17.45	$20.00	11.							
12.	3 m gabardine @ $3.95	$15.00	12.							
13.	2.75 m velvet @ $8.95	$25.00	13.							
14.	4.5 m gingham @ $1.98	$10.00	14.							
15.	15 m^2 flooring @ $2.98	$50.00	15.							
16.	12 m^2 broadloom @ $18.95	$230.00	16.							
17.	2 shirts @ $6.75, 1 tie @ $3.50	$25.00	17.							
18.	3 cans fruit @ 32¢, 2 cans soup @ 19¢	$1.50	18.							
19.	2 reams paper @ $3.50	$10.00	19.							
20.	5 kg potatoes @ 9¢/kg, 500 g peas @ 42¢/kg	$1.00	20.							
21.	4 tires @ $24.99	$100.00	21.							
22.	4 pr. curtains @ $5.96	$25.00	22.							
23.	1 dress @ $22.95, 1 dress @ $17.95	$45.00	23.							
24.	2 record albums @ $3.79	$10.00	24.							
25.	1 coat @ $34.79, 1 suit @ $64.50	$100.00	25.							
26.	2 pr. overalls @ $7.79, 3 pr. socks @ 89¢	$20.00	26.							
27.	1 pr. slacks @ $9.50, 2 blouses @ $4.98	$20.00	27.							
28.	4 chairs @ $11.98, 1 table @ $24.49	$80.00	28.							
29.	12 drinking glasses @ 35¢	$5.00	29.							
30.	1 kg nails @ 72¢, 1 hammer @ $2.95	$5.00	30.							
31.	3 sheets @ $2.83, 8 pillow cases @ $1.50	$25.00	31.							
32.	1 doll house @ $6.77, 6 sets furniture @ $3.49	$30.00	32.							
33.	1 purse @ $5.95, 1 hat @ $4.98	$15.00	33.							
34.	5 bats @ $3.50, 9 baseballs @ $2.75	$50.00	34.							
35.	10 rose bushes @ $2.49	$25.00	35.							
36.	7 storm windows @ $23.75									
37.	2 storm doors @ $57.50	$300.00	36.							

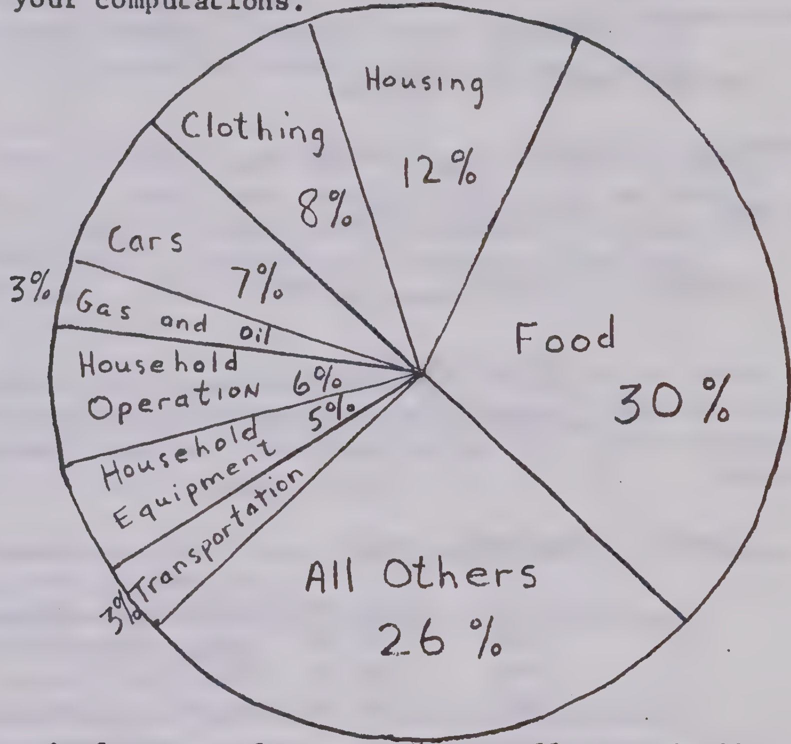
G. Using Circle Graphs

Study the circle graph at the right below. It shows how the American people spend the money they have left after taxes and savings are deducted.

1. What percent is represented by the complete circle? ______

Complete the chart at the left below showing how much of its net income of $5500 an average family spends on the following items. Use the work space at the bottom of the page for your computations.

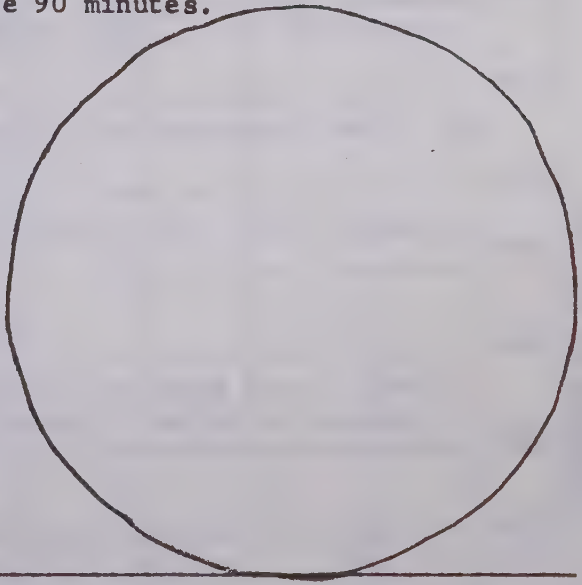
	Item	Amount
2.	Food	______
3.	Housing	______
4.	Clothing	______
5.	Automobiles	______
6.	Gasoline and Oil	______
7.	Household Equipment	______
8.	Household Operation	______
9.	Transportation	______
10.	All others	______



Jane has 90 minutes between morning and afternoon classes. She usually spends 30 minutes walking home and back, 20 minutes eating lunch, 15 minutes resting, and 25 minutes talking to her friends and playing. In the circle at the right below construct a circle graph to show how Jane spends the 90 minutes.

First complete the chart below. Then use a protractor to mark off the degrees, and complete the graph.

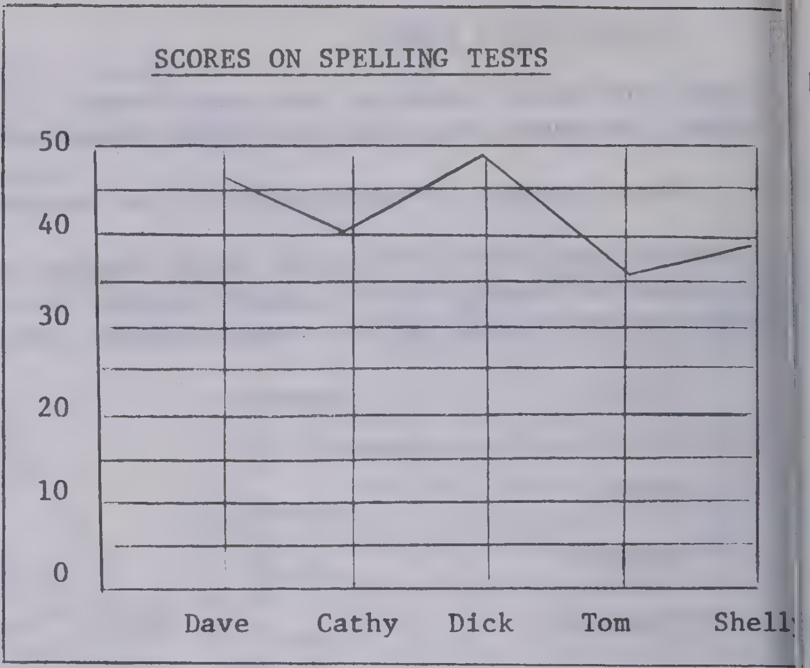
	Activity	Fraction	Degrees
11.	Walking	$\frac{30}{90} = \frac{1}{3}$	$\frac{1}{3}$ of 360° = ______
12.	Eating		
13.	Resting		
14.	Playing		



Work space

J. Using Line Graphs

The line graph at the right shows the test scores of some pupils on a 50-word spelling test. To tell how many words John had right, find his name. Follow the vertical line above John's name until it meets the graph line. This point is a little above the horizontal line marked 45. Dave had 46 words right.



1. How many words were spelled

correctly by Cathy? _____ words.

By Tom? _____ words. By Sally?

_____ words. By Dick? _____ words.

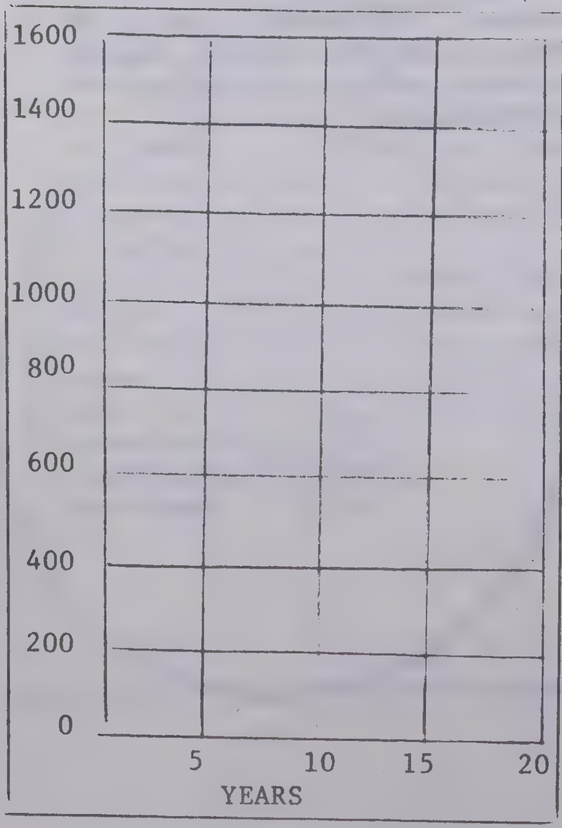
The table below gives some interesting information about a systematic savings plan. A family decided to save $5 weekly and place it in a savings account that paid interest at the rate of 7%. The table shows how the total amount grows when interest is added to savings.

Years	You Save	Total
1	$ 260.	$ 287.93
2	520.	816.08
3	780.	1114.96
4	1040.	1564.46

2. Use the ruled paper at the left below to help you make a double line graph. Plot one line to show the amount of money saved at $5 per week. Plot anothe line to show the total (amount saved plus earnings). Make this line a dotted line. The first part of each graph line has been drawn to show you how.

The distance between the two graph lines represent the amount of earnings on savings. You can estimate this amount for any period by estimating the amount represented by the distance between the two graph lin at that period.

3. Estimate the amount of earnings for each period and write it in the chart below.



Period	Amount of Earnings		Difference
	Estimate	Exact	
5 years			
10 years			
15 years			
20 years			

You can find the exact amount of earnings for any per by subtracting the amount saved from the total.

4. Find the exact amount of earnings for each period and write it in the chart above.

5. Find the differences between your estimates and the exact amounts and write them in the chart.

K. What Does A Car Cost?

Material needed: Newspaper with classified car ads.

Procedure: From a classified ad pick a car you might like to buy. Assume 5% of the price is the down payment if one is not given. Compute the balance, which is the remainder of the cost after you make the down payment.

To pay the balance, a person usually makes a payment every month for 1½, 2, 2½, or 3 years. The size of the payment depends on the balance that he owes.

But there is another cost. There is extra work when a car is sold on credit, and there is the risk that the buyer won't pay. So the buyer pays an extra amount - the interest - which is added to each monthly payment. The interest might be 15% of the balance each year. Suppose the balance is 2500 and payments will be made for two years. Then the total amount to pay would be:

$$\underset{\substack{\uparrow \\ \text{Balance}}}{\$2500} \quad + \quad \underset{\substack{\uparrow \\ \text{No. of Years}}}{2} \quad \text{X} \quad \underset{\substack{\uparrow \\ \text{Interest For Each Year}}}{(15\% \text{ of } \$2500)}$$

That equals: $2500 + 2 X $375, or $3250 (total cost)

To pay this in equal monthly payments in two years, you would pay $135 a month.

1. From the balance on the car you picked, find the interest for each year. Find the total cost for 1½, 2, 2½ and 3 years.

2. What does the monthly payment for each total cost? Which is the lowest monthly payment? Which is the lowest total cost? Which would be best and why?

3. Talk to some drivers or a car salesman. Find out about how many kilometres it can go on a litre of gas. What does gas cost? What would the monthly gas cost be?

4. How often does a car need oil, grease, tune-ups? What does each cost? How much would each cost per month?

5. What does insurance cost? What does a license cost? What would these cost per month?

6. What would be the total monthly cost of a car?

7. Can you think of extra expenses you might have if you owned a car? How much would it cost to lease a car? Which is better - buying or leasing a car?

L. Wages

1. Mr. Jones works 40 hours each week. If he receives $7.60 per hour, how much does he earn each week?

1. ____________

2. Find the weekly earnings for each of the following:

2. ____________

____

____

No. of hours	32	40	37
Rate per hour	$4.25	$3.96	$7.64

3. If the overtime rate is 1½ times the regular rate, what is the hourly overtime rate corresponding to each of the following regular hour rates: (a) $6.80? (b) $5.56? (c) $3.15?

3. ____________

____

____

4. Find the weekly earnings (time and a half rate over 40 hr.) for each of the following:

4. ____________

____

____

No. of hours	42	45	50½
Rate per hour	$3.90	$6.14	$4.80

Using "Career Trends" from the Guidance Department or Library, choose an occupation or career; determine your weekly wages.

HISTORY

LEVEL 7

UNIT III

RATIONAL NUMBERS

REFERENCES

The Committee recommends the following references as primary sources of information for Junior High School teachers and students. We suggest that those books labelled (T) be available as teacher references and those labelled (S) be available in quantities of 3 - 5 for class use. Many of these books may be in your library now and extra copies may be borrowed from the Library Service Centre.

These references are numbered (1 - 14) for referral in the following outline:

1. Adler, Irving. *The Giant Golden Book of Mathematics*. Golden Press, New York, 1966
   510 Ad 59g (S)

2. Adler, Irving. *Readings in Mathematics* (book 1). Ginn and Company, Lexington, Massachusetts, 1972.
   510 Ad 59r (S)

3. Adler, Irving. *Readings in Mathematics* (book 2). Ginn and Company, Lexington, Massachusetts, 1972.
   510 Ad 59r (S)

4. Bell, E.T. *Men of Mathematics*. Simon and Schuster, New York, 1966.
   920 B 4134 (T)

5. Bergamini, David. *Mathematics* (Life Science Library). Time Inc., New York, 1966.
   510 B 452 (T) and (S)

6. Denholm, Richard A. *Mathematics: Man's Key to Progress* (Book A) Franklin Publications Inc., Chicago, 1970
   (S)

7. Denholm, Richard A. *Mathematics: Man's Key to Progress* (Book B) Franklin Publications Inc., Chicago, 1970.
   (S)

8. Halacy, Dan. *Charles Babbage: Father of the Computer*. Crowell-Collier Press, Toronto, Ontario, 1970.
   921 B 113h (T) or (S)

9. Hogben, Lancelot. The Wonderful World of Mathematics. Doubleday and Company, Inc., Garden City, N.Y. 1955
510 H 679 (S)

10. Muir, Hane. Of Men and Numbers. Dodd, Mead and Co., New York, 1963
920 M 896 (S)

11. Ripley, R.D. and Tait, George, E. Mathematics Enrichment. Copp Clark Publishing Company, Toronto, 1966 (S)

12. Rogers, James T. Story of Mathematics for Young People. Pantheon Books, Random House Inc., Toronto, 1966.
510.09 R 632 (S)

13. Shaw, H. Alan and Fuge, Keri. The Story of Mathematics. Fletcher and Son Ltd., Norwich, Great Britain, 1963.
510.09 S h 26 (S)

14. Terry, Leon. The Mathmen. McGraw-Hill, New York, 1964.
510.09 T 279 (S)

SUPPLEMENTARY REFERENCES

(These are additional references for teachers)

Fadiman, Clifton, Fantasia Mathematics, Simon and Schuster, New York, 1958.

James & James, Mathematics Dictionary, 3rd ed., D. Van Nostrand Company, Inc., Toronto, 1968.

519 K58 King, Amy C. and Read, Cecil B. Pathways to Probability, Holt, Rinehart and Winston, Inc., New York, 1963.

Marks, Robert W. The New Mathematics Dictionary and Handbook. Bantam Books, Inc., New York, 1964.

512 N213 N.C.T.M. Historical Topics in Algebra. National Council of Teachers of Mathematics, Washington, D.C., 1971.

Newman, James R. The World of Mathematics. (Vol. 1, 2, 3, 4) Simon and Schuster, New York, 1956.

Smith, D.E. History of Mathematics. (Vol. 1,2) Dover Publications, Inc., New York, 1958.

920 T849 Turnbull, H.W. The Great Mathematicians. New York University Press, New York, L969.

Black, Gerald J. Canada Goes Metric. Doubleday Canada Ltd., Toronto, 1974.

Posters

1. Walch, J.W. (Publisher) "Posters on Famous Mathematics". Available on loan from the Library Service Centre.

2. I.B.M., Timeline "Men of Mathematics", available from I.B.M. Library, Calgary. Ask for Item #5050003. (Free)

Busts

"Mathematicians of the Century", available from Moyer. Available on loan from the Library Service Centre. (Price $48.00)

Movies

CK 10591 "Possibly So Pythagoras". Available on loan from Instructional Aids Department.

CK 538 "Donald Duck in Math Magic Land". Instructional Aids.

Games

1. Euclid. Western Educational Activities. For advanced students.

The resource list on Posters, Busts, Movies and Games was taken from Men of Mathematics - A Resource Unit developed by J. Barnes.

E. T. V. Math Series (Produced in Ontario). Available from Central Office.

Tape #3 part (a) Square Root: Newton's Method. (Time 20 min., 275 ft.)

Useful for introducing square roots in grades 8 or 9.

Tape #5 part (b) History of Computers.

Useful as a motivational unit.

Tape #5 part (f) Number Systems.

Useful for introducing number theory, grade 7.

Tape #6 part (a) History of Numerals

Useful in grade 7 whole numbers.

Tape #6 part (b) History of π.

Grade 9 geometry.

Tape #6 part (c) From Time to Time

Development of calendar.

Tape #6 part (f) History of India(n) Mathematics

Laid the basis for our present number system and useful in History of Math in an option.

Tape #7 part (a) Inverse Variation

Grade 9 functions.

Tape #7 part (b) Graphs

Grade 8 coordinate system (Descarte).

Tape #9 part (a) Fibonacci Sequence

Grade 8 real numbers.

Tape #9 part (b) The Divine Proportion: Golden Section

Grade 9 geometry.

Tape #9 part (c) Map Making

Useful for upper ability students in grade 9 solid geometry.

Tape #10 part (c) What are Numbers

History of development of number systems. Useful as an introduction to grade 7 number systems.

LEVEL 7 RATIONAL NUMBERS

UNIT III

OBJECTIVES	Reference Activities
A. Factors and Multiples	
Students should be able to:	
1. Maintain all previously developed skills and ideas: primes, composites, factors, multiples, and divisibility.	1
2. List the set of factors for whole numbers (Limit: 100)	2, 3
3. Write the prime factorization for whole numbers. (Limit: 100)	7
*4. Obtain the G.C.F. for a set of four whole numbers. (Limit: 100)	
*5. Obtain the L.C.M. for a set of four whole numbers. (Limit: 100)	
*6. Determine if a pair of whole numbers is relatively prime. (e.g. prime factorization) (Limit: 100)	
	4, 5 6

UNIT III RATIONAL NUMBERS

A. NUMBER THEORY (Factors and Multiples)

RESOURCES

Pythagoras (567 - 497 B.C.) - Prime and Composite Numbers, Male and Female numbers, Perfect numbers, Triangular numbers and Pythagorean triples are a few of his interests.

Reference #14, pages 57-66
Reference #13, pages 20-23
Reference #10, pages 7-8

Leonard Fibonacci (1170 - 1250) - Introduced Algebra and Arabic numbers to Italy in 1202. Fibonacci sequence.

Reference #3, pages 59-61
Reference #10, page 28
Reference #1, pages 30-32

Blaise Pascal (1623 - 1662) - Known for study of chance and probability.

Reference #1, pages 71-73
Reference #7, pages 41-60
Reference #12, pages 84-89
Reference #10, pages 94-95
Reference #4, pages 73-89

Eratosthenes (274 - 194 B.C.) - Prime number sieve.

Reference #1, pages 16-17
Reference #12, pages 58-60
Reference #14, pages 189-191

Euclid (350 - 275 B.C.) - Made important contributions to Theory of Numbers.

Reference #14, pages 157-162

Pierre de Fermat (1601 - 1665) - Theory of prime numbers and development of probability.

Reference #12, page 84
Reference #5, pages 128-147

Mesopotamian numbers - a base 60 system.

Reference #5, pages 16-17
Reference #4, pages 56-72

UNIT III RATIONAL NUMBERS

A. NUMBER THEORY (Factors and Multiples)

ACTIVITIES

1. Pythagoras divided numbers into two groups, male numbers and female numbers.

   What numbers are male?

   Which are female?

   Why was 5 for marriage?

   Why do some people consider 7 and 11 to be lucky?

   The Mathman (reference #14) page 61
   The Story of Mathematics (reference #13) page 20

2. (a) Pythagoreans thought some numbers were friendly (amicable). For example he thought 220 and 284 were friendly. Why?

   (b) Show that the following are friendly:

   1184 and 1210
   2620 and 2924
   5020 and 5564
   6232 and 6368

   The Mathmen (reference #14) pages 62-63
   Of Men and Numbers (reference #10) pages 7-8

3. Pythagoras made the exciting discovery of perfect numbers.

   (a) What are perfect numbers?

   (b) Give a few examples.

   (c) Deficient numbers are numbers whose factors added together (excluding itself) are less than the number and Abundant numbers are numbers whose factors added together (excluding itself) are greater than itself.

   List the abundant and deficient numbers from 1 to 30. Look for patterns.

   The Mathmen (reference #14) pages 61-62

4. Figurate numbers are numbers that have shapes. The Pythagoreans spent considerable time studying these numbers. Make a poster to illustrate figurate numbers.

The Mathmen (reference #14) pages 63-66
The Story of Mathematics (reference #13) page 22
The Giant Golden Book of Mathematics (reference #1) pages 18-20

5. Read about Fibonacci numbers from at least two of the following references and make a poster to illustrate:

Applications of this sequence in nature or
the sequence and spirals or
patterns in the sequence or
the sequence and the golden section (rectangle) or
the sequence and art

Reading in Mathematics Book 2 (reference #3) pages 57-65
The Giant Golden Book of Mathematics (reference #1) pages 30-32
Mathematics Enrichment (reference #11) pages 120-125
Mathematics (reference #5) pages 92-97

6. (a) Construct Pascal's triangle. Find patterns in the triangle and list applications of the triangle.

(b) Make a pinball machine and demonstrate how Pascal's triangle can be used to predict the outcome. (see reference #11)

(c) Pascal made a machine which was the ancestor of modern desk calculators. Could you make a working model of this machine?

The Story of Math for Young People (reference #12) pages 84-89
Of Men and Numbers (reference #10) pages 94-96
The Giant Golden Book of Math (reference #1) pages 71-73
Mathematics (reference #5) pages 144-145
Men of Mathematics (reference #4) pages 87-89
Mathematics Enrichment (reference #11) pages 129-130

7. Eratosthenes invented a prime number sieve. Make a chart to illustrate the sieve.

The Mathmen (reference #14) pages 189-190
The Giant Golden Book of Mathematics (reference #1) page 17

8. Some ancient countries used number systems with bases other than 10.

   (a) Learn to write the numbers from 1 - 50 in another base. e.g. base 2 or base 5.

   (b) The Babylonians used a base 60 system. Can you write numbers in base 60? What would be some advantages of using such a large base? Some disadvantages? What influence does base 60 have on our present systems of measurement.

   (c) Complete the investigation on page 46 of Math: Man's Key to Progress (reference #6).

   References:

   Math: Man's Key to Progress (reference #6) pages 44-46
   The Giant Golden Book of Math (reference #1) page 13
   Mathematics (reference #5) pages 16, 17

9. (a) The magic! Magic square in the painting on the following page was invented by Albrech Durer (1471 - 1528). The numbers do not show clearly so it is reproduced below. Find as many ways as you can of getting the magic number (34) in this square.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

   (b) Durer also managed to slip in the year he completed the painting in the magic square. What was the year?

   Refer to:

   Mathematics: A Human Endeavor (reference #15) p.35

LEVEL 7

RATIONAL NUMBERS

UNIT III

OBJECTIVES	Reference Activities
B. Fractional Numerals	
Students should be able to:	
1. Maintain previously developed skills and ideas: numerators, denominators, basic fraction, equivalent fractions, mixed numerals, improper fractions, and the order of fractions.	2
2. Demonstrate need for fractional numbers (e.g. solve condition of the type $3x = 8$)	1
3. Recognize fractional numerals of the form $\frac{a}{b}$ as: a. part of a whole b. part of a group c. a measure d. a ratio e. division	
* 4. Demonstrate that the whole numbers are a subset of the fractional numbers using Venn diagrams.	
5. Demonstrate knowledge of the fractional numbers by plotting a given set on a number line.	6
6. Demonstrate the use of identity elements and zero property. (De-emphasize terminology)	
7. Generate sets of equivalent fractions. (Limit: 5)	
*8. Find the L.C.D. (e.g. using L.C.M.)	
9. Reduce any fraction to its basic form (e.g. using G.C.F.)	
10. Perform the operations of addition and subtraction with fractional numbers. (Limit: 1 digit denominators)	5
11. Demonstrate the use of reciprocals to obtain the identity element.	
12. Perform the operations multiplication and division with fractional numbers. (Limit: 1 digit denominators)	
13. Demonstrate that division by zero is undefined.	
14. Evaluate an expression by using the properties to produce short cuts in computation. (De-emphasize terminology) e.g. $\frac{1}{2} + \frac{1}{4} + \frac{1}{2} = 1 + \frac{1}{4} = 1\frac{1}{4}$	

15. Evaluate expressions involving order of operations

    e.g. $\left(\frac{3}{4} \times \frac{4}{5} + \frac{6}{12} \div \frac{3}{4} = \right)$

16. Order fractional numerals using the symbols $<, >, =$.

C. Decimals

Students should be able to:

1. Maintain previously developed skills and ideas: change fractions to decimals by use of equivalent fractions, place value to 1 000ths. 4

2. Write any decimal numeral in expanded form. (use exponential notation only if previously taught).

3. Write any decimal numeral in standard form given expanded form. (use exponential notation only if previously taught).

4. Order decimal numerals using the symbols $<, >, =$.

5. Demonstrate knowledge of decimals by plotting a given set on the number line. (Limit: 100th)

6. Perform operations of addition and subtraction with decimal numerals.

7. Round off decimal numerals. (Limit: 1 000 ths)

8. Multiply and divide decimal numerals by powers of 10.

9. Perform operations of multiplication and division with decimal numerals.

10. Convert fractions to decimals by dividing the numerator by the denominator. (Limit: denominators of 100).

11. Convert decimals to fractions. (Limit: repeating zero and 3rds.)

12. Use symbols and notation illustrated below. (Use fractions and decimals)

    (a) Multiplication
    (i) $0.25 \times 0.1 = (0.25)(0.1)$
    (ii) $0.3y = (0.3)(y) = 0.3(y)$

    (b) Division
    (i) $\frac{0.5}{0.4} = 0.5 \div 0.4 = 0.4\overline{)\,.5} =$
    (ii) $\frac{0.2y}{0.5} = \frac{0.2}{0.5}y = 0.2y \div 0.5 = \frac{0.2}{0.5}(y)$

13. Evaluate expressions by substituting for the variable. (Use fractions , decimals and one variable.)

Reference Activities

14. Solve the following type of conditions (Use fractions and decimals)

(a) $A + x = B$ (c) $Ax + Bx = C$

(b) $Ax = B$ (d) $Ax + B = C$

15. Verify solutions of conditions by substitution.

16. Write English sentences for mathematical sentences.

17. Write mathematical sentences for English sentences.

18. Solve problems using an organized approach. (Use decimals and fractions).

UNIT III RATIONAL NUMBERS (NON-NEGATIVE)

FRACTIONS AND DECIMALS

RESOURCES

Blaise Pascal (1623 - 1662) - First calculating machine - could add, subtract, multiply and divide.

Reference #10, Page 88

John Napier (1550 - 1616) - The first man to use the decimal point in its modern context.

Reference #5, Page 85
Reference #5, Page 18

Egyptian Fractions

Reference #9, Page 15

UNIT III RATIONAL NUMBERS (NON-NEGATIVE)

FRACTIONS AND DECIMALS

ACTIVITIES

1. Egyptian Fractions were always one part of something.

   a) 1 digit was what fraction of a palm?

   b) 1 palm was what fraction of a cubit?

   c) What symbols were used to represent these two fractions?

   Reference #9, page 15 Wonderful World of Mathematics

2. Using encyclopedia or "History of Mathematics" by D. E. Smith, explore the development of fractions in Babylonian times, Greek times, Roman times, Chinese times up to present times.

3. Give an application of Sexagesimal

   Reference #5, pages 15 - 18 Life Science Mathematics

4. When did the decimal points in its modern context come into being? Who is given the credit?

   Reference #5, Page 18

5. Construct a slide rule for addition and subtraction of fractions. Activity #41 Mathematics for Enrichment

   Reference #11, Page 46

6. Draw a line to represent 1 unit. Divide it in half and label the point ½. Divide remaining portion in half and label the new point.

   Repeat five times labelling each new point.

LEARNING PACKAGE

LEVEL SEVEN

UNIT III

RATIONAL NUMBERS

UNIT III - RATIONAL NUMBERS

PERFORMANCE OBJECTIVES

A. Factors and Multiples:

Students should be able to:

1. Maintain all previously developed skills and ideas: primes, composites, factors, multiples, and divisibility.
2. List the set of factors for whole numbers. (Limit: 100)
3. Write the prime factorization for whole numbers. (Limit: 100)

*4. Obtain the G.C.F. for a set of four whole numbers. (Limit: 100).

*5. Obtain the L.C.M. for a set of four whole numbers. (Limit: 100).

*6. Determine if a pair of whole numbers is relatively prime. (e.g. prime factorization.) (Limit: 100).

B. Fractional Numerals:

Students should be able to:

1. Maintain previously developed skills and ideas: numerators, denominators, basic fractions, equivalent fractions, mixed numerals, improper fractions, and the order of fractions.
2. Demonstrate the need for fractional numbers (e.g. solve condition of the type 3x = 8).
3. Recognize fractional numerals of the form $\frac{a}{b}$ as:
   (a) part of a whole
   (b) part of a group
   (c) a measure
   (d) a ratio
   (e) division

*4. Demonstrate that the whole numbers are a subset of the fractional numbers using Venn diagrams.

5. Demonstrate knowledge of the fractional numbers by plotting a given set on a number line.
6. Demonstrate the use of identity elements and zero property. (De-emphasize terminology)
7. Generate sets of equivalent fractions. (Limit: 5).

B. *8. Find the L.C.D. (e.g. using L.C.M.)

9. Reduce any fraction to its basic form. (e.g. using G.C.F.)

10. Perform the operations of addition and subtraction with fractional numbers. (limit: 1 digit denominators.)

11. Demonstrate the use of reciprocals to obtain the identity element.

12. Perform the operations multiplication and division with fractional numbers. (Limit: 1 digit denominators.)

13. Demonstrate that division by zero is undefined.

14. Evaluate an expression by using the properties to produce short cuts in computation. (De-emphasize terminology)

e.g. $\frac{1}{2} + \frac{1}{4} + \frac{1}{2} = 1 + \frac{1}{4} = 1\frac{1}{4}$

15. Evaluate expressions involving order of operations.

e.g. $(\frac{3}{4} \times \frac{4}{5} + \frac{6}{12} \div \frac{3}{4} =)$

16. Order fractional numerals using the symbols: <, >, =.

C. Decimals:

Students should be able to:

1. Maintain previously developed skills and ideas; change fractions to decimals by use of equivalent fractions, place value to 1 000ths.

2. Write any decimal numeral in expanded form. (Use exponential notation only if previously taught.

3. Write any decimal numeral in standard form given expanded form. (Use exponential notation only if previously taught.)

4. Order decimal numerals using the symbols: <, >, =.

5. Demonstrate knowledge of decimals by plotting a given set on the number line. (Limit: 100th)

6. Perform operations of addition and subtraction with decimal numerals.

7. Round of decimal numerals. (Limit: 1 000ths)

8. Multiply and divide decimal numerals by powers of 10.

9. Perform operations of multiplication and division with decimal numerals.

C. 10. Convert fractions to decimals by dividing the numerator by the denominator. (Limit: deonominators of 100.)

11. Convert decimals to fractions. (Limit: repeating zero and 3rds.)

12. Use symbols and notation illustrated below. (Use fractions and decimals.)

(a) Multiplication (i) $0.25 \times 0.1 = (0.25)(0.1)$

(ii) $0.3y = (0.3)(y) = 0.3(y)$

(b) Division (i) $\frac{0.5}{0.4} = 0.5 \div 0.4 = 0.4\overline{).5} =$

(ii) $\frac{0.2y}{0.5} = \frac{0.2}{0.5}\, y = 0.2y \div 0.5 = \frac{0.2}{0.5}\,(y)$

13. Evaluate expressions by substituting for the variable. (Use fractions and decimals and one variable.)

14. Solve the following type of conditions. (Use fractions and decimals).

(a) $A + x = B$ (c) $Ax + Bx = C$

(b) $Ax = B$ (d) $Ax + B = C$

15. Verifty solutions of conditions by substitution.

16. Write English sentences for mathematical sentences.

17. Write mathematical sentences for English sentences.

18. Solve problems using an organized approach. (Use decimals and fractions.)

DEVELOPMENT AND EXERCISES

STRAND: *Rational Numbers* LEVEL: *7*

UNIT: *III* OBJECTIVE NUMBER: *A-1*

OBJECTIVE: *Maintain all previously developed skills and ideas: primes, composites, factors, multiples, and divisibility.*

SUGGESTED DEVELOPMENT:

1. A factor is a whole number which divides exactly into a second whole number.

   EXAMPLE: Find all factors of 72.

   Numbers that divide 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72.

   NOTE: *Factors are written in ascending order. When finding all factors it is useful to have students list them in pairs. Numbers that divide 72: 1, 72, 2, 36, 3, 24, 4, 18, 6, 12, 8, 9.*

2. Develop a table to list the factors of the whole numbers from 1 to 20. The students will use the table to identify those numbers having 1, 2, 3 or more factors.

   DEFINITION: (a) *Prime number - a prime number has exactly 2 factors (one and itself).*

   (b) *Composite number - a composite number has 3 or more factors.*

   NOTE: *1 is neither prime nor composite since it has only one factor.*

3. Have the students count by 3's up to 30, count by 5's to 50, and by 11's to 121. Define these sets to be the multiples of 3, 5, 11, respectively.

   Point out that the same result could be obtained by multiplying by the whole numbers in order.

4. Do Eratosthenes Sieve to find prime numbers less than 100.

5. Develop tests of divisibility for certain whole numbers.

   (a) 2 - all even numbers.

   (b) 3 - if the sum of the digits is divisible by 3.

   (c) 4 - if the number formed by the last two digits is divisible by 4.

   (d) 5 - if the number ends in 0 or 5.

   (e) 6 - if the number is even and the sum of the digits is divisible by 3.

(f) 8 - if the number is even and the number formed by the last 3 digits is divisible by 8.

(g) 9 - if the sum of the digits is divisible by 9.

(h) 10 - if the number ends in 0.

(i) 11 - if the difference between the sums of alternate digits is zero or a multiple of 11.

e.g. 131: 1 + 1 = 2, 2 - 2 = 0

9394: 9 + 9 = 18, 3 + 4 = 7, 18 - 7 = 11

NOTE: *Knowledge of these tests of divisibility will assist students in computations involved in the rest of the unit.*

6. Other Suggestions:

(a) Test Goldbach's conjecture: All even numbers greater than two can be written as the sum of a pair of prime numbers: 4 = 2 + 2, 6 = 3 + 3, 8 = 5 + 3,
10 = 7 + 3 or 5 + 5

(b) Look for and list twin primes. Twin primes are two prime numbers that have exactly one composite number between them. e.g. 3 and 5, 5 and 7, 11 and 13.

PRIME NUMBERS

DEFINITION

A NUMBER THAT HAS
EXACTLY 2 FACTORS, ONE AND ITSELF

P = THE SET OF PRIME NUMBERS
$P = \{ 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 \ldots \}$

COMPOSITE NUMBERS

DEFINITION

A NUMBER WITH 3 OR MORE FACTORS

$4 = 2 \times 2 \times 1$ $\quad 6 = 2 \times 3 \times 1$ $\quad 8 = 2 \times 2 \times 2 \times 1$

$9 = 3 \times 3 \times 1$ $\quad 56 = 2 \;\; 2 \times 2 \times 7 \times 1$

THE NUMBER ONE IS NEITHER
PRIME NOR COMPOSITE AS
IT ONLY HAS ONE FACTOR

THERE IS ONLY ONE EVEN PRIME
NUMBER, THE NUMBER TWO

FACTOR

DEFINITION

A FACTOR IS A WHOLE NUMBER WHICH DIVIDES EXACTLY INTO A SECOND WHOLE NUMBER

EG. $\left.\begin{array}{l} 6 \div 2 = 3 \\ 6 \div 3 = 2 \end{array}\right\} \therefore$ 2 AND 3 ARE FACTORS OF 6

FIND ALL THE FACTORS OF 24

NUMBERS THAT DIVIDE 24

$= \{ \qquad\qquad\qquad \}$

MULTIPLES

DEFINITION

A NUMBER THAT HAS ANOTHER NUMBER AS A FACTOR IS A MULTIPLE OF THAT NUMBER

EG. 2 IS A FACTOR OF 4, 6, 8

SO 4, 6, 8 ARE MULTIPLES OF 2

$\therefore$ THE SET OF MULTIPLES

OF $\{2 = 2, 4, 6, 8, 10, 12, \ldots\}$

WRITE THE SET OF MULTIPLES OF 6

$= \{ \qquad\qquad\qquad \}$

RAT 7 OBJ A-1

1. (a) List 5 multiples of 3. { 3, 6, 9, 12, 15, ···}
   (b) List 5 multiples of 7. { 7, 14, 21, 28, 35, ···}
   (c) List 5 multiples of 2. {2, 4, 6, 8, 10, ···}
   (d) Find and list ten numbers which are multiples of both 2 and 5.
   (e) Find and list ten numbers which are multiples of both 2 and 3.

Note: Answers may vary considerably.

(d) {10, 20, 30, 40, 50, 60, 70, 80, 90, 100, ···}
(e) {6, 12, 18, 24, 30, 36, 42, 48, 54, 60, ···}

2. Write each of the following as a product of two factors. Try to factor in two ways.
   e.g. 36 = 6 x 6 or 36 = 9 x 4
   (a) 14 1×14, 2×7 (b) 24 1×24, 2×12, 3×8, 4×6 (c) 8 1×8, 2×4 (d) 15 1×15, 3×5
   (e) 21 1×21, 3×7 (f) 35 1×35, 5×7 (g) 45 1×45, 3×15, 5×9 (h) 96 1×96, 2×48, 3×32, 4×24, 6×16, 8×12

3. (a) Which of these numbers are divisible by 2, 3, 4 or 5? (Do this by using the test for divisibility.)
   15, 36, 17, 45, 50, 91, 32, 196, 100
   2 - 36, 50, 32, 196, 100
   3 - 15, 36, 45
   4 - 36, 32, 196, 100
   5 - 15, 45, 50, 100
   (b) Which of these numbers are divisible by 6, 8, 10, or 11?
   6, 48, 20, 121, 143, 184, 102, 176, 2002, 33 000.
   6 - 6, 48, 102, 33000
   8 - 48, 184, 176, 33000
   10 - 20, 33000
   11 - 121, 176, 143, 2002, 33000

4. Identify each of the following numbers as prime or composite. List three factors of all composite numbers. Do not use one as a factor.
   e.g. 12, composite, 12, 6, 2
   7, prime

   Note: Answers may vary

   (a) 6 C - 2, 3, 6 (b) 19 P (c) 918 C - 2, 3, 6, 9, 17, 18, 27, 34, 51, 54, 102, 153, 306, 459, 918 (d) 121 C - 1, 11, 121
   (e) 13 P (f) 57 C - 3, 19, 57 (g) 91 (h) 71 P

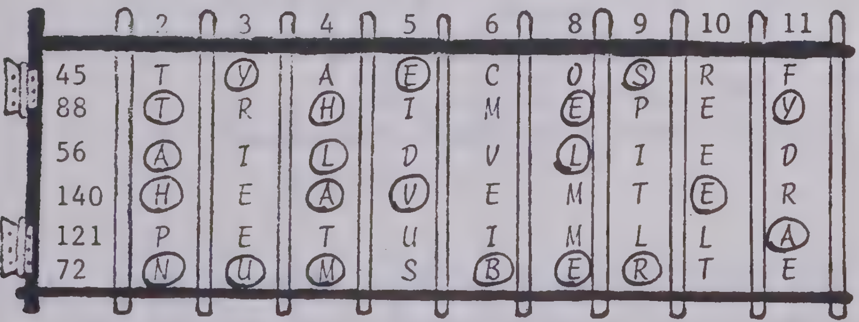
   (g) 91 - C - 7, 13, 91

5. Which number is a factor of all numbers?
   one

6. Using the tests of divisibility determine the factors of each of the numbers in the left hand column.

   a. Circle the letters in the appropriate columns to find the answer to the following question.

CAN YOU TELEPHONE PRISONERS IN JAIL?



	2	3	4	5	6	8	9	10	11
45	T	Ⓨ	A	Ⓔ	C	O	Ⓢ	R	F
88	Ⓣ	R	Ⓗ	I	M	Ⓔ	P	E	Ⓨ
56	Ⓐ	I	Ⓛ	D	V	Ⓛ	I	E	D
140	Ⓗ	E	Ⓐ	Ⓥ	E	M	T	Ⓔ	R
121	P	E	T	U	I	M	L	L	Ⓐ
72	Ⓝ	Ⓤ	Ⓜ	S	Ⓑ	Ⓔ	Ⓡ	T	E

b. Unscramble the letters which were not circled in each line to answer the following questions.

Line 1 - 6 is a factor of 96
Line 2 - Two is the only even prime
Line 3 - Knowing these tests will help you divide
Line 4 - An SI unit of linear measure. metre
Line 5 - 14 is a multiple of 7 and 2
Line 6 - A definite collection of objects set

DEVELOPMENT AND EXERCISES

STRAND: Rational Numbers
LEVEL: 7
UNIT: III
OBJECTIVE NUMBER: A-2
OBJECTIVE: List the set of factors for whole numbers. (Limit: 100)

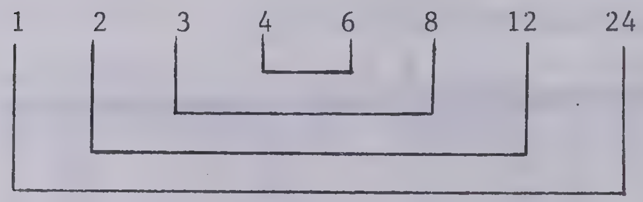
SUGGESTED DEVELOPMENT:

1. Point out to students one way to find all of the factors of a number is to begin with 1 and use the whole numbers in order until the factors begin to repeat, or until you arrive at that given number.

   e.g.
   $$\begin{array}{c|c}\downarrow & \uparrow\\ 1 \times & 24 = 24\\ 2 \times & 12 = 24\\ 3 \times & 8 = 24\\ 4 \times & 6 = 24\end{array}$$

   Notice when you try 6, the factors begin to repeat. The set of factors of 24 is 1, 2, 3, 4, 6, 8, 12, 24. To list the factors in order go down the first column and up the second (see arrows in above example). If a number is a square do not list the square root twice.

2. Another way to obtain all the factors of a number is to list the factors in pairs. An example would be 3 and its factor pair 8.



Each pair of factors has been joined by lines. All numbers will be paired except numbers which are squares of a whole number. In this case, the square root will not be paired with another number.

FACTORS
OF A NUMBER

TO LIST ALL THE NUMBERS
THAT ARE FACTORS OF A NUMBER:

1. FIND THE PRIME FACTORIZATION
2. COMBINE THE PRIME NUMBERS IN ALL POSSIBLE COMBINATIONS

TO LIST ALL THE FACTORS OF **72**

$72 = 2 \times 2 \times 2 \times 3 \times 3$

1 - FACTOR OF EVERY NUMBER
2 - PRIME FACTOR
3 - PRIME FACTOR
4 - 2×2
6 - 2×3
8 - $2 \times 2 \times 2$
9 - 3×3
12 - $2 \times 2 \times 3$
18 - $2 \times 3 \times 3$
24 - $2 \times 2 \times 2 \times 3$
36 - $2 \times 2 \times 3 \times 3$
72 - $2 \times 2 \times 2 \times 2 \times 3$

1. For each number, list the set of factors. Then write P if the number is prime, or C if it is composite.

(a) 14 1,2,7,14 C (b) 24 1,2,3,4,6,8, 12,24 C (c) 8 1,2,4,8 C (d) 16 1,2,4,8,16

(e) 4 1,2,4 C (f) 5 1,5 P (g) 10 1,2,5,10 C (h) 3 1,3 P

2. For each number list the set of factors.

(a) 6 1,2,3,6 (b) 15 1,3,5,15 (c) 18 1,2,3,6,9,18 (d) 12 1,2,3,4,6,12

(e) 25 1,5,25 (f) 21 1,3,7,21 (g) 26 1,2,13,26 (h) 30 1,2,3,5,6,10,

(i) 245 1,5,7,35, 49,245 (j) 360 1,2,3,4,5,6,8,9,10,12,15,18,20,24,30,36,40,45,60,72, 90,120,180,360

3. Use the set of whole numbers between 20 and 31. List the factors of each number. Tell whether the number is prime or composite.

A factor is a whole number which divides exactly into a second whole number.

e.g. 36

1 x 36 The factors of 36 are:
2 x 18 1, 2, 3, 4, 6, 9, 12, 18, 36
3 x 12
4 x 9
6 x 6

Stop when a factor reoccurs

21 - 1,3,7,21 C
22 - 1,2,11,22 C
23 - 1,23 P
24 - 1,2,3,4,6,8,12,24 C
25 - 1,5,25 C
26 - 1,2,13,26 C
27 - 1,3,9,27 C
28 - 1,2,4,7,14,28 C
29 - 1,29 P
30 - 1,2,3,5,6,10,15,30 C

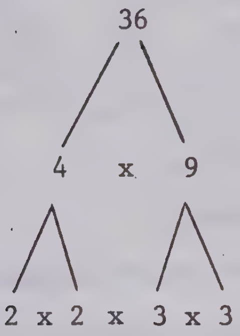
DEVELOPMENT AND EXERCISES

TRAND: *Rational Numbers* LEVEL: 7

NIT: *III* OBJECTIVE NUMBER: A-3

BJECTIVE: *Write the prime factorization for whole numbers. (Limit: 100)*

SUGGESTED DEVELOPMENT: METHOD 1: Tree Diagram



The tree is complete when the last row contains only prime numbers.

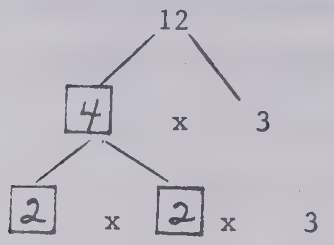
METHOD 2: Repeated Division Method

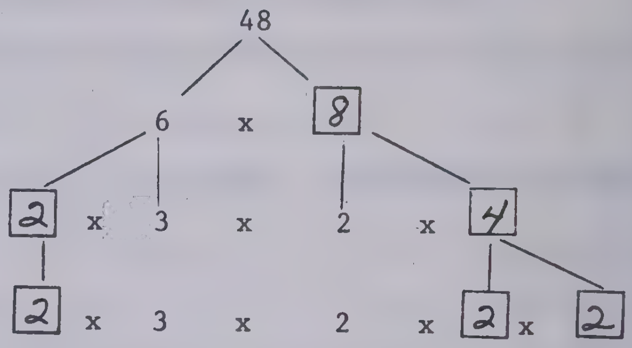
2/ 36
2/ 18
3/ 9
3/ 3
1

Stress attempting to divide the number which is being factorized by the least possible prime first, then use successive primes.

1. Complete the following tree diagrams:

(a)





The prime factorization of 12 is 2 × 2 × 3

The prime factorization of 48 is 2 × 2 × 2 × 2 × 3

2. Using the tree diagram method, prime factorize the following numbers:

(a) 42 2 × 3 × 7 (b) 72 2 × 2 × 2 × 3 × 3 (c) 27 3 × 3 × 3 (d) 84 2 × 2 × 3 × 7

(e) 255 3 × 5 × 17

Note: Tree diagrams may vary

3. Complete the following by placing the correct number in the blanks provided:

(a)
2 ⌊56
2 ⌊28
2 ⌊14
7 ⌊7
1

(b)
2 ⌊126
3 ⌊63
3 ⌊21
7 ⌊7
1

(c)
3 ⌊441
7 ⌊147
3 ⌊21
7 ⌊7
1

4. Using repeated division, prime factorize the following:

(a) 78 (b) 144 (c) 484 (d) 525

(e) 990 (f) 936

(a)
2 ⌊78
3 ⌊39
13 ⌊13
1

(b)
2 ⌊144
2 ⌊72
2 ⌊36
2 ⌊18
3 ⌊9
3 ⌊3
1

(c)
2 ⌊484
2 ⌊242
11 ⌊121
11 ⌊11
1

(d)
3 ⌊525
5 ⌊175
5 ⌊35
7 ⌊7
1

or

5 ⌊525
5 ⌊105
3 ⌊21
7 ⌊7
1

(e)
2 ⌊990
3 ⌊495
3 ⌊165
5 ⌊55
11 ⌊11
1

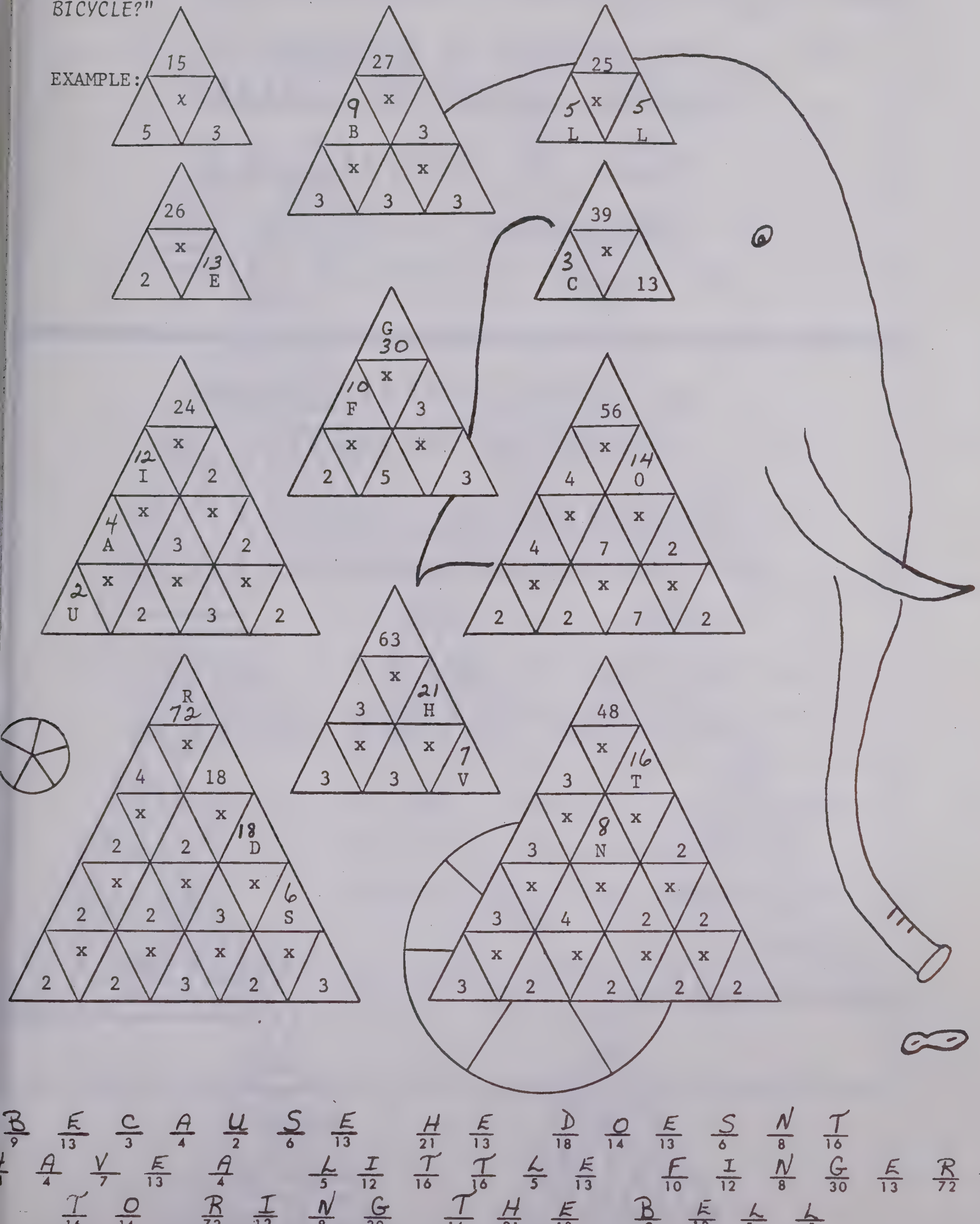
or

2 ⌊990
5 ⌊495
3 ⌊99
3 ⌊33
11 ⌊11
1

(f)
2 ⌊936
2 ⌊468
2 ⌊234
3 ⌊117
3 ⌊39
13 ⌊13
1

Find the numbers which would be used to replace the letters in the following factor tents.

Use the number code to discover . . . *"WHY CAN'T AN ELEPHANT RIDE A BICYCLE?"*


EXAMPLE:
15 x 5 3
27 x 9 B 3 x x 3 3 3
25 5 x 5 L L
26 x 2 13 E
39 3 x C 13
G 30 x 10 F 3 x x 2 5 3
24 x 12 I 2 x x 4 A 3 2 x x x 2 U 2 3 2
56 x 4 14 O x x 4 7 2 x x x 2 2 7 2
63 x 3 21 H x x 7 3 3 V
R 72 x 4 18 x x 2 2 18 D x x x 2 2 3 6 S x x x x 2 2 3 2 3
48 x 3 16 T x 8 x 3 N 2 x x x 3 4 2 2 x x x x 3 2 2 2 2

B E C A U S E   H E   D O E S N T
9 13 3 4 2 6 13   21 13   18 14 13 6 8 16

A V E   A   L I T T L E   F I N G E R
4 7 13   4   5 12 16 16 5 13   10 12 8 30 13 72

T O   R I N G   T H E   B E L L
16 14   72 12 8 30   16 21 13   9 13 5 5

6. Determine the prime factorization of each number in the list and place its factorization in an appropriate factor tent.

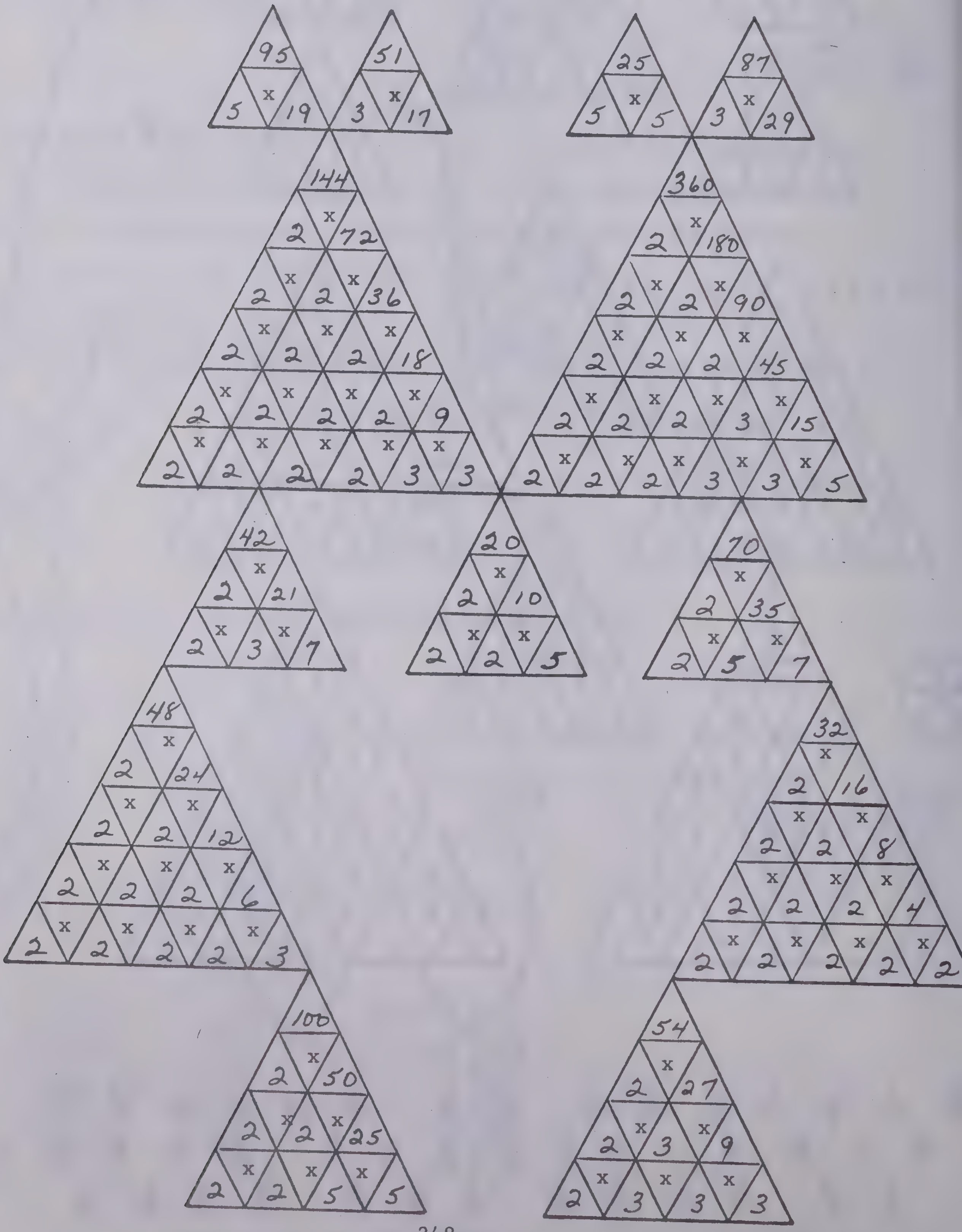
95 32 51

144 25

20 42 54

70 48

87 100 360



PRIME FACTORIZATION

DEFINITION

TO EXPRESS A NUMBER AS A PRODUCT OF PRIME NUMBERS

EG. 24 = 2 × 2 × 2 × 3

2 × 2 × 2 × 3 IS THE PRIME FACTORIZATION OF 24

PRIME FACTORIZATION BY REPEATED DIVISION

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

- CHOOSE THE SMALLEST PRIME THAT IS A FACTOR OF THE NUMBER
- DIVIDE BY THAT PRIME AS MANY TIMES AS POSSIBLE
- CHOOSE THE NEXT SMALLEST PRIME THAT IS A FACTOR
- REPEAT THE PROCESS UNTIL ONE REMAINS

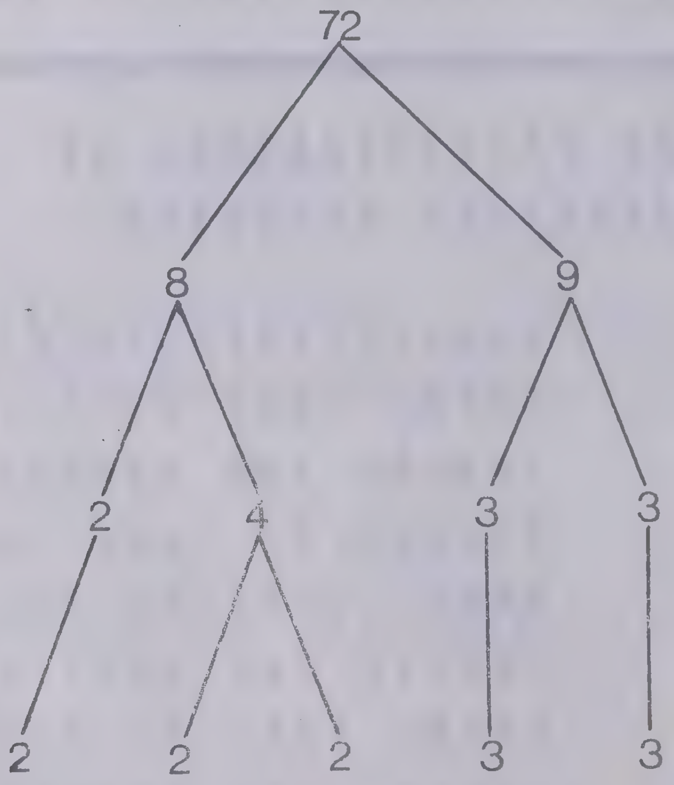
THE PRIME FACTORIZATION OF 72 IS 2 × 2 × 2 × 3 × 3

PRIME FACTORIZATION

BY TREE DIAGRAMS

1. WRITE THE NUMBER AS A PRODUCT OF TWO NUMBERS

2. CONTINUE THIS PROCESS UNTIL ALL NUMBERS ARE PRIME



THE PRIME FACTORIZATION OF
72 = 2 × 2 × 2 × 3 × 3

RAT 7 OBJ. A-3

DEVELOPMENT AND EXERCISES

TRAND: *Rational Numbers* LEVEL: 7

NIT: III OBJECTIVE NUMBER: A-4

BJECTIVE: **Obtain the G.C.F. for a set of four whole numbers. (Limit: 100)*

SUGGESTED DEVELOPMENT: METHOD 1: List the set of factors of two composite numbers: e.g. 24 and 36.

24: 1, 2, 3, 4, 6, 8, 12, 24

36: 1, 2, 3, 4, 6, 9, 12, 18, 36.

List the set of common factors of 24 and 36: 1, 2, 3, 4, 6, 12. The greatest member of this set is 12. e.g. 12 is the G.C.F. of 24 and 36.

NOTE: G.C.F. must now be explained

METHOD 2:

$24 = \boxed{2} \times \boxed{2} \times 2 \times \boxed{3}$

$36 = \boxed{2} \times \boxed{2} \times 3 \times \boxed{3}$

2, 2, and 3 are the common factors

$\therefore 2 \times 2 \times 3 = 12$ is the G.C.F.

METHOD 3: Algorithm Method.

2 /24 36

2 /12 18

3 / 6 9

2 3 - relatively prime, hence process is complete.

NOTE: The divisors must be one or primes and common factors of all members.

G.C.F. is calculated by multiplying vertically.

$2 \times 2 \times 3 = 12$

This method can be extended to finding the G.C.F. of three and four numbers.

2 /42 126 140

7 /21 63 70

3 9 10

NOTE: The process ends when there is no factor common to all remainders.

G.C.F. $2 \times 7 = 14$

LEAST COMMON MULTIPLE (LCM)

- THE SMALLEST NUMBER THAT IS A MULTIPLE OF TWO OR MORE GIVEN NUMBERS IS THE LCM OF THOSE NUMBERS
- THE LCM OF 6 AND 8 IS 24

GREATEST COMMON FACTOR (GCF)

- THE LARGEST NUMBER THAT IS A FACTOR OF TWO OR MORE GIVEN NUMBERS IS THE GCF OF THOSE NUMBERS
- THE GCF OF 8 AND 20 IS 4

RELATIVELY PRIME

- IF THE GCF OF TWO NUMBERS IS ONE THEN THE TWO NUMBERS ARE RELATIVELY PRIME
- THE GCF OF 14 AND 25 IS 1 SO THEY ARE RELATIVELY PRIME

RAT 7 OBJ. A-4,5,6

G.C.F. - The largest number that is a factor of two or more given numbers is the G.C.F. of those numbers.
- The G.C.F. of 15 and 25 is 5

. Complete the following:

(a)
$$\begin{array}{r|rr} 2 & 30 & 72 \\ \hline 3 & 15 & \boxed{36} \\ \hline & \boxed{5} & 12 \end{array}$$

G.C.F. of 30 and 72 is: 6

(b)
$$\begin{array}{r|rrr} 5 & 75 & 100 & 125 \\ \hline \boxed{5} & 15 & 20 & \boxed{25} \\ \hline & 3 & \boxed{4} & \boxed{5} \end{array}$$

G.C.F. of 75, 100, 125 is: 25

(c)
$$\begin{array}{r|rrrr} \boxed{2} & 72 & 108 & 252 & 672 \\ \hline \boxed{2} & 36 & \boxed{54} & \boxed{126} & \boxed{336} \\ \hline \boxed{3} & \boxed{18} & \boxed{27} & 63 & \boxed{168} \\ \hline & 6 & 9 & 21 & 56 \end{array}$$

G.C.F. of 72, 108, 252, 672 is: 12

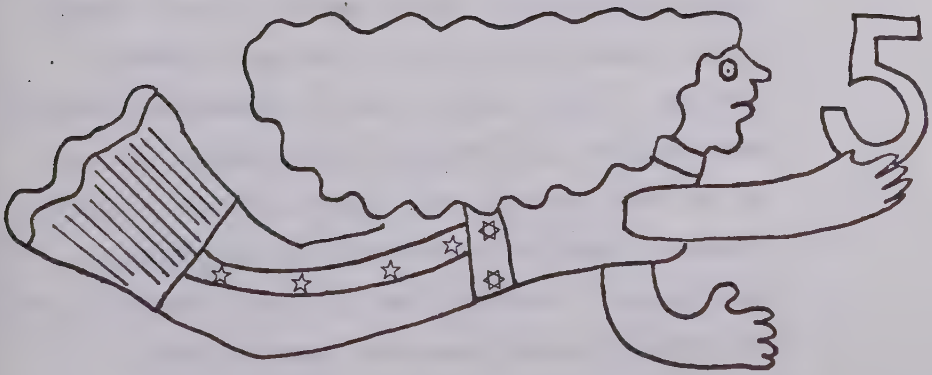
(d)
$$\begin{array}{r|rrr} \boxed{1} & 8 & 12 & 15 \\ \hline & \boxed{8} & \boxed{12} & 15 \end{array}$$

G.C.F. of 8, 12, 15 is: 1

. Find the G.C.F. of each set of numbers.

(a) 3, 6 3
(b) 12, 18 6
(c) 14, 21, 35 7
(d) 60, 84 12
(e) 27, 36, 72 9

(f) 36, 90, 210 6
(g) 24, 48, 132, 240 12
(h) 306, 936 18
(i) 37, 74, 148, 296 37



DEVELOPMENT AND EXERCISES

STRAND: Rational Numbers LEVEL: 7
UNIT: III OBJECTIVE NUMBER: A-5
OBJECTIVE: *Obtain the L.C.M. for a set of four whole numbers. (Limit: 100).

SUGGESTED DEVELOPMENT: METHOD 1:

1) Present students with two numbers and have them list the multiples.
2) Ask students to circle the common multiples.
3) Have students choose the Least Common Multiple. (L.C.M.)
4) Discuss why the L.C.M. is used for common denominators in fraction computation (i.e. ease of computation)

e.g. To find the L.C.M. of 6 and 8

Multiples of 6 = {6, 12, 18, ㉔, 30, 36, 42, ㊽, 54, 60 ...}

Multiples of 8 = {8, 16, ㉔, 32, 40, ㊽, 56, 64 ...}

Common Multiples: 24 and 48

Least Common Multiple: 24

METHOD 2: Prime Factorization.

Present the students with the following numbers: 16, 24, 32, and ask them to list all prime factors of each.

Circle all factors which appear in any three of the numbers, but which are not already circled. (List such a factor separately below).

Next, circle all factors which appear in any two of the numbers, but which are not already circled. Add such a factor to the separate list below. Finally, add to your separate list all factors which have not been circled.

L.C.M. is the product of the factors in the separate list.

e.g. 16 = ② x ② x ② x ②
24 = ② x ② x ② x 3
32 = ② x ② x ② x ② x 2

Factors common to three numbers: 2, 2, 2
Factors common to two numbers: 2,
Leftovers: 3, 2

L.C.M. 2 x 2 x 2 x 2 x 3 x 2 = 96

METHOD 3: Algorithm.

Use the algorithm for G.C.F. with two exceptions:

1) A factor may be listed in the vertical column if it is a factor of at least 2 numbers.
2) L.C.M. is calculated by finding the product of its vertical column and the horizontal column.

NOTE: If a number does not have the given factor, repeat the number in the next column.

EXAMPLE:

L.C.M. of 8, 18, 24, 32

2 /8 18 24 32
2 /4 9 12 16
2 /2 9 6 8 - Note carrying 9 to the next
3 /1 9 3 4 row.
1 3 1 4

L.C.M. 2 x 2 x 2 x 3 x 1 x 3 x 1 x 4 = 288

1. Use your knowledge of the L.C.M. method to fill in the boxes in each question below.

(a)
2 /12 28
2 /$\boxed{6}$ 14
3 $\boxed{7}$

L.C.M. of 12, 28: 84

(b)
3 /6 15 24
2 /$\boxed{2}$ 5 $\boxed{8}$
1 $\boxed{5}$ 4

L.C.M. of 6, 15, 24: 120

(c)
3 /12 $\boxed{15}$ 35
5 /$\boxed{4}$ 5 $\boxed{35}$
4 1 7

L.C.M. of 12, $\boxed{15}$, 35: 420

(d)
3 /9 $\boxed{21}$ 24
$\boxed{3}$ 7 $\boxed{8}$

L.C.M. of 9, $\boxed{21}$, 24: 504

(e)
2 /8 $\boxed{12}$ 20 $\boxed{30}$
2 /$\boxed{4}$ 6 $\boxed{10}$ 15
$\boxed{5}$ /$\boxed{2}$ 3 5 $\boxed{15}$
3 /2 3 1 3
$\boxed{2}$ 1 $\boxed{1}$ 1

L.C.M. of 8, ☐ , 20, ☐ : 120

(f)
$\boxed{2}$ /8 24 40
2 /4 12 20
2 /2 $\boxed{6}$ $\boxed{10}$
$\boxed{1}$ 3 5

L.C.M. of 8, 24, 40: 120

2. Find the L.C.M. of the following groups of numbers:
e.g. 12, 15, 24

3 /12, 15, 24
2 / 4, 5 8
2 / 2 5 4
1 5 2

3 x 2 x 2 x 1 x 5 x 2 = 120
L.C.M. = 120

(a) 45, 60 180
(b) 33, 22 66
(c) 16, 28, 40 560
(d) 15, 12, 6 60
(e) 8, 9, 70, 100 12600
(f) 21, 90, 33 6930
(g) 25, 100, 160, 1000 4000
(h) 11, 8, 32, 60 5280

L.C.M. Means Lowest Common Multiple

DEVELOPMENT AND EXERCISES

TRAND: Rational Numbers LEVEL: 7

NIT: III OBJECTIVE NUMBER: A-6

BJECTIVE: *Determine if a pair of whole numbers are relatively prime. (e.g. prime factorization) (Limit: 100)

SUGGESTED DEVELOPMENT:

1. DEFINITION: If the G.C.F. of two numbers is one, then the two numbers are relatively prime.

2. EXAMPLES: Using any of the methods of Objective A-3, show examples of relatively prime numbers.
   e.g. 16, 27; 4, 9; 23, 12; etc.

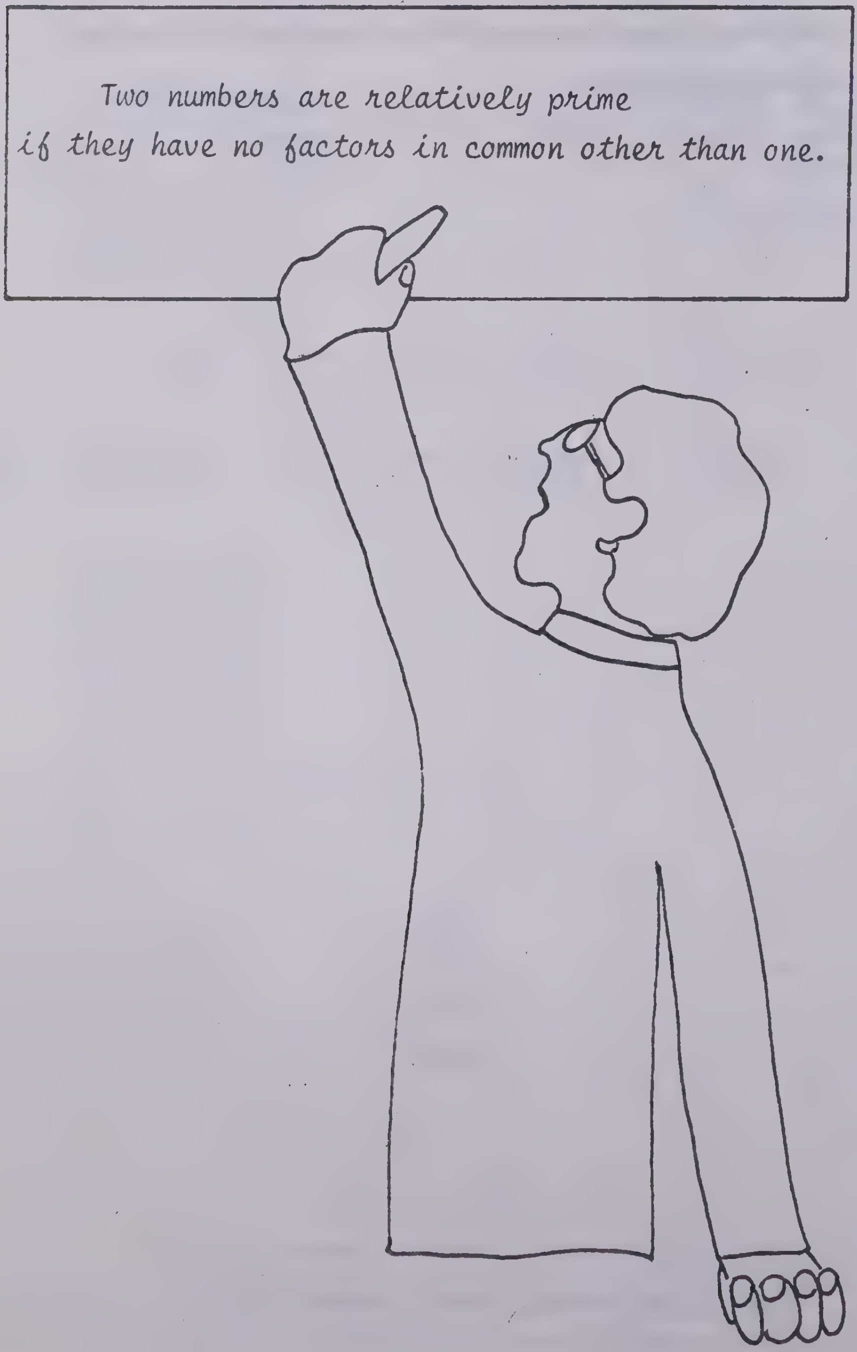
   Also show some examples of numbers which are not relatively prime.
   e.g. 9, 15; 8, 18; 210, 75; etc.

1. Determine whether or not the following pairs of numbers are relatively prime:

(a) 17, 14 relatively prime (b) 16, 48 not relatively prime
(c) 25, 49 relatively prime (d) 64, 97 relatively prime
(e) 21, 68 relatively prime (f) 36, 37 relatively prime
(g) 19, 57 not relatively prime (h) 69, 153 not relatively prime
(i) 17, 91 relatively prime (j) 184, 258 not relatively prime

2. From the above exercises, find the G.C.F. for those which are NOT relatively prime.

	G.C.F
(b) 16, 48 -	16
(g) 19, 57 -	19
(h) 69, 153 -	3
(j) 184, 258 -	2



Two numbers are relatively prime if they have no factors in common other than one.

DEVELOPMENT AND EXERCISES

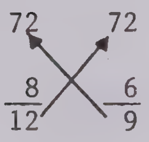
TRAND: *Rational Numbers* LEVEL: *7*

NIT: *III* OBJECTIVE NUMBER: *B-1*

BJECTIVE: *Maintain previously developed skills and ideas: denominators, basic fractions, equivalent fractions, mixed numerals, improper fractions, and the order of fractions.*

SUGGESTED DEVELOPMENT: DEFINITIONS:

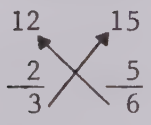
1. Numerators and Denominators: In any fraction $\frac{a}{b}$, the bottom number "b" tells how many equal parts a unit has been divided into. This is called the denominator. The number "a" refers to the number of parts we take. This is called the numerator.
2. Basic Fraction: If the numerator and denominator of a fraction have no common factors other than one, it is called a basic fraction.
3. Equivalent Fractions:
   (a) Two fractions that have the same basic fraction are called equivalent fractions.
   (b) Test for equivalence: Find the cross product.



Since 72 = 72,

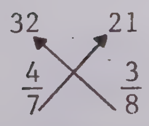
$\frac{8}{12}$ is equivalent to $\frac{6}{9}$

4. Improper Fractions: Any fraction whose numerator is greater than its denominator is called an improper fraction.
5. Order of Fractions:
   (a) To determine which of two fractions is larger, find cross-products.



$12 < 15$ so $\frac{2}{3} < \frac{5}{6}$

CAUTION: Ensure that the product is written above the fraction.



$32 > 21$ so $\frac{4}{7} > \frac{3}{8}$

   (b) Fractions can be put in order by getting equivalent fractions with common denominators.

$\frac{7}{8}$, $\frac{3}{4}$, $\frac{1}{2}$

Common denominator is 8.

$\frac{7}{8}, \frac{6}{8}, \frac{4}{8}$

Therefore, $\frac{4}{8} < \frac{6}{8} < \frac{7}{8}$

6. Mixed Numeral: Any improper fraction which is written as a whole number and a fraction is called a mixed numeral.

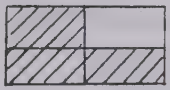
e.g. $\frac{27}{4} = 6\frac{3}{4}$

Improper Fraction	Mixed Numeral

EXERCISES: OBJECTIVE NO. B-1

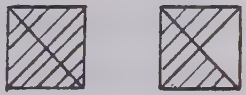
1. Below are five diagrams. Give the fraction that is represented by the shaded area.

(a)



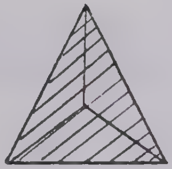
$\frac{3}{4}$

(d)



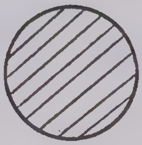
$\frac{7}{4}$

(b)



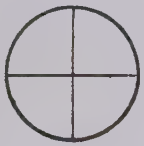
$\frac{3}{3}$

(e)



$\frac{1}{1}$

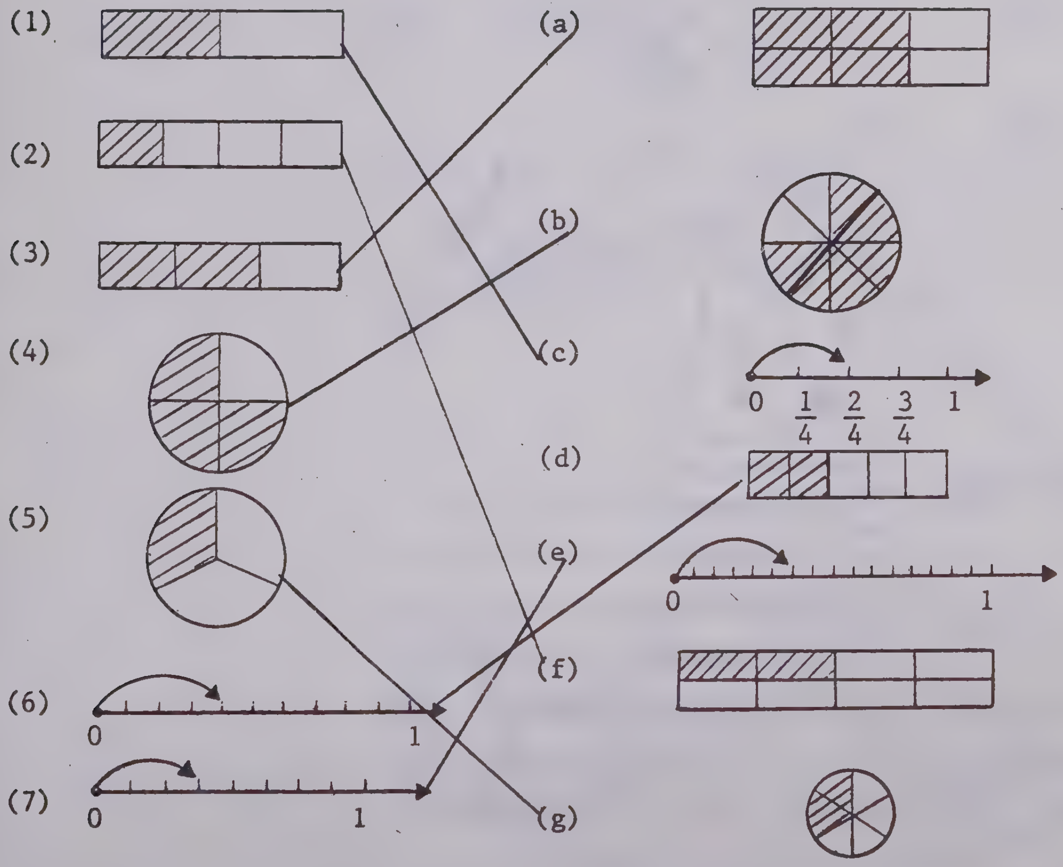
(c)



$\frac{0}{4}$

2. The numerator is 6. The denominator is 11. Make a fraction that would be represented by the above numbers. $\frac{6}{11}$

3. Draw a line matching two picture models to show equivalent fractions.



4. Decide whether the following are equivalent.

(a) $\frac{7}{8}, \frac{3}{4}$ No

(b) $\frac{0}{5}, \frac{0}{8}$ Yes

(c) $\frac{4}{3}, \frac{12}{9}$ Yes

(d) $\frac{2}{3}, \frac{6}{7}$ No

(e) $\frac{1}{4}, \frac{2}{7}$ No

5. Find the basic fraction for each.

(a) $\frac{27}{30}$ $\frac{9}{10}$

(b) $\frac{20}{25}$ $\frac{4}{5}$

(c) $\frac{14}{16}$ $\frac{7}{8}$

(d) $\frac{110}{121}$ $\frac{10}{11}$

(e) $\frac{12}{15}$ $\frac{4}{5}$

6. Write the following as mixed numbers.

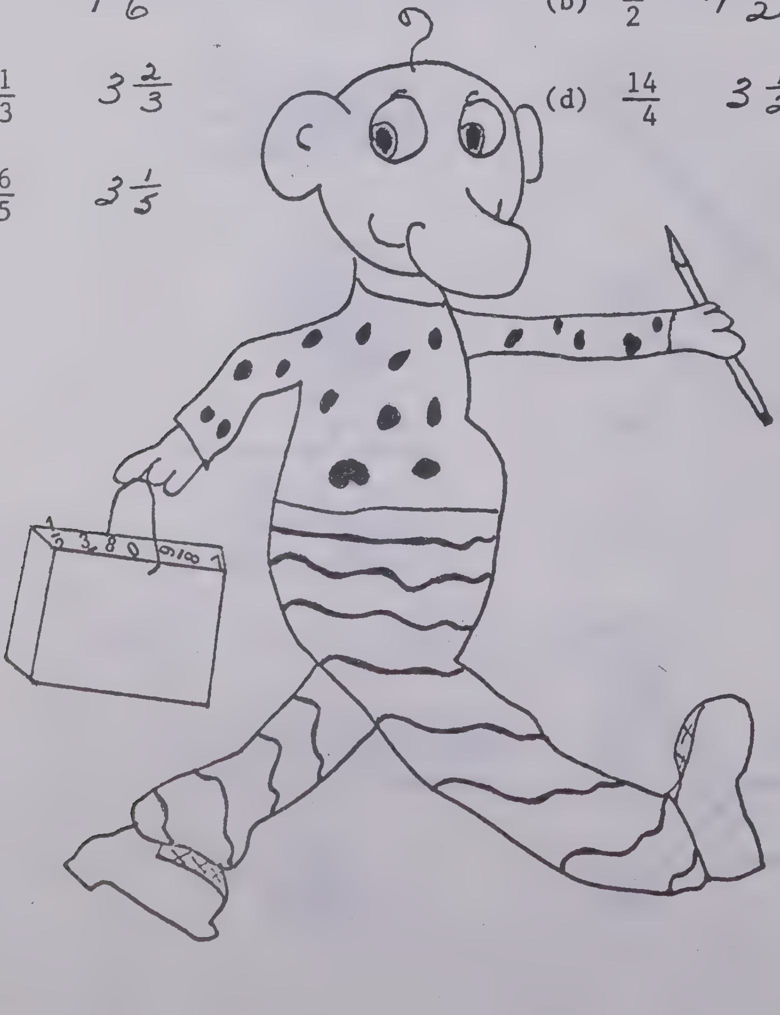
(a) $\frac{7}{6}$ $1\frac{1}{6}$

(b) $\frac{9}{2}$ $4\frac{1}{2}$

(c) $\frac{11}{3}$ $3\frac{2}{3}$

(d) $\frac{14}{4}$ $3\frac{1}{2}$

(e) $\frac{16}{5}$ $3\frac{1}{5}$



DEVELOPMENT AND EXERCISES

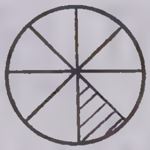
STRAND: Rational Numbers LEVEL: 7
UNIT: III OBJECTIVE NUMBER: B-2, B-3
OBJECTIVE: 2. Demonstrate need for fractional numbers (e.g. Solve conditions of the type 3x = 8)
3. Recognize fractional numerals of the form $\frac{a}{b}$ as (a) part of a whole, (b) part of a group, (c) a measure, (d) a ratio, and (e) division.

SUGGESTED DEVELOPMENT: Present the following conditions and ask the students to find the solutions.

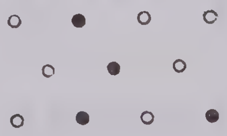
(a) $2x = 12$
(b) $3x = 15$
(c) $3x = 8$

Point out that the first two are solvable using whole numbers but the third is not. Hence, there is a need for fractional numbers.

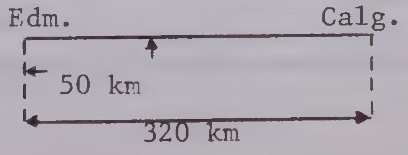
a) Present the class with the picture below. Develop through discussion that the fraction $\frac{1}{8}$ could be used to represent this picture. (See Objective B-1).



b) Present the class with the picture below. Develop through discussion that the fraction $\frac{4}{11}$ could be used to represent this picture. What other fraction is possible?



c) Present the following picture. Develop through discussion the idea that the location of the arrow could be represented as being $\frac{50}{300}$ of the distance from Edmonton to Calgary. i.e. Measurements are often made in terms of fractions. e.g. That house is two and $\frac{1}{2}$ times as large as this one. My brother is half as old as me.

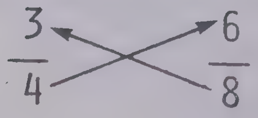


BASIC FRACTION

- A FRACTION IN WHICH THE NUMERATOR AND DENOMINATOR HAVE NO COMMON FACTORS OTHER THAN ONE
- $\frac{3}{4}$, $\frac{7}{8}$, $\frac{10}{13}$ ARE BASIC FRACTIONS
- $\frac{4}{6}$ IS NOT A BASIC FRACTION AS 4 AND 6 HAVE A COMMON FACTOR

EQUIVALENT FRACTIONS

- TWO OR MORE FRACTIONS THAT HAVE THE SAME BASIC FRACTION
- $\frac{6}{8}$, $\frac{75}{100}$, $\frac{9}{12}$ ARE EQUIVALENT FRACTIONS AS THEY ALL HAVE $\frac{3}{4}$ AS A BASIC FRACTION
- TWO FRACTIONS ARE EQUIVALENT IF THEIR CROSS PRODUCTS ARE EQUAL



$$3\,(8) = 4\,(6)$$

$$24 = 24$$

(d) Ask the students to write fractions to represent the following rates:

1) 10 marbles sell for 14¢.
2) 12 oranges sell for 99¢.
3) 160 km is covered in 3 hours.
4) 27 chickens lay 3 eggs every 5 hours.
5) Our school uses 250 sheets of paper for each student in one year.
6) 110 km/h

Point out that fractions are the numerical way of writing rates.

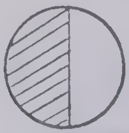
(e) Ask the class to write in symbols, "sixteen divided by 4."

Most students will write 16 ÷ 4 or $4\overline{)16}$. Point out that $\frac{16}{4}$ is also acceptable.

NOTE: *This must be stressed as it causes difficulties later.*

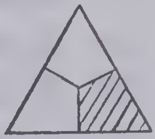
1. Tell what part of each is shaded using fractions of the form $\frac{a}{b}$.

(a)



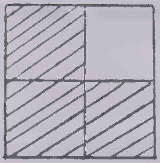
$\frac{1}{2}$

(d)



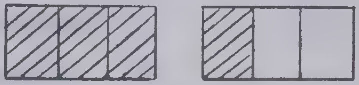
$\frac{1}{3}$

(b)



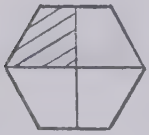
$\frac{3}{4}$

(e)



$\frac{4}{3}$

(c)



$\frac{1}{4}$

(f) In the fraction $\frac{a}{b}$, a is called the numerator and b is called the denominator

2. What fraction is expressed when:

(a) 50 of a group of 100 children are boys. $\frac{50}{100}$

(b) of 120 stamps, 37 are Canadian stamps. $\frac{37}{120}$

(c) Bob spent 18 of his 20 dollars. $\frac{18}{20}$

(d) 8 red markers out of a group of 120 markers. $\frac{8}{120}$

(e) 10 students in a class of 35 had their homework completed. $\frac{10}{35}$

Note: Answers have not been given in basic form so as to stress the concept of the use of fractions. Basic fractions would be considered correct.

3. What fraction is expressed when:

(a) a ruler is marked off with 10 parts to the centimetre and you consider 7 of them. $\frac{7}{10}$

(b) the large hand of a clock moves through 30 minutes. $\frac{30}{60}$

(c) 4 eggs out of a dozen are needed for a cake. $\frac{4}{12}$

(d) the point is half the distance between one and two on the number line. $\frac{3}{2}$

(e) if a container of water is divided into eight equal parts and four of them are poured out, how much is left? $\frac{4}{8}$

4. What fraction is expressed when:

(a) 2 cans of soup sell for 45¢. $\frac{2}{45}$ or $\frac{45}{2}$

(b) a car travels 240 kilometres in 3 hours. $\frac{240}{3}$ or $\frac{3}{240}$

(c) Paul has 300 stamps. John has 100. $\frac{300}{100}$ or $\frac{100}{300}$

(d) 150 people donated $1500. $\frac{150}{1500}$ or $\frac{1500}{150}$

(e) for every 10 people 3 have blue eyes. $\frac{3}{10}$ or $\frac{10}{3}$

. Write fractions as division. NOTE: What is being divided.

(a) One lunch must be shared by two friends. $\frac{1}{2}$

(b) 15 lockers must be shared by 45 girls. $\frac{15}{45}$

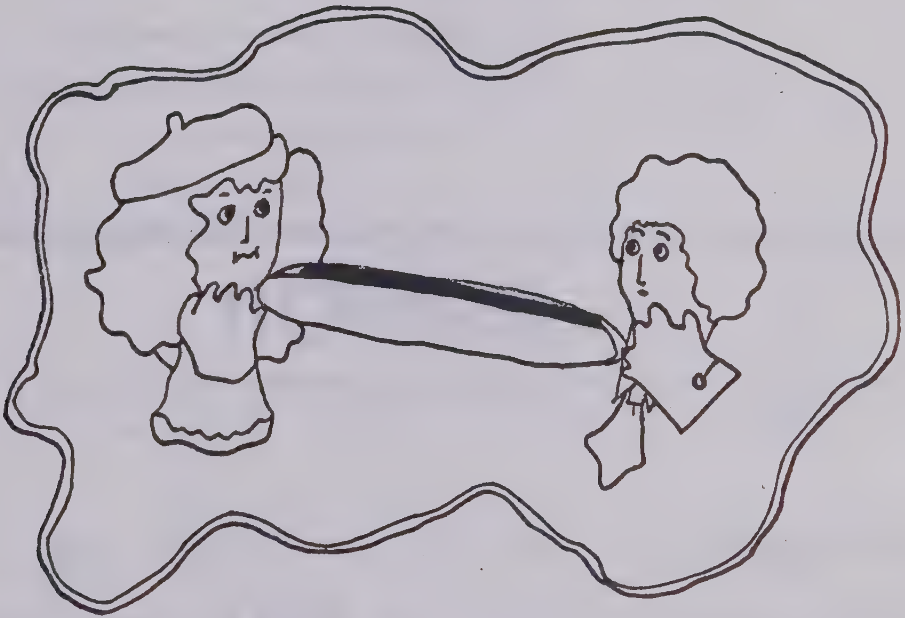
(c) A cake is cut up for 16 people. $\frac{1}{16}$

(d) 5 bags of candy are handed out to 26 children. $\frac{5}{26}$

(e) $95 will buy four chairs. $\frac{95}{4}$

A FRACTION CAN BE USED TO SHOW

1. Part of a whole
2. Part of a group
3. Measure
4. Ratio
5. Division



WHAT IS A FRACTION

RATIO

THE RATIO OF CANS OF PEAS TO MONEY CAN BE EXPRESSED AS $\frac{4}{69}$

SALE

4 cans of peas for 69¢

DIVISION

$16 \div 5$

$= 5\overline{)16}$

$= \frac{16}{5}$

A FRACTION CAN BE USED TO REPRESENT DIVISION

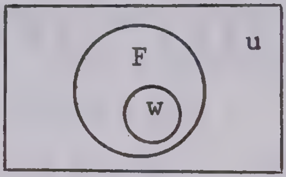
DEVELOPMENT AND EXERCISES

STRAND: *Rational Numbers* LEVEL: 7

UNIT: *III* OBJECTIVE NUMBER: B-4

OBJECTIVE: **Demonstrate that the whole numbers are a subset of the fractional numbers using Venn diagrams.*

SUGGESTED DEVELOPMENT: Present the following Venn diagram illustrating the set of whole numbers:



Through discussion, develop the idea that when whole numbers are divided, fractions are formed.

By examining some special divisions, lead the class to accept that every whole number is a fraction, but not every fraction is a whole number.

This can be shown by drawing another larger circle which includes the whole numbers.

i.e. The whole numbers form a proper subset of the set of fractions.

EXAMPLE:

$$\left\{ 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \right\}$$

$$\div\, 3 \longrightarrow \left\{ \frac{1}{3} \quad \frac{2}{3} \quad \frac{3}{3} \quad \frac{4}{3} \quad \frac{5}{3} \quad \frac{6}{3} \quad \frac{7}{3} \quad \frac{8}{3} \quad \frac{9}{3} \quad \frac{10}{3} \right\}$$

NOTE: $\frac{3}{3}$, $\frac{6}{3}$ and $\frac{9}{3}$ are in fact merely whole numbers 1, 2, 3.

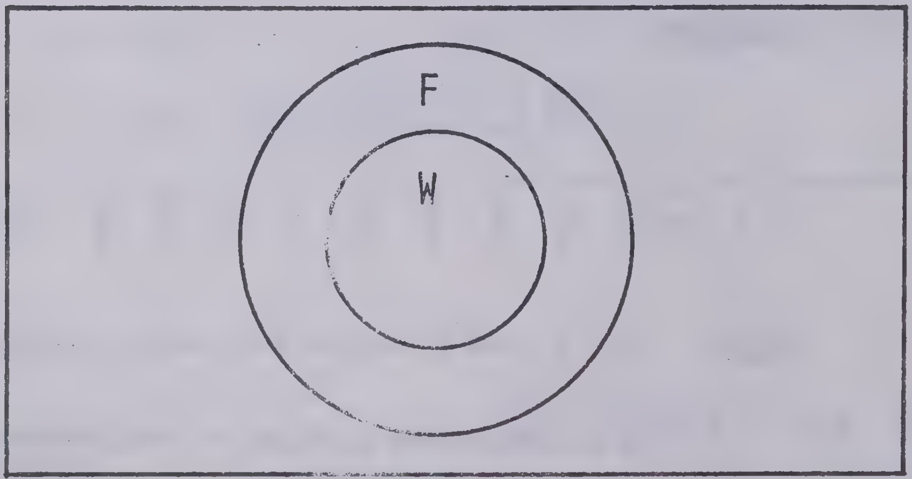
i.e. They form a subset of the fraction set.

FRACTIONS ⟷ WHOLES

ANY WHOLE NUMBER CAN
BE EXPRESSED AS A
FRACTIONAL NUMBER

$$3 = \frac{3}{1} = \frac{6}{2} = \frac{9}{3} = \frac{12}{4} \quad \ldots$$

$$15 = \frac{15}{1} = \frac{30}{2} = \frac{45}{3} = \frac{60}{4} \quad \ldots$$



W IS A SUBSET OF F

$W \subset F$

RAT 7 OBJ. B-4

STRAND: *Rational Numbers* LEVEL: 7

UNIT: III OBJECTIVE NUMBER: B-5

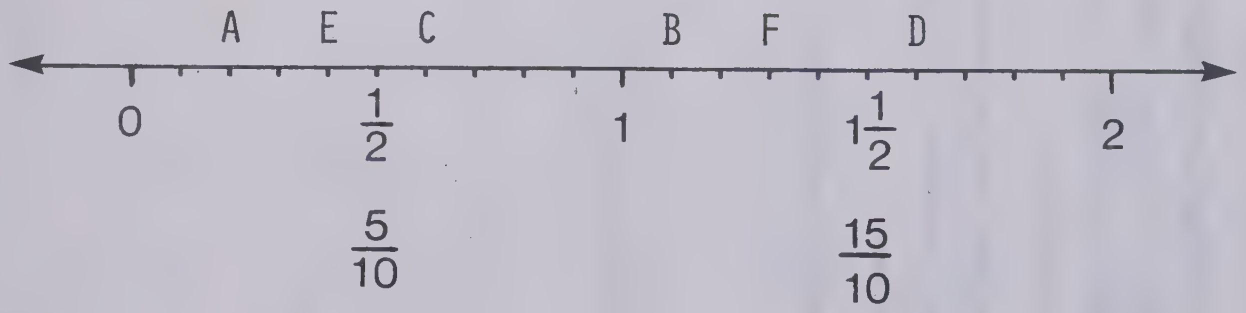
OBJECTIVE: *Demonstrate knowledge of the fractional numbers by plotting a given set on a number line.*

SUGGESTED DEVELOPMENT: On a whole number line, divide each segment into two equal parts. Have students label each point. The same line can be divided into quarters, eighths, etc. Make sure each point is labelled.

The line can be used to show sets of equivalent fractions and the whole numbers as fractions. Procedure can be repeated for other fractions.

NOTE: *All divisions must be made equal.*

FRACTIONAL NUMBER LINE



WHAT FRACTIONAL NUMBER IS ASSOCIATED WIITH:

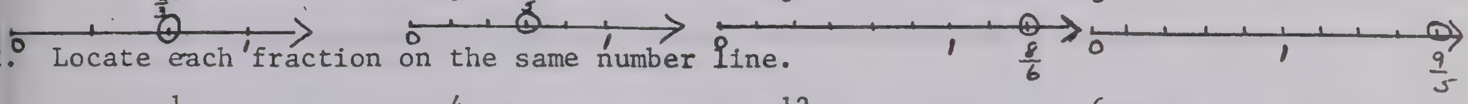
A ____________ D ____________

B ____________ E ____________

C ____________ F ____________

. Locate each fraction on a separate number line.

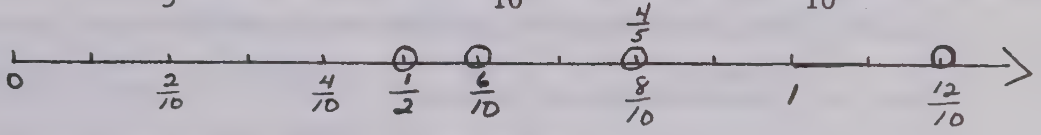
(a) $\frac{2}{3}$ (b) $\frac{3}{5}$ (c) $\frac{8}{6}$ (d) $\frac{9}{5}$



. Locate each fraction on the same number line.

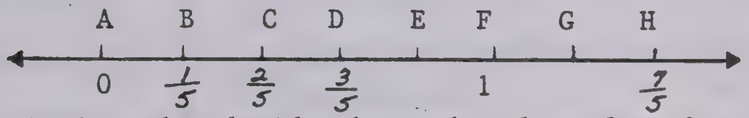
(a) $\frac{1}{2}$ (b) $\frac{4}{5}$ (c) $\frac{12}{10}$ (d) $\frac{6}{10}$

(e) $\frac{8}{10}$

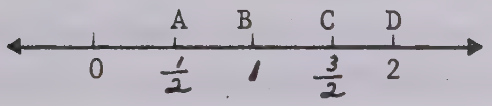


. Plot the given fractions on the number line below.

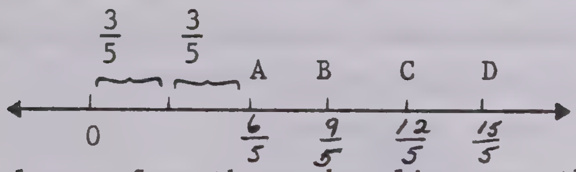
$\frac{1}{5}$, $\frac{3}{5}$, $\frac{5}{5}$, $\frac{2}{5}$, $\frac{7}{5}$



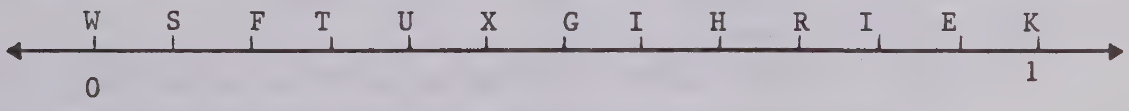
. Points A, B, C, and D divide the number line from 0 to 2. What fraction is represented by each point?



. What fractions are represented by points A, B, C, and D?



. Place the letter from the number line over the fraction that it represents.



I FLUNKED!	I	T	F	I	G	U	R	E	S
	$\frac{7}{12}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{3}{4}$	$\frac{11}{12}$	$\frac{1}{12}$

BE SURE:

1. *the divisions on your number line always represent the same unit.*
2. *the points on your number line are labelled.*

DEVELOPMENT AND EXERCISES

STRAND: *Rational Numbers* LEVEL: *7*

UNIT: *III* OBJECTIVE NUMBER: *B-6*

OBJECTIVE: *Demonstrate the use of identity elements and zero property.*
(De-emphasize terminology)

SUGGESTED DEVELOPMENT: Present the following to the class.

Column A	*Column B*	*Column C*
$1 \times 3 = 3$	$0 + 5 = 5$	$0 \times 9 = 0$
$1 \times 75 = 75$	$0 + \frac{2}{3} = \frac{2}{3}$	$0 \times 1\frac{1}{3} = 0$
$\frac{3}{4} \times 1 = \frac{3}{4}$	$2\frac{1}{2} + 0 = 2\frac{1}{2}$	$\frac{3}{5} \times 0 = 0$
$6\frac{5}{8} \times 1 = 6\frac{5}{8}$	$527 + 0 = 527$	$229 \times 0 = 0$
$2\ 147\ 258 \times 1 = 2\ 147\ 258$	$6721 + 0 = 6721$	$0 \times 3841 = 0$

Through discussion, lead the class to observe that

(a) Column A answers remain unchanged from the starting amount or they retain their identity.

(b) Column B answers remain unchanged from the starting amount or they retain their identity.

(c) Column C answers all are zero.

From these observations, have the class identify the terms

(a) *identity element of addition is zero* $(a + 0 = a)$

(b) *identity element of multiplication is one* $(a \times 1 = a)$

(c) *zero property.* $(a \times 0 = 0)$

NOTE: *The important idea is to get the correct answer when you use one and zero. DO NOT memorize the definitions.*

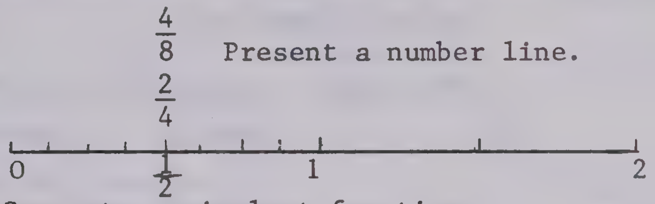
DEVELOPMENT AND EXERCISES

STRAND: *Rational Numbers* LEVEL: 7

UNIT: *III* OBJECTIVE NUMBER: *B-7*

OBJECTIVE: *Generate sets of equivalent fractions. (Limit: 5).*

SUGGESTED DEVELOPMENT: I. Construct fractional diagrams (use transparency B-7) to show the meaning of equivalent fractions.



Present a number line.

Generate equivalent fractions.

II. OBTAINING ONE EQUIVALENT FRACTION USING MULTIPLICATION AND DIVISION.

The student knows that $\frac{1}{2} = \frac{2}{4}$ and $\frac{2}{8} = \frac{1}{4}$

Discuss how $\frac{2}{4}$ was obtained from $\frac{1}{2}$

Discuss how $\frac{1}{4}$ was obtained from $\frac{2}{8}$

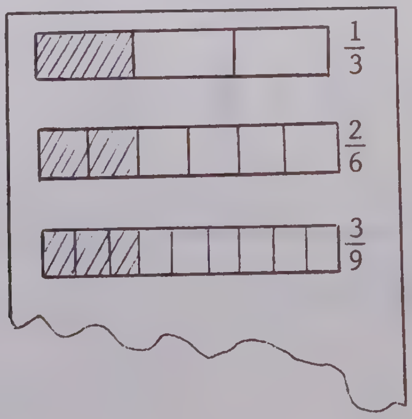
Point out that when the numerator and denominator of a fraction are multiplied or divided by the same number, an equivalent fraction is obtained.

$$\frac{1}{2} = \frac{1 \times \boxed{2}}{2 \times \boxed{2}} = \frac{2}{4} \qquad \frac{2}{8} = \frac{2 \div \boxed{2}}{8 \div \boxed{2}} = \frac{1}{4}$$

NOTE: $\frac{2}{2} = 1$ When you multiply by one, or a fraction equivalent to one, the resulting fraction will be an equivalent fraction.

III. Generate a set of equivalent fractions.

(a) Present a chart similar to the one shown here. Think of this chart as going on without end (infinite).



(b) List the set of equivalent fractions.

$$\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9} \ldots\right\}$$

Discuss how each new fraction $\frac{2}{6}$, $\frac{3}{9}$ is obtained.

What could be the next equivalent fraction? How did you obtain it?

CONCLUSION: To obtain a set of equivalent fractions, multiply the numerator and denominator of the basic fraction by the same natural number.

IV. EQUIVALENT - BASIC FRACTIONS (using division).

(a) The students know that through division equivalent fractions are obtained.

(b) Use a fraction. e.g. $\frac{18}{24}$ Ask the students to obtain equivalent fractions through division.

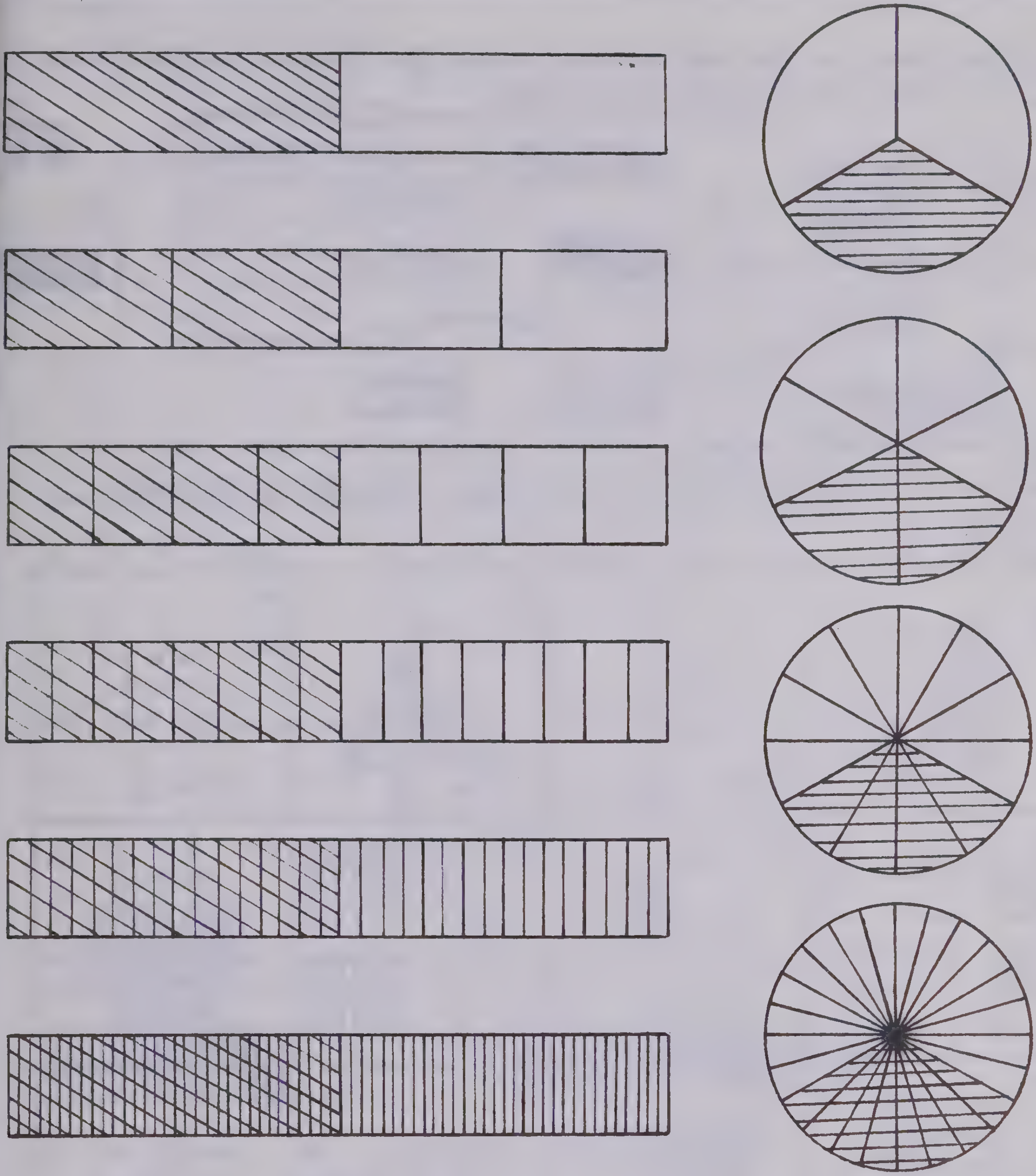
$$\frac{18}{24} = \frac{9}{12} = \frac{3}{4}$$

(c) CONCLUSION: When 1 is the only common divisor or factor of the numerator or denominator, that fraction is BASIC.

(c) Discuss a faster way of obtaining the BASIC fraction - DIVIDING the numerator and denominator by the G.C.F.

V. A basic fraction is in the form $\frac{a}{b}$ $\therefore$ a mixed number is not a basic fraction.

EQUIVALENT FRACTIONS



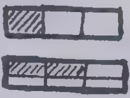
$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16} = \frac{16}{32} = \frac{32}{64}$

$\frac{1}{3} = \frac{2}{6} = \frac{4}{12} = \frac{8}{24}$

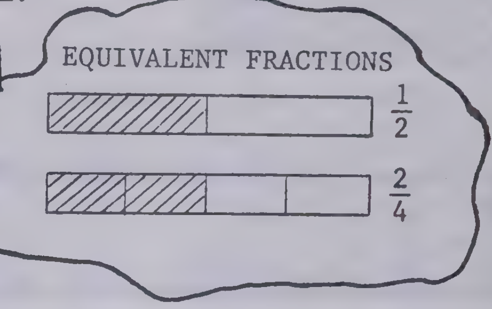
RAT 7 OBJ B-7

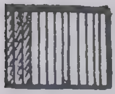
I. For each exercise, draw two pictures to show that the fractions are equivalent. KEEP YOUR DIAGRAMS THE SAME SIZE.

A) $\frac{2}{6}$, $\frac{1}{3}$

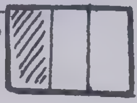


B) $\frac{2}{10}$, $\frac{1}{5}$





C) $\frac{5}{15}$, $\frac{1}{3}$



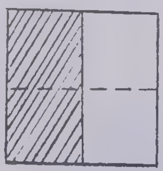
D) $\frac{8}{8}$, $\frac{4}{4}$

E) $\frac{8}{12}$, $\frac{4}{6}$

F) $\frac{2}{8}$, $\frac{1}{4}$

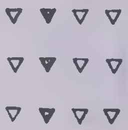
II. Each exercise below suggests a pair of equivalent fractions. Give a pair of equivalent fractions for each figure. ONE MUST BE THE BASIC FRACTION.

1



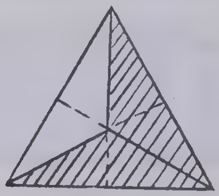
$\frac{1}{2}$, $\frac{2}{4}$

2



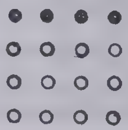
$\frac{1}{4}$, $\frac{3}{12}$

3



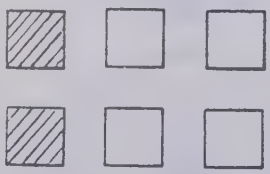
$\frac{2}{3}$, $\frac{4}{6}$

4



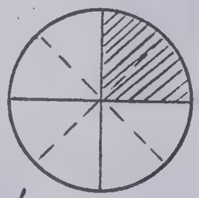
$\frac{1}{4}$, $\frac{4}{16}$

5



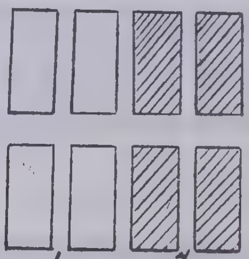
$\frac{1}{3}$, $\frac{2}{6}$

6



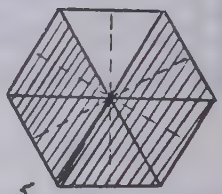
$\frac{1}{4}$, $\frac{2}{8}$

7



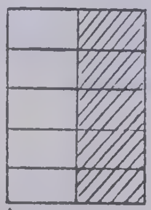
$\frac{1}{2}$, $\frac{4}{8}$

8



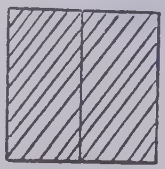
$\frac{5}{6}$, $\frac{10}{12}$

9



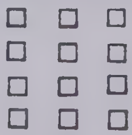
$\frac{1}{2}$, $\frac{5}{10}$

10



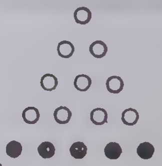
$\frac{1}{1}$, $\frac{2}{2}$

11



$\frac{0}{1}$, $\frac{0}{12}$

12



$\frac{1}{3}$, $\frac{5}{15}$

III. Give two equivalent fractions. Obtain one by multiplying and one by dividing. DO NOT WRITE AS MIXED NUMBERS. Note; Answers may vary

A) $\frac{12}{24}$ $\frac{24}{48}$, $\frac{1}{2}$ B) $\frac{8}{10}$ $\frac{16}{20}$, $\frac{4}{5}$ C) $\frac{28}{18}$ $\frac{56}{36}$, $\frac{14}{9}$ D) $\frac{2}{6}$ $\frac{4}{12}$, $\frac{1}{3}$

I. Cont'd.

E) $\frac{30}{25}$ $\frac{60}{50}$, $\frac{6}{5}$ F) $\frac{5}{5}$ $\frac{10}{10}$, $\frac{1}{1}$ G) $\frac{18}{3}$ $\frac{36}{6}$, $\frac{6}{1}$ H) $\frac{12}{15}$ $\frac{24}{30}$, $\frac{4}{5}$

$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$, $\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$, $\frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$

$\frac{1}{2} \times \frac{5}{5} = \frac{5}{10}$, $\frac{1}{2} \times \frac{6}{6} = \frac{6}{12}$ $\therefore$ The set of fractions equivalent to $\frac{1}{2}$ is

$$\left\{ \frac{2}{4}, \cancel{\frac{3}{4}}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12} \ldots \right\}$$

IV. For each exercise, specify if the fractions are equivalent.

A) $\frac{1}{2}$, $\frac{3}{5}$ No

B) $\frac{3}{9}$, $\frac{1}{3}$ Yes

C) $\frac{7}{6}$, $\frac{14}{10}$ No

D) $\frac{15}{3}$, $\frac{3}{2}$ No

E) $\frac{2}{3}$, $\frac{14}{21}$ Yes

F) $\frac{7}{8}$, $\frac{42}{48}$ Yes

V. Give the equivalent BASIC fraction. DO NOT WRITE AS A MIXED NUMBER.

A) $\frac{4}{8}$ $\frac{1}{2}$

B) $\frac{15}{10}$ $\frac{3}{2}$

C) $\frac{9}{9}$ $\frac{1}{1}$

D) $\frac{4}{10}$ $\frac{2}{5}$

E) $\frac{6}{21}$ $\frac{2}{7}$

F) $\frac{15}{9}$ $\frac{5}{3}$

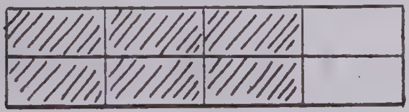
G) $\frac{33}{11}$ $\frac{3}{1}$

H) $\frac{54}{48}$ $\frac{9}{8}$

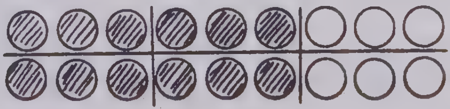
I) $\frac{10}{12}$ $\frac{5}{6}$

J) $\frac{50}{35}$ $\frac{10}{7}$

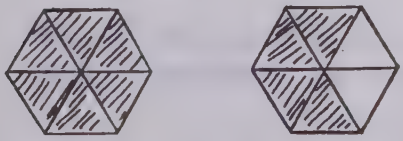
VI. For each picture, colour the amount described. Then give the basic fraction for this amount.



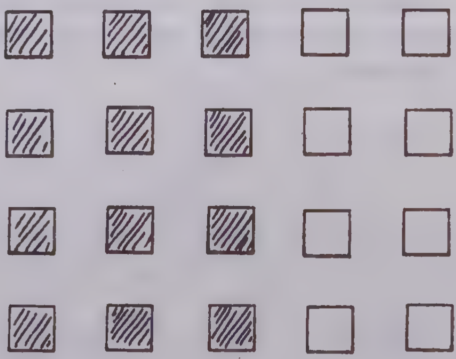
A) Six eighths $\frac{3}{4}$



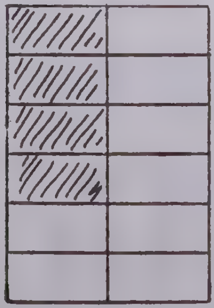
B) Four sixths $\frac{2}{3}$



C) Ten sixths $\frac{5}{3}$

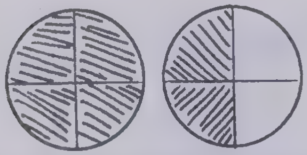


D) Three fifths $\frac{3}{5}$

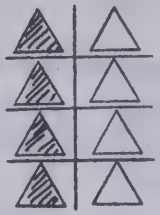


E) Four twelfths $\frac{1}{3}$

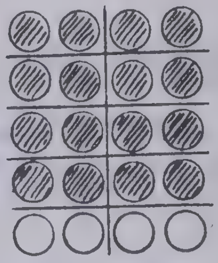
VI. (Cont'd.)



F) Six fourths $\frac{3}{2}$



G) Four eighths $\frac{1}{2}$



H) Eight tenths $\frac{4}{5}$

VII. Tabulate a set of equivalent fractions equivalent to the basic fraction. (Tabulate four). The first one is done for you. Note: Answers may vary

A) $\frac{3}{4}\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\}$ C) $\frac{9}{8}\{\frac{9}{8}, \frac{18}{16}, \frac{27}{24}, \frac{36}{32}, \ldots\}$ E) $\frac{7}{5}\{\frac{7}{5}, \frac{14}{10}, \frac{21}{15}, \frac{28}{20}, \ldots\}$ G) $\frac{2}{5}\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \ldots\}$

B) $\frac{1}{7}\{\frac{1}{7}, \frac{2}{14}, \frac{3}{21}, \frac{4}{28}, \ldots\}$ D) $\frac{3}{10}\{\frac{3}{10}, \frac{6}{20}, \frac{9}{30}, \frac{12}{40}, \ldots\}$ F) $\frac{1}{3}\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$ H) $\frac{2}{9}\{\frac{2}{9}, \frac{4}{18}, \frac{6}{27}, \frac{8}{36}, \ldots\}$

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8} \qquad \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

$$\frac{3 \times 4}{4 \times 4} = \frac{12}{16} \qquad \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

$$\frac{3}{4} = \left\{\frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}\right\}$$

VIII. Show how the first fraction was used to obtain the second fraction.

A) $\frac{5}{6}, \frac{10}{12}$ Multiply by 2 D) $\frac{2}{7}, \frac{12}{42}$ Multiply by 6 G) $\frac{3}{4}, \frac{27}{36}$ Multiply by

B) $\frac{2}{5}, \frac{8}{20}$ Multiply by 4 E) $\frac{6}{18}, \frac{1}{3}$ Divide by 6 H) $\frac{12}{15}, \frac{4}{5}$ Divide by

C) $\frac{8}{3}, \frac{24}{9}$ Multiply by 3 F) $\frac{20}{25}, \frac{4}{5}$ Divide by 5 I) $\frac{4}{16}, \frac{1}{4}$ Divide by

IX. *WHY WILL HARRY BE A FIGHT REFEREE?*

To find the answer to this question determine the value for each letter. Place the letter above that value in the code below the exercise. Your answer will appear.

e.g.
$\frac{4}{6} = \frac{16}{U}$
U = 24

1. $\frac{3}{18} = \frac{A}{6}$ A = 1
2. $\frac{2}{9} = \frac{Y}{45}$ Y = 10
3. $\frac{1}{2} = \frac{T}{8}$ T = 4
4. $\frac{14}{49} = \frac{2}{E}$ E = 7
5. $\frac{1}{4} = \frac{3}{H}$ H = 12
6. $\frac{3}{"O"} = \frac{36}{24}$ "O" = 2
7. $\frac{N}{32} = \frac{2}{8}$ N = 8
8. $\frac{C}{15} = \frac{1}{3}$ C = 5
9. $\frac{42}{56} = \frac{L}{8}$ L = 6

$\frac{H}{12}, \frac{E}{7}$ $\frac{C}{5}, \frac{A}{1}, \frac{N}{8}$ $\frac{O}{2}, \frac{N}{8}, \frac{L}{6}, \frac{Y}{10}$ $\frac{C}{5}, \frac{O}{2}, \frac{U}{24}, \frac{N}{8}, \frac{T}{4}$ $\frac{T}{4}, \frac{O}{2}$ $\frac{T}{4}, \frac{E}{7}, \frac{N}{8}$

What natural number can you use for each fraction. Give four other fractions you can use for this number. Note: Answers may vary

A) $\frac{2}{1} = 2 = \frac{4}{2} = \frac{6}{3} = \frac{8}{4}$

B) $\frac{7}{1} = 7 = \frac{14}{2} = \frac{21}{3} = \frac{28}{4} = \frac{35}{5}$

C) $\frac{8}{1} = 8 = \frac{16}{2} = \frac{24}{3} = \frac{32}{4} = \frac{40}{5}$

D) $\frac{200}{1} = 200 = \frac{400}{2} = \frac{600}{3} = \frac{800}{4} = \frac{1000}{5}$

E) $\frac{700}{1} = 700 = \frac{1400}{2} = \frac{2100}{3} = \frac{2800}{4} = \frac{3500}{5}$

F) $\frac{150}{1} = 150 = \frac{300}{2} = \frac{450}{3} = \frac{600}{4} = \frac{750}{5}$

For each exercise, give three equivalent fractions. Note: Answers may vary

A) $\frac{9}{12} = \frac{3}{4} = \frac{6}{8} = \frac{75}{100}$

B) $\frac{7}{7} = \frac{1}{1} = \frac{2}{2} = \frac{3}{3}$

C) $\frac{8}{14} = \frac{4}{7} = \frac{12}{21} = \frac{16}{28}$

D) $\frac{20}{30} = \frac{2}{3} = \frac{8}{12} = \frac{4}{6}$

E) $\frac{6}{15} = \frac{2}{5} = \frac{4}{10} = \frac{8}{20}$

F) $\frac{2}{9} = \frac{4}{18} = \frac{6}{27} = \frac{8}{36}$

G) $\frac{72}{54} = \frac{4}{3} = \frac{8}{6} = \frac{12}{9}$

H) $\frac{36}{42} = \frac{6}{7} = \frac{12}{14} = \frac{18}{21}$

I) $\frac{18}{6} = \frac{3}{1} = \frac{6}{2} = \frac{9}{3}$

DEVELOPMENT AND EXERCISES

STRAND: Rational Numbers LEVEL: 7

UNIT: III OBJECTIVE NUMBER: B-8

OBJECTIVE: *Find L.C.D. (e.g. using L.C.M.)

SUGGESTED DEVELOPMENT: Review the methods of calculating L.C.M. for two numbers. At this point, it may be best if you use only one of the methods of Objective A-5.

Point out that since you are working only with the denominators of fractions, you may use the term "L.C.D." in place of "L.C.M."

. A) List the multiples (to 54) of 6.
6, 12, 18, 24, 30, 36, 42, 48, 54

B) List the multiples (to 54) of 9.
9, 18, 27, 36, 45, 54

C) List the common multiples(to 54) of 6 and 9. 18, 36, 54

D) What is the L.C.M. of 6 and 9? 18

E) What is the L.C.D. of $\frac{1}{6}$ and $\frac{1}{9}$? 18

The multiples of 2 are 2,4,6,8,10 ...

The multiples of 5 are 5,10,15,20 ...

The common multiples of 2 and 5 are 10,20,30 ...

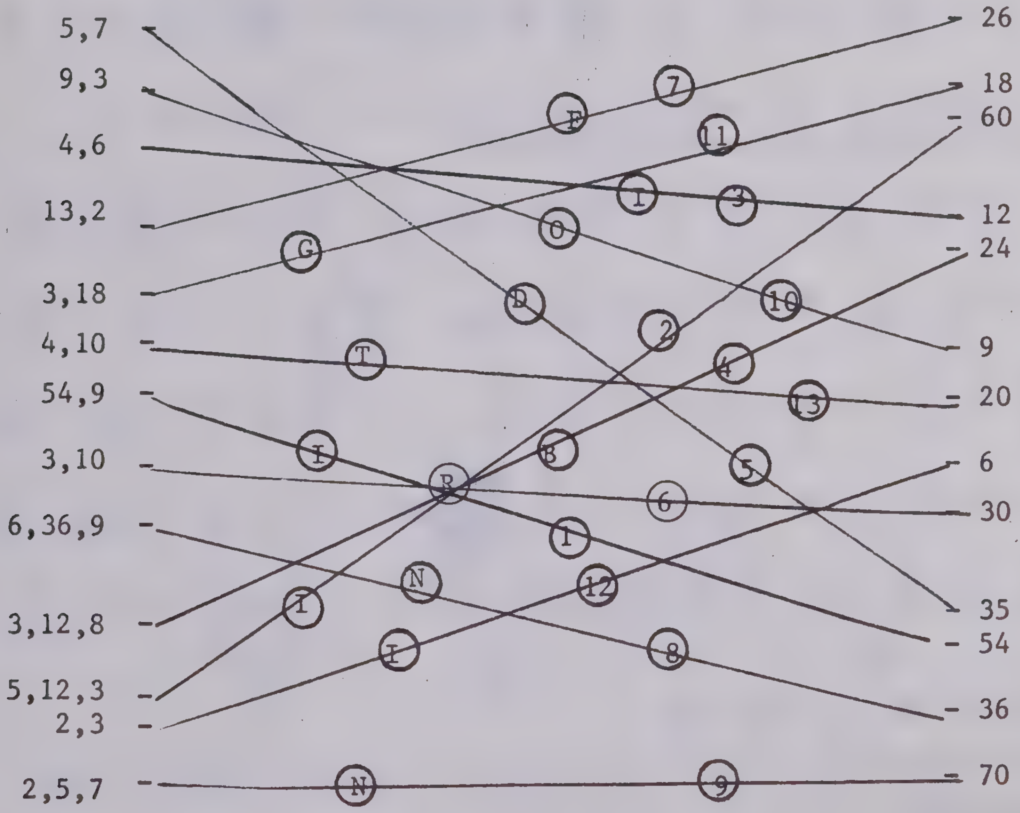
The L.C.M. of 2 and 5 is 10

The L.C.D. for $\frac{1}{2}$ and $\frac{1}{5}$ is 10

. MORE DRILL?

HOW DO YOU FIND THE DENTIST'S DRILL?

Figure out the L.C.M. for each set of numbers on the left. Draw a straight line connecting each problem with its answer. Each line will cross a number and a letter. The number tells you where to put the letter in the line below the questions.



I FIND IT BORING

I	F	I	N	D	I	T	B	O	R	I	N	G
2	7	3	8	5	1	13	4	10	6	12	9	11

III. Circle each number that you could use as a common denominator for the fractions named.

A) $\frac{2}{3}$ and $\frac{3}{4}$

8, 10, [✓ 12], 18, [✓ 24], 30

B) $\frac{5}{12}$ and $\frac{1}{3}$

[✓ 12], 15, 18, [✓ 24], [✓ 36], [✓ 48]

C) $\frac{5}{2}$ and $\frac{6}{7}$

7, [✓ 14], 21, [✓ 28], [✓ 42], 54

D) $\frac{7}{8}$ and $\frac{4}{3}$

8, [✓ 24], 30, 36, [✓ 96]

E) $\frac{5}{6}$ and $\frac{9}{8}$

8, 18, [✓ 24], 36, [✓ 48], [✓ 96]

F) $\frac{1}{10}$ and $\frac{7}{15}$

15, [✓ 30], 40, 50, [✓ 60], 80

G) $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{4}{3}$

[✓ 12], 16, 18, [✓ 24], 32, [✓ 36]

H) $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{1}{9}$

8, 12, [✓ 18], [✓ 36], 42, [✓ 54]

IV. For the fractions in each exercise, find the L.C.D. Then give the equivalent fractions using this denominator.

A) $\frac{2}{5}$, $\frac{1}{7}$ 35, $\frac{14}{35}$, $\frac{5}{35}$

B) $\frac{13}{11}$, $\frac{5}{2}$ 22, $\frac{26}{22}$, $\frac{55}{22}$

C) $\frac{9}{5}$, $\frac{3}{4}$ 20, $\frac{36}{20}$, $\frac{15}{20}$

D) $\frac{3}{2}$, $\frac{5}{9}$ 18, $\frac{27}{18}$, $\frac{10}{18}$

E) $\frac{6}{7}$, $\frac{1}{4}$ 28, $\frac{24}{28}$, $\frac{7}{28}$

F) $\frac{2}{3}$, $\frac{4}{5}$ 15, $\frac{10}{15}$, $\frac{12}{15}$

G) $\frac{1}{3}$, $\frac{11}{12}$ 12, $\frac{4}{12}$, $\frac{11}{12}$

H) $\frac{1}{4}$, $\frac{3}{8}$ 8, $\frac{2}{8}$, $\frac{3}{8}$

I) $\frac{17}{27}$, $\frac{5}{9}$ 27, $\frac{17}{27}$, $\frac{15}{27}$

J) $\frac{1}{6}$, $\frac{7}{10}$ 30, $\frac{5}{30}$, $\frac{21}{30}$

K) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ 12, $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$

L) $\frac{2}{7}$, $\frac{7}{4}$, $\frac{3}{28}$ 28, $\frac{8}{28}$, $\frac{49}{28}$, $\frac{3}{28}$

M) $\frac{19}{10}$, $\frac{7}{3}$, $\frac{5}{2}$ 30, $\frac{57}{30}$, $\frac{70}{30}$, $\frac{75}{30}$

N) $\frac{4}{5}$, $\frac{5}{6}$, $\frac{4}{15}$ 30, $\frac{24}{30}$, $\frac{25}{30}$, $\frac{8}{30}$

O) $\frac{7}{8}$, $\frac{2}{3}$, $\frac{11}{12}$ 24, $\frac{21}{24}$, $\frac{16}{24}$, $\frac{22}{24}$

P) $\frac{1}{25}$, $\frac{1}{10}$, $\frac{1}{4}$ 100, $\frac{4}{100}$, $\frac{10}{100}$, $\frac{25}{100}$

FOR X-PERTS ONLY

Q) $\frac{9}{18}$, $\frac{29}{60}$, $\frac{23}{45}$ 180

$\frac{90}{180}$ $\frac{87}{180}$ $\frac{92}{180}$

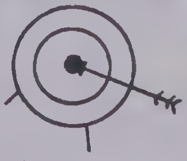
R) $\frac{7}{8}$, $\frac{9}{10}$, $\frac{13}{16}$ 80, $\frac{70}{80}$, $\frac{72}{80}$, $\frac{65}{80}$

S) $\frac{11}{12}$, $\frac{15}{16}$, $\frac{9}{10}$ 240

$\frac{220}{240}$, $\frac{225}{240}$, $\frac{216}{240}$

T) $\frac{32}{40}$, $\frac{19}{25}$, $\frac{21}{32}$, $\frac{39}{50}$ 800

$\frac{640}{800}$, $\frac{608}{800}$, $\frac{525}{800}$, $\frac{624}{800}$



V. For each pair of fractions, give the least common denominator. Then express each fraction as a fraction with this denominator.

	L.C.D.	First Fraction	Second Fraction
A) $\frac{1}{12}, \frac{5}{6}$	12	$\frac{1}{12} = \frac{1}{12}$	$\frac{5}{6} = \frac{10}{12}$

B) $\frac{2}{5}, \frac{4}{25}$ 25, $\frac{10}{25}$, $\frac{4}{25}$

C) $\frac{3}{4}, \frac{9}{10}$ 20, $\frac{15}{20}$, $\frac{18}{20}$

D) $\frac{7}{3}, \frac{3}{8}$ 24, $\frac{56}{24}$, $\frac{9}{24}$

E) $\frac{5}{6}, \frac{3}{4}$ 12, $\frac{10}{12}$, $\frac{9}{12}$

F) $\frac{5}{2}, \frac{4}{9}$ 18, $\frac{45}{18}$, $\frac{8}{18}$

G) $\frac{3}{5}, \frac{1}{6}$ 30, $\frac{18}{30}$, $\frac{5}{30}$

H) $\frac{7}{10}, \frac{5}{8}$ 40, $\frac{28}{40}$, $\frac{25}{40}$

I) $\frac{8}{9}, \frac{7}{6}$ 18, $\frac{16}{18}$, $\frac{21}{18}$

J) $\frac{4}{5}, \frac{3}{4}$ 20, $\frac{16}{20}$, $\frac{15}{20}$

K) $\frac{4}{9}, \frac{1}{12}$ 36, $\frac{16}{36}$, $\frac{3}{36}$

L) $\frac{1}{17}, \frac{3}{5}$ 85, $\frac{5}{85}$, $\frac{51}{85}$

M) $\frac{6}{7}, \frac{4}{9}$ 63, $\frac{54}{63}$, $\frac{28}{63}$

N) $\frac{5}{8}, \frac{7}{6}$ 24, $\frac{15}{24}$, $\frac{28}{24}$

O) $\frac{1}{13}, \frac{1}{7}$ 91, $\frac{7}{91}$, $\frac{13}{91}$

DEVELOPMENT AND EXERCISES

STRAND: *Rational Numbers* LEVEL: 7

UNIT: *III* OBJECTIVE NUMBER: *B-9*

OBJECTIVE: *Reduce any fraction to its basic form. (e.g. using G.C.F.)*

SUGGESTED DEVELOPMENT: DEFINITION: A fraction is in its *basic form* or in *lowest terms* when the G.C.F. of the numerator and denominator is 1.

A. Find the G.C.F. of the numerator and the denominator and then dividing both numerator and denominator by its common factors. (A review of finding G.C.F. may be advantageous).

If we had to reduce the fraction $\frac{30}{42}$, we could find the G.C.F. and then divide both the numerator and denominator by 6.

$$\frac{30}{42} = \frac{30 \div 6}{42 \div 6} = \frac{5}{7}$$

B. Alternately, one could express the numerator and the denominator as their respective prime factorizations, and simply cancel the common factors (the G.C.F. in its factored form).

$$\frac{30}{42} = \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{3}} \times 5}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{3}} \times 7} = \frac{5}{7}$$

C. Repeated division by common factors until numerator and denominator are relatively prime.

EXAMPLE:

$$\frac{28}{70} = \frac{28 \div 2}{70 \div 2} = \frac{14 \div 7}{35 \div 7} = \frac{2}{5}$$

XERCISES: OBJECTIVE NO. B-9

30 = 2 x 3 x 5
42 = 2 x 3 x 7
110 = 2 x 5 x 11
140 = 2 x 2 x 5 x 7
300 = 2 x 2 x 3 x 5 x 5

1. Find the prime factorizations of 30, 42, 110, 140, 300

2. Find the G.C.F. of 36, 60; 600, 180; 35, 200
12, 60, 5

3. *HOW DID YOU GET THAT GLASS OF ORANGE JUICE?*

To discover the answer reduce the following fractions. Find your answer in the answer section. The number in front of the answer tells you where to put the letter of the problem in the row of boxes at the bottom of the answers.

Ⓐ $\frac{3}{9} = \frac{1}{3}$ Ⓘ $\frac{64}{12} = \frac{16}{3}$ Ⓣ $\frac{16}{72} = \frac{2}{9}$ Ⓣ $\frac{10}{16} = \frac{5}{8}$

Ⓘ $\frac{14}{21} = \frac{2}{3}$ Ⓐ $\frac{28}{56} = \frac{1}{2}$ Ⓔ $\frac{35}{200} = \frac{7}{40}$ Ⓔ $\frac{6}{48} = \frac{1}{8}$

Ⓖ $\frac{40}{50} = \frac{4}{5}$ Ⓢ $\frac{70}{28} = \frac{5}{2}$ Ⓗ $\frac{49}{21} = \frac{7}{3}$ Ⓣ $\frac{15}{40} = \frac{3}{8}$

Ⓤ $\frac{45}{60} = \frac{3}{4}$ Ⓠ $\frac{21}{36} = \frac{7}{12}$ Ⓩ $\frac{144}{200} = \frac{18}{25}$

Ⓔ $\frac{24}{27} = \frac{8}{9}$ Ⓦ $\frac{85}{100} = \frac{17}{20}$ Ⓢ $\frac{25}{3000} = \frac{1}{120}$

ANSWERS

① $\frac{2}{3}$ ⑤ $\frac{5}{2}$ ⑨ $\frac{4}{5}$ ⑬ $\frac{7}{12}$ ⑰ $\frac{18}{25}$

② $\frac{2}{9}$ ⑥ $\frac{1}{2}$ ⑩ $\frac{7}{3}$ ⑭ $\frac{3}{4}$ ⑱ $\frac{1}{8}$

③ $\frac{17}{20}$ ⑦ $\frac{5}{8}$ ⑪ $\frac{3}{8}$ ⑮ $\frac{8}{9}$

④ $\frac{1}{3}$ ⑧ $\frac{16}{3}$ ⑫ $\frac{1}{120}$ ⑯ $\frac{7}{40}$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
I	T	W	A	S	A	T	I	G	H	T	S	Q	U	E	E	Z	E

DEVELOPMENT AND EXERCISES

STRAND: Rational Numbers

LEVEL: 7

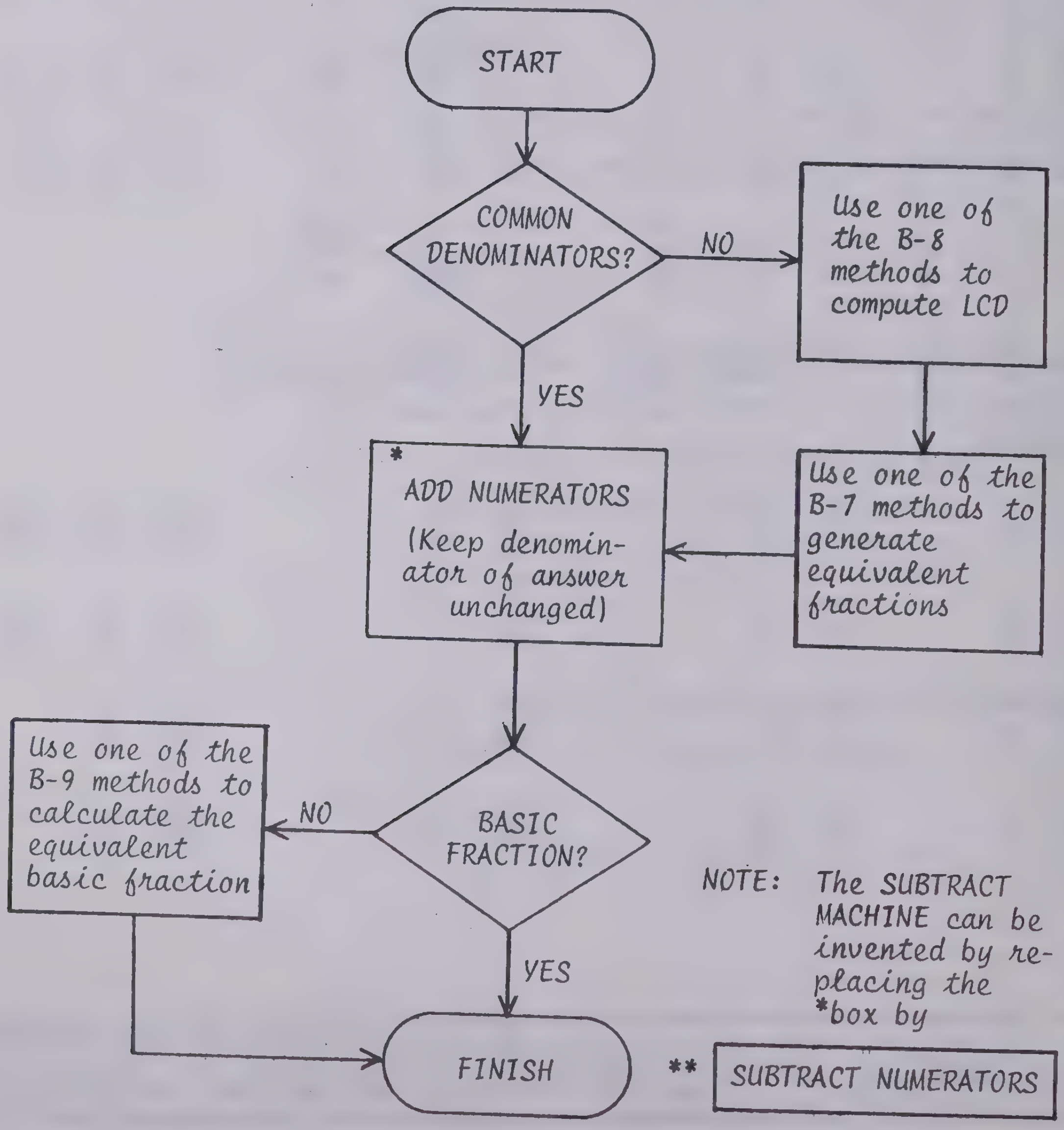
UNIT: III

OBJECTIVE NUMBER: B-10

OBJECTIVE: Perform the operations of addition and subtraction with fractional numbers. (Limit: 1 digit denominator.)

SUGGESTED DEVELOPMENT: Present the following flow chart as the method for adding (and subtracting) fractions. A short review of Objective B-7, B-8, and B-9 may be needed.

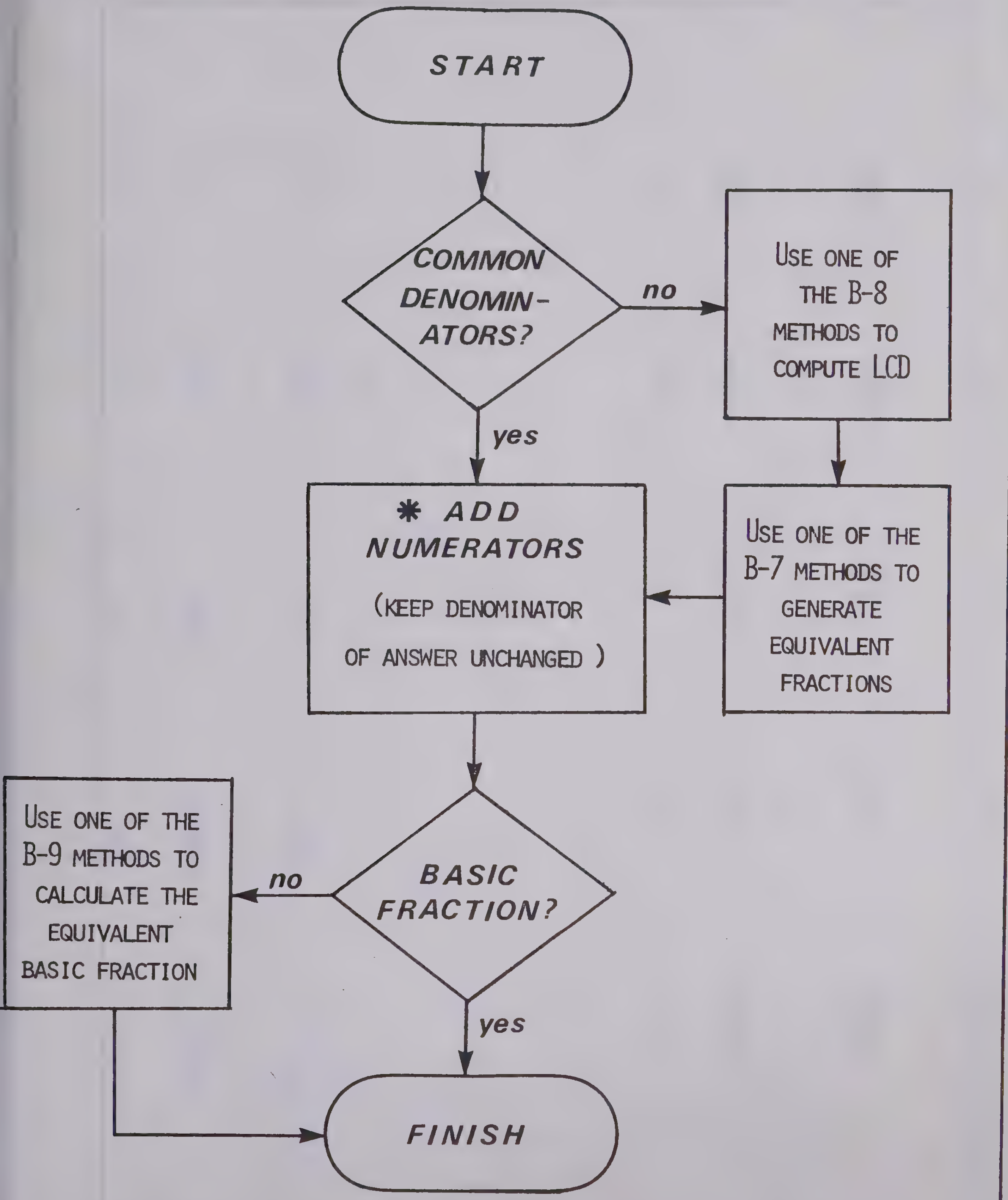
THE ADD MACHINE


START
COMMON DENOMINATORS?
NO
Use one of the B-8 methods to compute LCD
YES
*
ADD NUMERATORS (Keep denominator of answer unchanged)
Use one of the B-7 methods to generate equivalent fractions
Use one of the B-9 methods to calculate the equivalent basic fraction
NO
BASIC FRACTION?
YES
FINISH
**
SUBTRACT NUMERATORS

NOTE: The SUBTRACT MACHINE can be invented by replacing the *box by

ADDITION OF FRACTIONS

+ THE ADD MACHINE +


START
COMMON DENOMINATORS?
no
Use one of the B-8 methods to compute LCD
yes
Use one of the B-7 methods to generate equivalent fractions
✱ ADD NUMERATORS
(KEEP DENOMINATOR OF ANSWER UNCHANGED)
BASIC FRACTION?
no
Use one of the B-9 methods to calculate the equivalent basic fraction
yes
FINISH

SUBTRACTION OF FRACTIONS

1. $\frac{5}{8} - \frac{3}{8} =$

2. $\frac{3}{4} - \frac{1}{4} =$

3. $\frac{9}{10} - \frac{1}{4} =$

4. $\frac{5}{7} - \frac{1}{3} =$

5. $\frac{2}{3} - \frac{1}{9} =$

ADDITION OF FRACTIONS

1. $\frac{3}{7} + \frac{2}{7} =$

2. $\frac{4}{5} + \frac{3}{5} =$

3. $\frac{2}{3} + \frac{1}{6} =$

4. $\frac{1}{5} + \frac{7}{10} =$

5. $\frac{3}{4} + \frac{2}{5} =$

EXERCISES: OBJECTIVE NO. B-10

1. ADD:

(a) $\frac{3}{4} + \frac{5}{4} = \frac{8}{4} = \frac{2}{1} = 2$ (b) $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$ (c) $\frac{3}{9} + \frac{4}{9} + \frac{7}{9} = \frac{14}{9} = 1\frac{5}{9}$

(d) $\frac{5}{3} + \frac{7}{3} = \frac{12}{3} = \frac{4}{1} = 4$ (e) $\frac{3}{8} + \frac{5}{8} = \frac{8}{8} = 1$ (f) $\frac{2}{7} + \frac{1}{7} + \frac{3}{7} = \frac{6}{7}$

2. What is the L.C.D. for each set?

(a) $\frac{2}{3} + \frac{1}{6} =$ 6 (b) $\frac{3}{5} + \frac{3}{10} =$ 10 (c) $\frac{1}{6} + \frac{1}{9} =$ 18

(d) $\frac{7}{8} + \frac{3}{4} =$ 8 (e) $\frac{2}{3} + \frac{3}{9} + \frac{1}{6} =$ 18

3. ADD:

(a) $\frac{5}{6} + \frac{1}{4} = \frac{13}{12} = 1\frac{1}{12}$ (b) $\frac{1}{5} + \frac{3}{4} = \frac{19}{20}$ (c) $\frac{5}{8} + \frac{1}{4} = \frac{7}{8}$

(d) $\frac{5}{6} + \frac{2}{3} = \frac{9}{6} = \frac{3}{2} = 1\frac{1}{2}$ (e) $\frac{3}{5} + \frac{2}{3} + \frac{1}{10} = \frac{41}{30} = 1\frac{11}{30}$

4. ADD:

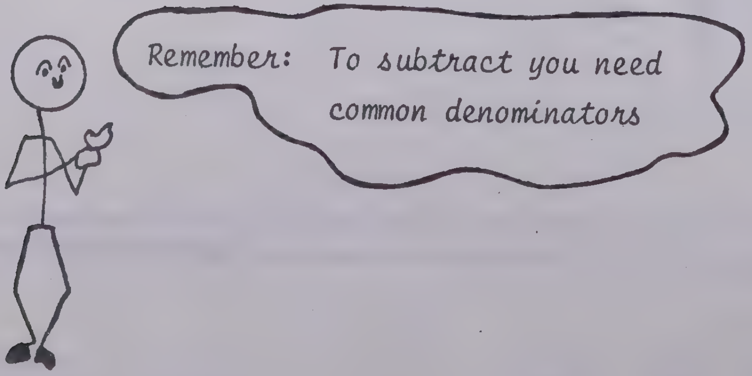
(a) $\frac{5}{8} + \frac{3}{4} = \frac{11}{8} = 1\frac{3}{8}$ (b) $\frac{3}{9} + \frac{7}{6} = \frac{27}{18} = \frac{3}{2} = 1\frac{1}{2}$ (c) $\frac{1}{2} + \frac{4}{5} = \frac{13}{10} = 1\frac{3}{10}$

(d) $\frac{2}{9} + \frac{5}{9} = \frac{7}{9}$

5. SUBTRACT:

(a) $\frac{4}{5} - \frac{3}{5} = \frac{1}{5}$ (b) $\frac{9}{10} - \frac{0}{10} = \frac{9}{10}$ (c) $\frac{7}{9} - \frac{3}{9} = \frac{4}{9}$

(d) $\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$ (e) $\frac{13}{4} - \frac{11}{8} = \frac{15}{8} = 1\frac{7}{8}$ (f) $\frac{8}{9} - \frac{1}{3} = \frac{5}{9}$


Remember: To subtract you need common denominators

6. What is the title of this picture?

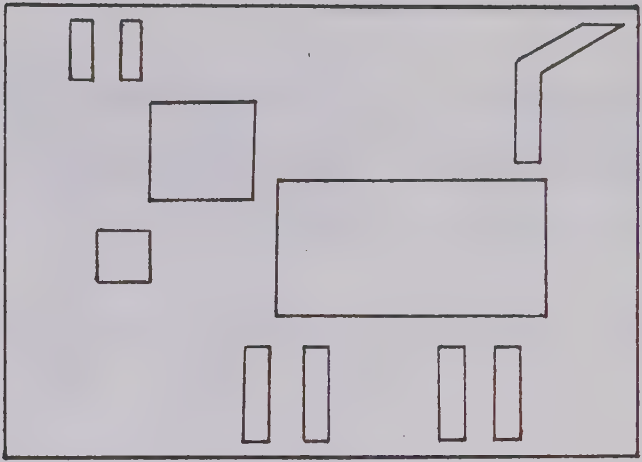
TO DECODE THE TITLE:

(a) Work the problems below.

(b) Find the answer in the code.

(c) Each time the answer appears in the code, write the letter that matches the answer in the blank.

(d) Work until you have discovered the title.



$L = \frac{1}{2} + \frac{1}{5} = \frac{7}{10}$ $K = \frac{7}{9} - \frac{1}{3} = \frac{4}{9}$

$I = \frac{1}{3} + \frac{1}{7} = \frac{10}{21}$ $O = \frac{3}{4} - \frac{1}{6} = \frac{7}{12}$

$F = \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$ $B = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

$E = \frac{2}{3} + \frac{2}{7} = \frac{20}{21}$ $R = \frac{3}{3} - \frac{1}{5} = \frac{4}{5}$

$U = \frac{3}{8} + \frac{4}{9} = \frac{59}{72}$ $N = \frac{3}{9} - \frac{1}{3} = 0$

$T = \frac{2}{5} + \frac{5}{8} = 1\frac{1}{40}$ $A = \frac{3}{4} - \frac{5}{7} = \frac{1}{28}$

$S = \frac{1}{3} + \frac{4}{5} = 1\frac{2}{15}$ $G = \frac{5}{1} - \frac{3}{4} = 4\frac{1}{4}$

$Q = \frac{3}{5} + \frac{2}{7} = \frac{31}{35}$ $D = \frac{3}{5} - \frac{3}{8} = \frac{9}{40}$

$\underset{\frac{4}{9}}{\underline{K}}$ $\underset{\frac{10}{21}}{\underline{I}}$ $\underset{1\frac{1}{40}}{\underline{T}}$ $\underset{\frac{5}{12}}{\underline{F}}$ $\underset{\frac{7}{12}}{\underline{O}}$ $\underset{\frac{4}{5}}{\underline{R}}$

$\underset{\frac{1}{6}}{\underline{B}}$ $\underset{\frac{59}{72}}{\underline{U}}$ $\underset{\frac{10}{21}}{\underline{I}}$ $\underset{\frac{7}{10}}{\underline{L}}$ $\underset{\frac{9}{40}}{\underline{D}}$ $\underset{\frac{10}{21}}{\underline{I}}$ $\underset{0}{\underline{N}}$ $\underset{4\frac{1}{4}}{\underline{G}}$

$\underset{\frac{1}{28}}{\underline{A}}$ $\underset{1\frac{2}{15}}{\underline{S}}$ $\underset{\frac{31}{35}}{\underline{Q}}$ $\underset{\frac{59}{72}}{\underline{U}}$ $\underset{\frac{1}{28}}{\underline{A}}$ $\underset{\frac{4}{5}}{\underline{R}}$ $\underset{\frac{20}{21}}{\underline{E}}$

$\underset{\frac{9}{40}}{\underline{D}}$ $\underset{\frac{7}{12}}{\underline{O}}$ $\underset{4\frac{1}{4}}{\underline{G}}$

DEVELOPMENT AND EXERCISES

STRAND: Rational Numbers
LEVEL: 7
UNIT: III
OBJECTIVE NUMBER: B-11
OBJECTIVE: Demonstrate the use of reciprocals to obtain the identity element.

PREREQUISITE SKILL: Primary knowledge of multiplying fractions.

SUGGESTED DEVELOPMENT: Present the following multiplication problems:

(a) $\frac{3}{4} \times \frac{4}{3} =$
(b) $\frac{2}{3} \times \frac{3}{2} =$
(c) $\frac{1}{3} \times \frac{3}{1} =$
(d) $\frac{1}{5} \times 5 =$
(e) $2\frac{1}{2} \times \frac{2}{5} =$
(f) $2\frac{1}{4} \times \frac{4}{9}$

Derive from the class that the answer to each problem is 1. Point out that 1 has a special name - identity element for multiplying.

Define: - The product of reciprocals is the identity element.

RECIPROCAL

DEFINITION

IF THE PRODUCT OF TWO NUMBERS IS ONE, THEN THE NUMBERS ARE RECIPROCALS

$$\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$$

$\therefore \frac{3}{4}$ AND $\frac{4}{3}$ ARE RECIPROCALS OF EACH OTHER

GIVE THE RECIPROCAL OF EACH OF THE FOLLOWING:

1. $\frac{1}{2}$
2. $\frac{5}{6}$
3. $1\frac{3}{4}$
4. $2\frac{1}{3}$

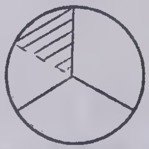
DEVELOPMENT AND EXERCISES

STRAND: *Rational Numbers* LEVEL: *7*
UNIT: *III* OBJECTIVE NUMBER: *B-12*
OBJECTIVE: *Perform the operations of multiplication and division with fractional numbers. (Limit: 1 digit denominator)*

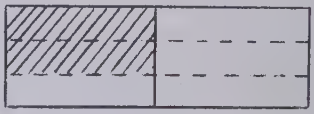
PREREQUISITE SKILLS: Reducing fractions
Basic fractions

SUGGESTED DEVELOPMENT: (1) Try to illustrate the meaning of multiplication by use of models. Select fractions like the following:

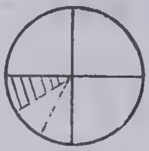
$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$



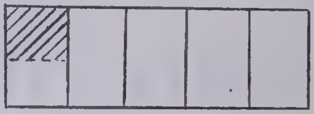
$\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$



$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$



$\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$



(2) To multiply any two or more fractions, you multiply the numerators to find the numerator of the product, and you multiply the denominators to find the denominator of the product.

$$\frac{2}{3} \times \frac{4}{7} = \frac{2 \times 4}{3 \times 7} = \frac{8}{21}$$

(3) A whole number multiplied by a fraction. The whole number can be written as a fraction.

$$4 \times \frac{1}{3} = \frac{4}{1} \times \frac{1}{3} = \frac{4 \times 1}{1 \times 3} = \frac{4}{3}$$

$$\frac{6}{7} \times 5 = \frac{6}{7} \times \frac{5}{1} = \frac{6 \times 5}{7 \times 1} = \frac{30}{7}$$

(4) To multiply mixed numerals, convert these to fractions.

$$1\frac{4}{5} \times 3\frac{1}{3} = \frac{9}{5} \times \frac{10}{3} = \frac{90}{15} = \frac{6}{1} = 6$$

(5) When multiplying fractions, the product can be obtained in basic form if the factors are reduced before multiplying (cancelling).

NOTE: 1. *This procedure can be used only when multiplying.*

2. *Factors reduced must be in numerator and denominator position.*

$$\frac{3}{4} \times \frac{2}{5} = \frac{3 \times \overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}} \times 5} = \frac{3}{10}$$

$$\frac{5}{7} \times \frac{14}{10} = \frac{\overset{1}{\cancel{5}} \times \overset{2}{\cancel{14}}}{\underset{1}{\cancel{7}} \times \underset{2}{\cancel{10}}} = \frac{2}{2} = 1$$

DIVISION

METHOD A: Both fractions have a common denominator.

$$\frac{5}{2} \div \frac{3}{7} = \frac{35 \div 6}{14 \div 14} = \frac{35 \div 6}{1} = 5\frac{5}{6}$$

METHOD B: To find the quotient of two fractions, change the operation from division to multiplication and replace the divisor by its reciprocal. Show that the following statements are equivalent:

$32 \div 4 = 8$ and $32 \times \frac{1}{4} = 8$

$18 \div 3 = 6$ and $18 \times \frac{1}{3} = 6$

DEFINITION:

If the product of two numbers is 1, then the numbers are reciprocals.

The same procedure can be followed for mixed numerals. However, the mixed numerals should be changed to improper fraction first.

$$2\frac{2}{9} \div 6\frac{1}{3} = \frac{20}{9} \div \frac{19}{3} = \frac{20}{9} \times \frac{3}{19} = \frac{20}{57}$$

DIVISION
OF FRACTIONS

1. $\frac{5}{8} \div \frac{3}{7} =$

2. $\frac{4}{5} \div \frac{7}{10} =$

3. $\frac{5}{2} \div \frac{10}{14} =$

4. $\frac{2}{3} \div 1\frac{1}{2} =$

5. $6\frac{7}{8} \div 2\frac{1}{3} =$

RAT 7 OBJ B-12

MULTIPLICATION OF FRACTIONS

1. $\frac{5}{6} \times \frac{3}{10} =$

2. $\frac{1}{5} \times \frac{2}{3} =$

3. $1\frac{1}{2} \times \frac{1}{3} =$

4. $2\frac{3}{4} \times \frac{4}{5} =$

5. $\frac{3}{5} \times \frac{10}{18} \times \frac{4}{5} =$

REMOVING COMMON FACTORS

MULTIPLICATION

$$\frac{3}{8} \times \frac{4}{9}$$

$$= \frac{\cancel{3}^{1}}{2 \times \cancel{4}_{1}} \times \frac{\cancel{4}^{1}}{\cancel{3}_{1} \times 3}$$

$$= \frac{1}{2} \times \frac{1}{3}$$

$$= \frac{1}{6}$$

WHEN MULTIPLYING FRACTIONS ANY FACTORS COMMON TO THE NUMERATOR AND DENOMINATOR MAY BE REMOVED

$$1\frac{1}{2} \times 4$$

$$= \frac{3}{2} \times \frac{4}{1}$$

$$= \frac{3}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{1}$$

$$= \frac{6}{1} = 6$$

$$\frac{3}{5} \times \frac{10}{21} \times \frac{7}{8}$$

$$=$$

1. Carry out the operations

EXAMPLES: i) $\frac{2}{3} \times \frac{1}{8} = \frac{\overset{1}{\cancel{2}} \times 1}{3 \times \cancel{8}_4} = \frac{1}{12}$

ii) $2\frac{1}{2} \times 3\frac{1}{4} = \frac{5}{2} \times \frac{13}{4} = \frac{5 \times 13}{2 \times 4} = \frac{65}{8} = 8\frac{1}{8}$

iii) $\frac{5}{2} \div \frac{3}{7} = \frac{35 \div 6}{14 \div 14} = \frac{35 \div 6}{1} = 5\frac{5}{6}$

iv) $\frac{5}{2} \div \frac{3}{7} = \frac{5}{2} \times \frac{7}{3} = \frac{5 \times 7}{2 \times 3} = \frac{35}{6} = 5\frac{5}{6}$

v) $6\frac{1}{8} \div 2\frac{1}{3} = \frac{49}{8} \div \frac{7}{3} = \frac{\overset{7}{\cancel{49}}}{8} \times \frac{3}{\cancel{7}_1} = \frac{21}{8} = 2\frac{5}{8}$

a) $\frac{2}{3} \times \frac{6}{7} = \frac{4}{7}$

b) $\frac{3}{5} \times \frac{9}{10} = \frac{27}{50}$

c) $8 \times \frac{2}{3} = \frac{16}{3} = 5\frac{1}{3}$

d) $4 \times \frac{5}{8} = \frac{5}{2} = 2\frac{1}{2}$

e) $\frac{7}{8} \times 1\frac{1}{4} = \frac{35}{32} = 1\frac{3}{32}$

f) $1\frac{4}{5} \times 3\frac{1}{3} = \frac{90}{15} = 6$

g) $2\frac{1}{2} \times 1\frac{1}{5} = 3$

h) $1\frac{1}{2} \times \frac{4}{5} \times 2\frac{1}{6} = \frac{13}{5} = 2\frac{3}{5}$

i) $\frac{3}{4} \div \frac{5}{8} = \frac{6}{5} = 1\frac{1}{5}$

j) $\frac{2}{3} \div \frac{1}{5} = \frac{10}{3} = 3\frac{1}{3}$

k) $\frac{3}{8} \div \frac{5}{7} = \frac{21}{40}$

l) $6 \div 4\frac{1}{2} = \frac{4}{3} = 1\frac{1}{3}$

m) $\frac{7}{8} \div 2 = \frac{7}{16}$

n) $5\frac{5}{9} \div 10 = \frac{5}{9}$

o) $12 \div 4\frac{1}{2} = \frac{8}{3} = 2\frac{2}{3}$

p) $\frac{3}{4} \div 1\frac{3}{5} = \frac{15}{32}$

q) $3\frac{1}{5} \div 1\frac{1}{3} = \frac{12}{5} = 2\frac{2}{5}$

r) $2\frac{2}{3} \div 1\frac{5}{9} = \frac{12}{7} = 1\frac{5}{7}$

FIND THE BINGO

FIRST Work out any problem below in lowest terms.

SECOND: Find the answer in the bingo card and cross out the space it is in.

THIRD: Any five correct answers in a straight line (horizontally, vertically, or diagonally) completes the exercise.

$\frac{2}{11}$	6	$\frac{4}{7}$	4	$2\frac{1}{3}$
$\frac{5}{7}$	$\frac{7}{22}$	$\frac{1}{9}$	$\frac{3}{20}$	2
$2\frac{2}{7}$	$\frac{3}{19}$	$5\frac{1}{2}$	$\frac{7}{16}$	$\frac{2}{3}$
$\frac{1}{2}$	$\frac{6}{25}$	$\frac{1}{8}$	$4\frac{3}{4}$	$\frac{1}{7}$
$\frac{1}{12}$	$\frac{1}{3}$	$1\frac{1}{15}$	$1\frac{1}{2}$	12

$\frac{7}{10} \div 2\frac{1}{5} = \frac{7}{22}$

$\frac{6}{5} \times \frac{10}{3} = 4$

$\frac{3}{8} \times 4 = 1\frac{1}{2}$

$\frac{7}{8} \div 2 = \frac{7}{16}$

$1 \div 7 = \frac{1}{7}$

$\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

$\frac{1}{8} \div \frac{1}{4} = \frac{1}{2}$

$2\frac{1}{2} \times \frac{4}{5} = 2$

$\frac{8}{10} \times \frac{3}{10} = \frac{6}{25}$

$\frac{4}{5} \div \frac{3}{4} = 1\frac{1}{15}$

$\frac{7}{3} \times 1 = \frac{7}{3} = 2\frac{1}{3}$

$16 \div 7 = 2\frac{2}{7}$

$\frac{3}{5} \times \frac{1}{4} = \frac{3}{20}$

$\frac{4}{7} \times 1\frac{1}{4} = \frac{5}{7}$

$\frac{4}{9} \div 4 = \frac{1}{9}$

$\frac{1}{4} \div 3 = \frac{1}{12}$

$5 \div \frac{5}{8} = 8$

$\frac{11}{10} \div \frac{1}{5} = 5\frac{1}{2}$

$8 \times \frac{3}{4} = 6$

$\frac{3}{5} \div \frac{9}{10} = \frac{2}{3}$

$4\frac{3}{4} \div 1 = 4\frac{3}{4}$

REVIEW EXERCISES: OBJECTIVE NO'S. B-10 - B-12

Solve the questions below to find the answer to the riddle. Place the letter for each question in the space above the answer.

1. If a hen-and-a-half layed an egg-and-a-half in a day and a half what would you have in five days?

R	O	A	S	T		C	H	I	C	K	E	N
4	$\frac{1}{18}$	$1\frac{1}{10}$	$\frac{9}{28}$	$\frac{1}{2}$		$\frac{3}{4}$	$\frac{7}{12}$	$\frac{2}{3}$	$\frac{3}{4}$	$5\frac{3}{4}$	$\frac{11}{12}$	$\frac{3}{8}$

$\frac{3}{4} + \frac{1}{6} = E$ $\frac{11}{12}$

$\frac{6}{7} \div \frac{3}{14} = R$ 4

$\frac{5}{8} \times \frac{4}{5} = T$ $\frac{1}{2}$

$2\frac{1}{2} + 3\frac{1}{4} = K$ $5\frac{3}{4}$

$\frac{3}{10} \div \frac{2}{5} = C$ $\frac{3}{4}$

$\frac{3}{4} - \frac{3}{7} = S$ $\frac{9}{28}$

$\frac{1}{3} + \frac{3}{5} + \frac{1}{6} = A$ $1\frac{1}{10}$

$5\frac{1}{4} - 4\frac{2}{3} = H$ $\frac{7}{12}$

$\frac{1}{4} \div \frac{2}{3} = N$ $\frac{3}{8}$

$\frac{5}{6} \times \frac{3}{5} \times 1\frac{1}{3} = I$ $\frac{2}{3}$

$\frac{1}{2} \times \frac{1}{9} = O$ $\frac{1}{18}$

2. What do we call a toe which has had an unfortunate experience with a buzz saw?

A	M	I	D	G	E	T		D	I	G	I	T
$\frac{7}{8}$	$\frac{2}{3}$	3	$\frac{1}{15}$	$\frac{1}{2}$	$\frac{8}{27}$	1		$\frac{1}{15}$	3	$\frac{1}{2}$	3	1

$T = \frac{7}{8} \times \frac{4}{5} \times 1\frac{3}{7} = 1$

$A = \frac{3}{8} + \frac{1}{2} = \frac{7}{8}$

$M = \frac{4}{9} \div \frac{2}{3} = \frac{2}{3}$

$G = \frac{9}{10} - \frac{2}{5} = \frac{1}{2}$

$D = \frac{2}{3} - \frac{3}{5} = \frac{1}{15}$

$E = \frac{2}{9} \div \frac{3}{4} = \frac{8}{27}$

$I = 2\frac{2}{5} \times 1\frac{1}{4} = 3$

3. Copy the following in your book and solve as quickly as possible. Time yourself. The first one is done for you.

x ÷	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{9}{10}$
$\frac{4}{7}$	$\frac{1}{14}$ $4\frac{4}{7}$	$\frac{3}{7}$ $\frac{16}{21}$	$\frac{2}{7}$ $1\frac{1}{7}$	$\frac{4}{21}$ $1\frac{5}{7}$	$\frac{8}{35}$ $1\frac{3}{7}$	$\frac{18}{35}$ $\frac{40}{63}$
$\frac{1}{6}$	$\frac{1}{48}$ $1\frac{1}{3}$	$\frac{1}{8}$ $\frac{2}{9}$	$\frac{1}{12}$ $\frac{1}{3}$	$\frac{1}{18}$ $\frac{1}{2}$	$\frac{1}{15}$ $\frac{5}{12}$	$\frac{3}{20}$ $\frac{5}{27}$
$\frac{5}{9}$	$\frac{5}{72}$ $4\frac{4}{9}$	$\frac{5}{12}$ $\frac{20}{27}$	$\frac{5}{18}$ $1\frac{1}{9}$	$\frac{5}{27}$ $1\frac{2}{3}$	$\frac{2}{9}$ $1\frac{1}{18}$	$\frac{1}{2}$ $\frac{50}{81}$
$\frac{2}{3}$	$\frac{1}{12}$ $5\frac{1}{3}$	$\frac{1}{2}$ $\frac{8}{9}$	$\frac{1}{3}$ $1\frac{1}{3}$	$\frac{2}{9}$ 2	$\frac{4}{15}$ $1\frac{2}{3}$	$\frac{3}{5}$ $\frac{20}{27}$
$\frac{6}{7}$	$\frac{3}{28}$ $6\frac{6}{7}$	$\frac{9}{14}$ $1\frac{1}{7}$	$\frac{3}{7}$ $1\frac{5}{7}$	$\frac{2}{7}$ $2\frac{4}{7}$	$\frac{12}{35}$ $2\frac{1}{7}$	$\frac{27}{35}$ $\frac{20}{21}$
$\frac{7}{8}$	$\frac{7}{64}$ 7	$\frac{21}{32}$ $1\frac{1}{6}$	$\frac{7}{16}$ $1\frac{3}{4}$	$\frac{7}{24}$ $2\frac{5}{8}$	$\frac{7}{20}$ $2\frac{3}{16}$	$\frac{63}{80}$ $\frac{35}{36}$

Number of Right Answers ______

Time ______

DEVELOPMENT AND EXERCISES

STRAND: Rational Numbers LEVEL: 7

UNIT: III OBJECTIVE NUMBER: B-13

OBJECTIVE: Demonstrate that division by zero is undefined.

SUGGESTED DEVELOPMENT:

1. Present the following and caution students to watch out for errors in reasoning.

   (a) Suppose $\frac{5}{0} = n$

   (b) Then this means $5 = n \times 0$

   CONCLUDE:

   This proves division by zero is possible, since an aswer can be found.

   Point out that the conclusion is erroneous because it is a false statement.
   ($0 \times n$ should be 0)

   This must mean the first statement has no answer, or is impossible.

2. $\frac{0}{0} = ?$

   Using the same approach as above, suppose $\frac{0}{0} = n$, then $0 = n \times 0$.

   However, "n" could be any number, so again division by zero is not allowed.

DEVELOPMENT AND EXERCISES

STRAND: *Rational Numbers* LEVEL: 7

UNIT: *III* OBJECTIVE NUMBER: *B-14*

OBJECTIVE: *Evaluate an expression by using the properties to produce short cuts in computation. (De-emphasize terminology)*

SUGGESTED DEVELOPMENT: A. COMMUTATIVE PROPERTY

Present the following examples:

1) 273 + 96 + 804 + 137 =

2) 347 + 151 + 293 + 112 + 361 =

Ask students to find the answer to the first problem as quickly as they can. After about 35 or 40 seconds, stop them and inquire if anyone is finished. Point out that a much quicker way to add columns of numbers is to seek groups of 10.

EXAMPLE 1:

- don't add 273 to 94 and then to 814, etc.
- note that 3,7 and 6,4 make groups of 10, even though they are not in order.
- follow through the whole example to show time saved by seeking groups of 10.

Ask students to answer Example 2 as quickly as possible. Stop after 35 or 40 seconds and proceed as before. (Hopefully this time more students will have finished).

B. ASSOCIATIVE PROPERTY

117 + 532 + 461 =

Proceed as before, but point out groups of 10 can be made using 3 or more numbers, not necessarily in order.

C. DISTRIBUTIVE PROPERTY

Ask students to compute 48 x 17 as quickly as they can. After 35 or 40 seconds, stop them and ask if anyone is finished.

Present the following quick method:

C. (Cont'd.)

$48 \times 17 \longrightarrow$ write $(50 - 2) \times 17$

$= 50 \times 17 - 2 \times 17$

$= 850 - 34$

$= 816$ (Time: 15 seconds)

EXAMPLE 2:

$27 \times 14 \longrightarrow$ write $(20 + 7) \times 14$

$= 20 \times 14 + 7 \times 14$

$= 280 + 98$

$= 378$ (Time: 9 seconds)

Doing some more examples involving fractions will surely prove your point:

$2\frac{1}{2} \times 6 \longrightarrow$ write $2 \times 6 + \frac{1}{2} \times 6$

$= 12 + 3$

$= 15$

1. Race the Clock:

Calculate the following as quickly as you can:

(a) $\frac{1}{2} \times \frac{3}{4} \div \frac{3}{4} =$ $\frac{1}{2}$

(b) $1 \div \frac{1}{2} =$ 2

(c) $13(12 - 10) =$ 26

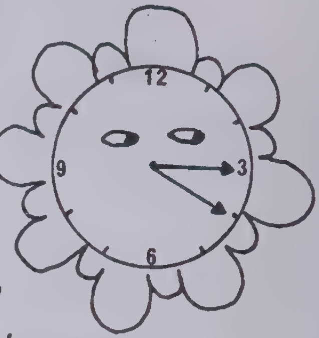
(d) $47 \times 12 =$ 564

(e) $\frac{1}{2} + \frac{2}{3} + \frac{1}{2} =$ $1\frac{2}{3}$

(f) $3\frac{1}{4} \times 8 =$ 26

(g) $2071 + 196 + 44 =$ 2311

(h) $4 + 7 - 3 - 4 =$ 4



Now, go back and use the following code to see if you are a winner against the clock.

ALLOWED TIME:

D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
2405	2	4	$\frac{7}{16}$	$1\frac{3}{5}$	26	$\frac{3}{4}$	39	$\frac{5}{8}$	$\frac{1}{2}$	$1\frac{2}{3}$	$10\frac{1}{4}$	502	$8\frac{1}{2}$	19	2311	$12\frac{1}{2}$	17	564

CODE:

Match the answers to the questions with the letters above, unscramble them, and the time spelled out is the qualifying time.

F I V E M I N S

2. Calculate the following as quickly as you can. As before, your time limit is spelled out below, using the same code as before.

(a) $2\frac{1}{2} \times 5 =$ $12\frac{1}{2}$

(b) $0 + 2\frac{1}{3} - \frac{1}{3} =$ 2

(c) $\frac{2}{3} \times \frac{3}{1} \times \frac{1}{4} =$ $\frac{1}{2}$

(d) $\frac{1}{3} \times 19 \times 3 =$ 19

(e) $\frac{2}{3}(\frac{3}{2} + 1) =$ $1\frac{2}{3}$

(f) $\frac{3}{4} \times \frac{1}{2} \times 26 \times \frac{2}{3} \times 4 =$ 26

(g) $\frac{1}{4} + \frac{3}{5} + \frac{3}{4} =$ $1\frac{3}{5}$

(h) $6 \times 2 \times 1 \times \frac{1}{6} =$ 2

T H R E E M I N

DEVELOPMENT AND EXERCISES

STRAND: Rational Numbers LEVEL: 7

UNIT: III OBJECTIVE NUMBER: B-15

OBJECTIVE: Evaluate expressions involving order of operations.

SUGGESTED DEVELOPMENT: Review order of operations with whole numbers.

RULES:

1) First perform operations within parentheses.

2) Second, perform all multiplications and divisions in left - to - right order.

3) Third, perform all additions and subtractions in left-to-right order.

Many examples of varied operations and length should be done in class. If the class is of higher ability, inclusion of a series of brackets may be challenging.

B - brackets
O - of with fractions } in the order in
M - multiplication } which they occur
D - division
A - addition } in the order in
S - subraction } which they occur

EXERCISES:

1. Evaluate the following:

(a) 5 + 4 x 2 = 13
(b) 4 - 2 x 2 = 0
(c) 6 + 3 ÷ 3 = 7
(d) 7 x 6 - 3 = 39
(e) 64 ÷ (7 + 9 - 8) x 3 = 24

(f) 7 x (5 + 3) ÷ 9 - 5 = $1\frac{2}{9}$
(g) 8 + 7 x (27 ÷ 3 - 4) = 43
(h) (28 ÷ 2 - 6) x (5 + 2) = 56
(i) (7 + 8) - (3 x 4 ÷ 2) = 9
(j) 12 - 3 [14 ÷ (3 + 4)] = 6

2. Solve:

(a) $\frac{2}{3} \times \frac{1}{2} \div \frac{1}{6} =$ 2
(b) $\frac{3}{4} \div \frac{1}{2} \times \frac{3}{5} =$ $\frac{9}{10}$
(c) $\frac{4}{5} \times \frac{1}{2} \div \frac{1}{5} \times \frac{3}{4} = \frac{3}{2} = 1\frac{1}{2}$

(d) $\frac{5}{7} \div \frac{1}{7} \times \frac{5}{3} \div \frac{1}{5} = \frac{125}{3} = 41\frac{2}{3}$
(e) $\frac{1}{3} \times \frac{1}{4} \div \frac{1}{5} \times \frac{1}{6} \div \frac{1}{7} =$ $\frac{35}{72}$

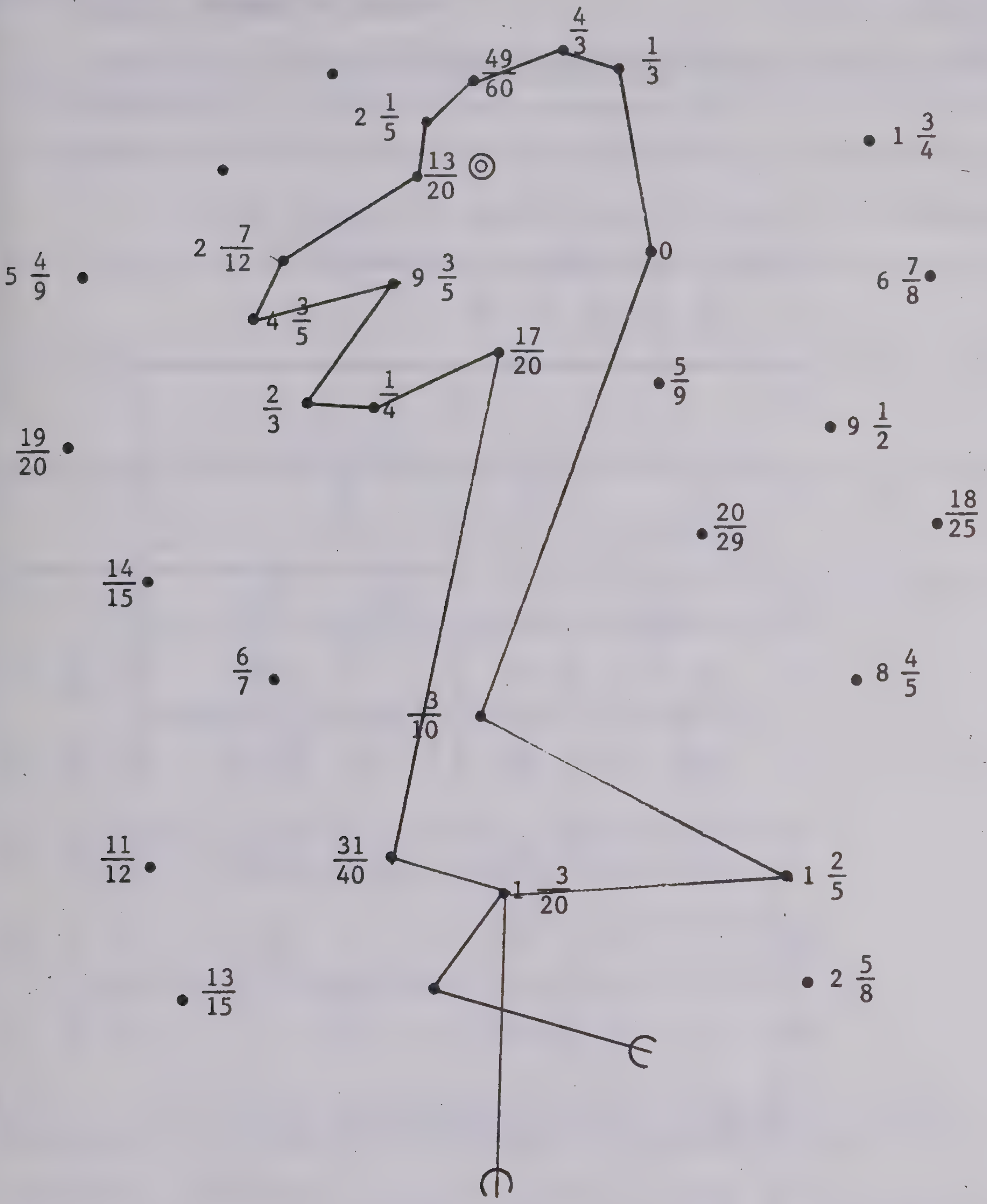
RULES: *1. Do all work in parentheses.*

2. Do all multiplications and divisions in order from left to right.

3. Do all addition and subtraction in order from left to right.

3. Solve the following problems and locate the answers on the next page. If you connect the dots for each correct answer in the same order as the problems are numbered, a cartoon will appear. (There are more answers than there are problems).

1. $\frac{1}{2} + \frac{2}{3} \times \frac{1}{4} =$ $\frac{2}{3}$
2. $\frac{3}{4} - \frac{3}{2} \times \frac{1}{3} =$ $\frac{1}{4}$
3. $\frac{4}{5} \div \frac{1}{2} - \frac{3}{4} =$ $\frac{17}{20}$
4. $\frac{2}{5} + \frac{1}{2} \div \frac{4}{3} =$ $\frac{31}{40}$
5. $\frac{1}{3} \times \frac{1}{4} + \frac{4}{5} \div \frac{3}{4} =$ $1\frac{3}{20}$
6. $\frac{3}{4} + (\frac{2}{5} + \frac{1}{4}) =$ $1\frac{2}{5}$
7. $(\frac{2}{3} - \frac{1}{6}) \times \frac{3}{5} =$ $\frac{3}{10}$
8. $1 - (\frac{1}{2} + \frac{1}{3}) \times \frac{6}{5} =$ 0
9. $\frac{3}{4} - (\frac{3}{5} \times \frac{5}{4}) + \frac{1}{3} =$ $\frac{1}{3}$
10. $(\frac{2}{3} \times \frac{5}{4} - \frac{1}{6}) \div \frac{1}{2} =$ $1\frac{1}{3}$
11. $\frac{1}{8} \div \frac{1}{4} \times \frac{5}{6} + \frac{2}{5} =$ $\frac{49}{60}$
12. $\frac{3}{8} \div (\frac{1}{4} \times \frac{5}{6}) + \frac{2}{5} =$ $2\frac{1}{5}$
13. $(\frac{4}{5} - \frac{1}{4}) + \frac{1}{2} \times \frac{1}{5} =$ $\frac{13}{20}$
14. $\frac{7}{16} \div \frac{3}{4} + \frac{6}{7} \times 2\frac{1}{3} =$ $2\frac{7}{12}$
15. $5 \div \frac{1}{2} - 6 + \frac{3}{5} =$ $4\frac{3}{5}$
16. $6\frac{2}{5} \div (4 \times \frac{1}{2}) \div \frac{1}{3} =$ $9\frac{3}{5}$



*4. Place in the required parenthesis and/or brackets to make the following true:

(a) $\left(\frac{1}{2} + \frac{1}{3}\right) x \frac{6}{5} = 1$

(b) $\left(\frac{1}{2} + \frac{1}{3} - \frac{1}{6}\right) x \frac{3}{4} = \frac{1}{2}$

(c) $\frac{1}{2} x \left(\frac{4}{5} + \frac{1}{5}\right) \div \frac{3}{2} = \frac{1}{3}$

(d) $\frac{1}{2} x \left(\frac{1}{2} + \frac{1}{2}\right) x \frac{1}{2} = \frac{1}{4}$

(e) $\left[\left(\frac{2}{3} - \frac{1}{3}\right) \div \frac{1}{3}\right] x \frac{1}{5} = \frac{1}{5}$

DEVELOPMENT AND EXERCISES

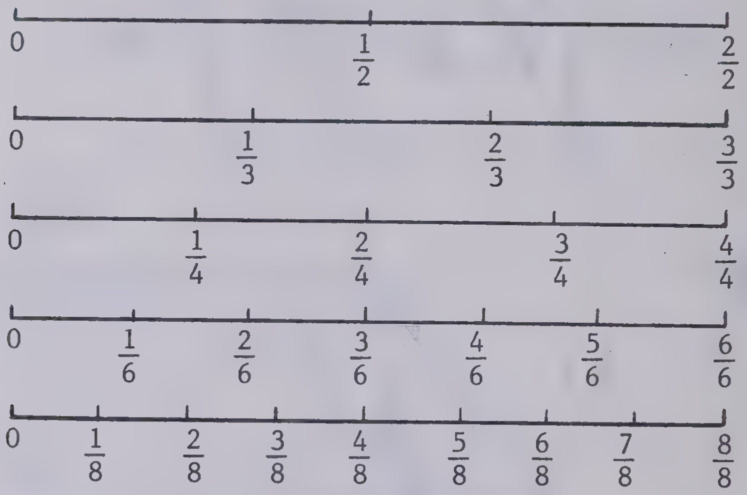
STRAND: Rational Numbers LEVEL: 7

UNIT: III OBJECTIVE NUMBER: B-16

OBJECTIVE: Order fractional numerals using the symbols <, >, =.

SUGGESTED DEVELOPMENT: 1. Have students construct five number lines. Mark off the number lines in order of

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$



Discuss the relative value of fractions, depending on location on number line. If "b" is to the right of "a", b > a or a < b.

2. Use the cross product to determine if one fraction is >, <, or = the other.

$\overset{40}{\frac{5}{6}} \times \overset{42}{\frac{7}{8}}$ 40 < 42, so $\frac{5}{6} < \frac{7}{8}$

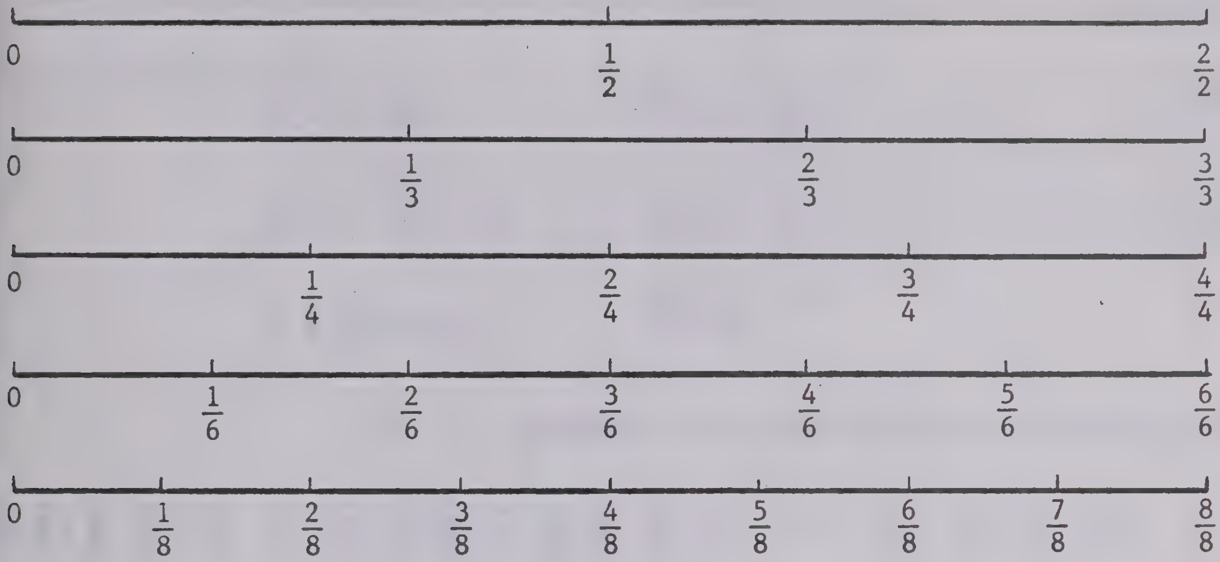
$\overset{15}{\frac{5}{6}} \times \overset{6}{\frac{1}{3}}$ 15 > 6, so $\frac{5}{6} > \frac{1}{3}$

$\overset{24}{\frac{3}{4}} \times \overset{24}{\frac{6}{8}}$ 24 = 24, so $\frac{3}{4} = \frac{6}{8}$

It is imperative that the cross product is written above the fractions as in the examples.

3. Given a series of fractions, their order can be obtained by converting each to equivalent fractions with common denominators and comparing their numerators.

. Using these number lines, indicate whether the following statements are true or false:



(a) $\frac{7}{8} > \frac{5}{8}$ [✓ T] F

(b) $\frac{3}{6} < \frac{5}{6}$ [✓ T] F

(c) $\frac{1}{4} = \frac{2}{8}$ [✓ T] F

(d) $\frac{1}{3} < \frac{2}{3}$ [✓ T] F

(e) $\frac{2}{3} = \frac{5}{6}$ T [✓ F]

(f) $\frac{4}{8} = \frac{2}{4}$ [✓ T] F

(g) $\frac{3}{4} > \frac{2}{3}$ [✓ T] F

(h) $\frac{3}{4} < \frac{5}{6}$ [✓ T] F

(i) $\frac{3}{8} < \frac{1}{3}$ T [✓ F]

(j) $\frac{5}{6} > \frac{7}{8}$ T [✓ F]

. Since it is not always practical to construct number lines to order fractions, you can use the Order Property of Rational Numbers.

Use the Order Property to prove if these statements are true or false.

(a) $\overset{3}{\frac{1}{3}} < \overset{6}{\frac{2}{3}}$ [✓ T] F

(b) $\frac{2}{5} < \frac{4}{5}$ [✓ T] F

(c) $\frac{8}{4} < \frac{7}{4}$ T [✓ F]

(d) $\frac{7}{6} < \frac{9}{6}$ [✓ T] F

(e) $\frac{3}{4} < \frac{5}{6}$ [✓ T] F

(f) $\frac{4}{6} < \frac{5}{7}$ [✓ T] F

(g) $\frac{2}{3} < \frac{2}{5}$ T [✓ F]

(h) $\frac{4}{5} < \frac{4}{6}$ T [✓ F]

(i) $\frac{3}{7} < \frac{2}{4}$ [✓ T] F

(j) $\frac{6}{9} < \frac{5}{7}$ [✓ T] F

3. Place a <, >, or = sign between the following pairs of fractions to make true statements.

(a) $\frac{6}{7} > \frac{5}{6}$

(b) $\frac{2}{5} < \frac{7}{10}$

(c) $\frac{4}{2} > \frac{3}{8}$

(d) $\frac{3}{8} < \frac{2}{5}$

(e) $\frac{3}{2} > \frac{7}{5}$

(f) $\frac{7}{9} < \frac{5}{6}$

(g) $\frac{1}{4} < \frac{2}{5}$

(h) $\frac{7}{8} < \frac{9}{10}$

(i) $\frac{6}{10} = \frac{3}{5}$

4. Order the fractions below from smallest to largest.

(a) $\frac{3}{5}, \frac{1}{2}, \frac{7}{10}$ $\frac{1}{2}, \frac{3}{5}, \frac{7}{10}$

(b) $\frac{5}{6}, \frac{2}{3}, \frac{3}{4}$ $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}$

(c) $\frac{8}{9}, \frac{2}{3}, \frac{5}{6}$ $\frac{2}{3}, \frac{5}{6}, \frac{8}{9}$

(d) $\frac{5}{9}, \frac{4}{7}, \frac{3}{5}$ $\frac{5}{9}, \frac{4}{7}, \frac{3}{5}$

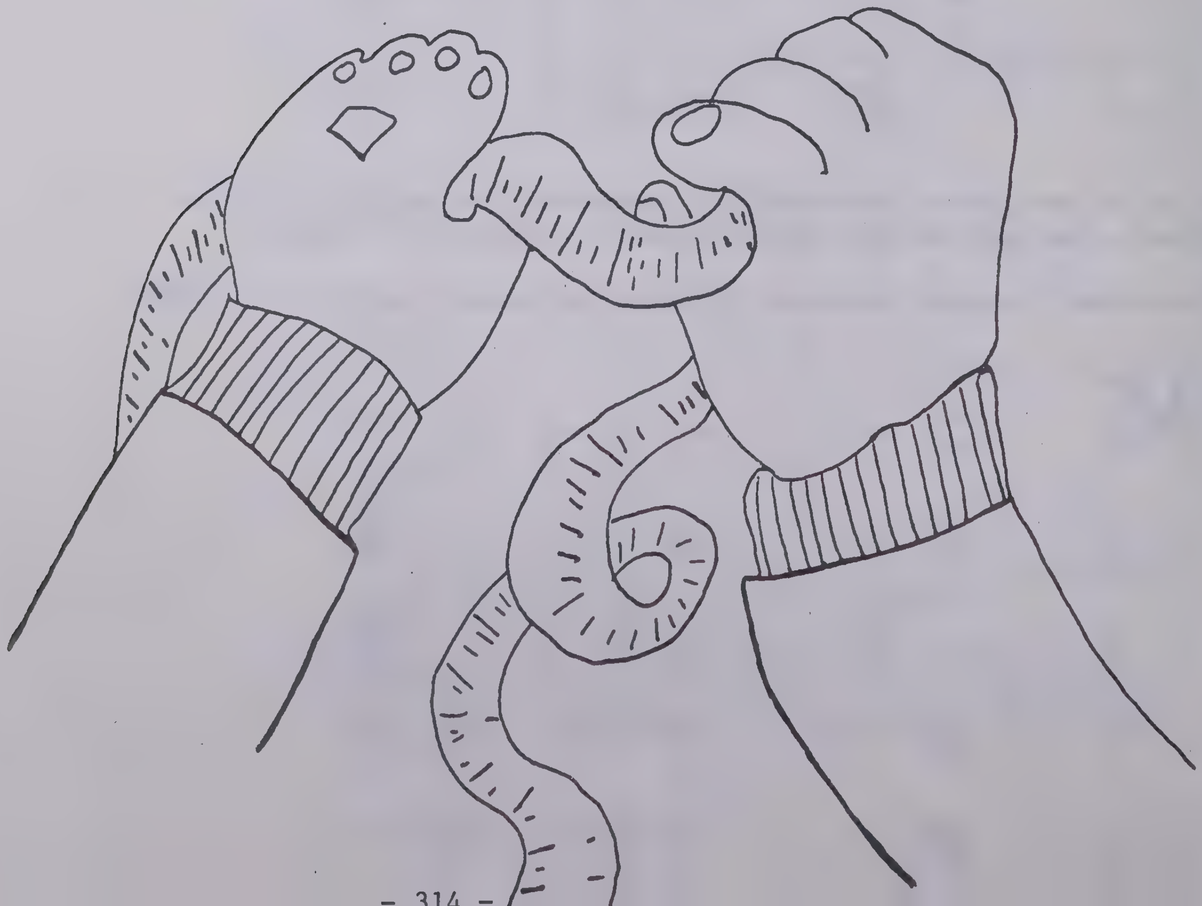
(e) $\frac{7}{8}, \frac{3}{4}, \frac{9}{10}$ $\frac{3}{4}, \frac{7}{8}, \frac{9}{10}$

(f) $\frac{5}{8}, \frac{4}{7}, \frac{3}{6}$ $\frac{3}{6}, \frac{4}{7}, \frac{5}{8}$

(g) $\frac{2}{3}, \frac{3}{4}, \frac{2}{5}, \frac{5}{6}$ $\frac{2}{5}, \frac{2}{3}, \frac{3}{4},$

(h) $\frac{3}{2}, \frac{4}{5}, \frac{5}{4}, \frac{3}{7}$ $\frac{3}{7}, \frac{4}{5}, \frac{5}{4},$

(i) $\frac{7}{6}, \frac{2}{8}, \frac{5}{9}, \frac{7}{4}$ $\frac{2}{8}, \frac{5}{9}, \frac{7}{6},$



STRAND: Rational Numbers LEVEL: 7

UNIT: III OBJECTIVE NUMBER: C-1

OBJECTIVE: Maintain previously developed skills and ideas: change fractions to decimals by use of equivalent fractions, place value to 1000ths.

SUGGESTED DEVELOPMENT: Restrict your questions to those with denominators that are factors of 10, 100, 1000. Take these fractions and obtain equivalent fractions whose denominators are 10, 100, 1000. Then change the numerator to decimal fraction.

EXAMPLES:

(1) $\frac{1}{2} = \frac{5}{10} = 0.5$

(2) $\frac{17}{25} = \frac{68}{100} = 0.68$

(3) $\frac{1}{20} = \frac{5}{100} = 0.05$

(4) $3\,\frac{2}{5} = 3\,\frac{4}{10} = 3.4$

(5) $\frac{9}{40} = \frac{225}{1000} = 0.225$

NOTE: The denominator determines the decimal value.

EXERCISES:

OBJECTIVE NO. C-1

1. Write decimals for:

(a) $\frac{9}{10}$ 0.9

(b) $\frac{37}{100}$ 0.37

(c) $\frac{254}{1000}$ 0.254

(d) $\frac{635}{100}$ 6.35

(e) $\frac{51}{100}$ 0.51

(f) $17\frac{5}{10}$ 17.5

(g) $15\frac{9}{10}$ 15.9

(h) $99\frac{99}{100}$ 99.99

(i) $3\frac{1}{5}$ 3.2

(j) $11\frac{6}{10}$ 11.6

2. Write decimals for:

(a) $\frac{4}{25}$ 0.16

(b) $\frac{7}{8}$ 0.875

(c) $\frac{2}{5}$ 0.4

(d) $\frac{1}{4}$ 0.25

(e) $\frac{13}{50}$ 0.26

(f) $\frac{3}{8}$ 0.375

(g) $\frac{7}{10}$ 0.7

(h) $\frac{3}{5}$ 0.6

(i) $\frac{17}{20}$ 0.85

(j) $\frac{1}{5}$ 0.2

HINT:

8ths and multiples of 8ths convert to 1000ths.

DEVELOPMENT AND EXERCISES

STRAND: Rational Numbers LEVEL: 7

UNIT: III OBJECTIVE NUMBER: C-2

OBJECTIVE: Write any decimal numeral in expanded form. (Use exponential notation only if previously taught)

SUGGESTED DEVELOPMENT: 1. Develop the place value chart for the decimal system.

(N.B. Use exponents only if previously covered)

TEN THOUSANDS	THOUSANDS	HUNDREDS	TENS	ONES		TENTHS	HUNDREDTHS	THOUSANDTHS	TEN-THOUSANDTHS	- Reading
10 000	1 000	100	10	1	•	0.1	0.01	0.001	0.0001	- Place value
10 000	1 000	100	10	1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\ 000}$	- Fraction
10^4	10^3	10^2	10^1	1	•	$\frac{1}{10^1}$	$\frac{1}{10^2}$	$\frac{1}{10^3}$	$\frac{1}{10^4}$	- Exponents

2. Write 972.564 in expanded form as:

$(9 \times 100) + (7 \times 10) + (2 \times 1) + (5 \times \frac{1}{10}) +$

$(6 \times \frac{1}{100}) + (4 \times \frac{1}{1000})$

$3.9 = (3 \times 1) + (9 \times \frac{1}{10})$

$6.254 = (6 \times 1) + (2 \times \frac{1}{10}) + (5 \times \frac{1}{100}) + (4 \times \frac{1}{1000}$

$25.004 = (2 \times 10) + (5 \times 1) + (0 \times \frac{1}{10}) + (4 \times \frac{1}{1000})$

EXPANDED NOTATION

350.621

$$= \left(3 \times 100\right) + \left(5 \times 10\right) + \left(6 \times \frac{1}{10}\right) + \left(2 \times \frac{1}{100}\right) + \left(1 \times \frac{1}{1000}\right)$$

45.327

=

2.4378

=

EXERCISES: OBJECTIVE NO. C-2

1. Complete the chart below.

	TEN THOUSANDS 10 000	THOUSANDS 1 000	HUNDREDS 100	TENS 10	ONES 1	.	TENTHS $\frac{1}{10}$	HUNDREDTHS $\frac{1}{100}$	THOUSANDTHS $\frac{1}{1000}$	TEN-THOUSANDTHS $\frac{1}{10\,000}$
17.2				1	7	.	2			
28.35				2	8	.	3	5		
27.381				2	7	.	3	8	1	
0.273					0	.	2	7	3	
273.01			2	7	3	.	0	1		
725.235		1	7	2	5	.	2	3	5	
1.136					1	.	1	3	6	
238.701			2	3	8	.	7	0	1	
0.017					0	.	0	1	7	
200.0015			2	0	0	.	0	0	1	5

2. Write in expanded form.

(a) 357.8 $3(100)+5(10)+7(1)+8(\frac{1}{10})$ (b) 21.37 $2(10)+1(1)+3(\frac{1}{10})+7(\frac{1}{100})$

(c) 4.392 $4(1)+3(\frac{1}{10})+9(\frac{1}{100})+2(\frac{1}{1000})$ (d) 69.401 $6(10)+9(1)+4(\frac{1}{10})+0(\frac{1}{100})+1(\frac{1}{1000})$

(e) 84.003 $8(10)+4(1)+0(\frac{1}{10})+0(\frac{1}{100})+3(\frac{1}{1000})$ (f) 10.01 $1(10)+0(1)+0(\frac{1}{10})+1(\frac{1}{100})$

(g) 2.075 $2(1)+0(\frac{1}{10})+7(\frac{1}{100})+5(\frac{1}{1000})$ (h) 82.6 $8(10)+2(1)+6(\frac{1}{10})$

(i) 77.707 01 (j) 0.000 03 $0(1)+0(\frac{1}{10})+0(\frac{1}{100})+0(\frac{1}{1000})+0(\frac{1}{10000})+3(\frac{1}{100000})$

$7(10)+7(1)+7(\frac{1}{10})+0(\frac{1}{100})+7(\frac{1}{1000})+0(\frac{1}{10000})+1(\frac{1}{100000})$

To expand a number multiply the face value of each digit (i.e. what you see) by its place value and join the products with '+' signs.

DEVELOPMENT AND EXERCISES

STRAND: *Rational Numbers* LEVEL: *7*

UNIT: *III* OBJECTIVE NUMBER: *C-3*

OBJECTIVE: *Write any decimal numeral in standard form, given expanded form. (Use exponential notation only if previously taught.)*

SUGGESTED DEVELOPMENT: Give a number in expanded form.

EXAMPLE 1:

$$3(10) + 7(1) + 1(\frac{1}{10}) + 3(\frac{1}{100}) + 4(\frac{1}{1000})$$

By multiplying and adding:

$$30 + 7 + \frac{1}{10} + \frac{3}{100} + \frac{4}{1000}$$

$$= 37.134$$

EXAMPLE 2:

$$(8 \times 100) + (0 \times 10) + (0 \times 1) + (0 \times \frac{1}{10})$$

$$+ (3 \times \frac{1}{100}) + (7 \times \frac{1}{1000})$$

$$= 800 + 0 + 0 + \frac{0}{10} + \frac{3}{100} + \frac{7}{1000}$$

$$= 800.037$$

NOTE: Use exponents only if previously covered.

EXPANDED NOTATION

$$\left(3 \times 10\right) + \left(9 \times 1\right) + \left(6 \times \frac{1}{100}\right) + \left(3 \times \frac{1}{1000}\right)$$

$$= 39.063$$

$$\left(4 \times 100\right) + \left(3 \times 10\right) + \left(8 \times 1\right) + \left(7 \times \frac{1}{10}\right) + \left(1 \times \frac{1}{100}\right)$$

=

$$\left(3 \times \frac{1}{10}\right) + \left(7 \times \frac{1}{100}\right) + \left(6 \times \frac{1}{10\,000}\right)$$

=

RAT 7 OBJ C-3

EXERCISES: OBJECTIVE NO. C-3

1. Write decimal numerals for each of the following:
The first one has been done for you.

(a) $(1 \times 1) + (3 \times \frac{1}{10}) + (4 \times \frac{1}{100}) = 1.34$

(b) $(0 \times 1) + (7 \times \frac{1}{10}) + (8 \times \frac{1}{100}) = 0.78$

(c) $(7 \times 1) + (6 \times \frac{1}{10}) + (8 \times \frac{1}{100}) + (9 \times \frac{1}{1000}) = 7.689$

(d) $(1 \times 10) + (1 \times 1) + (0 \times \frac{1}{10}) + (1 \times \frac{1}{100}) + (6 \times \frac{1}{1000}) = 11.016$

(e) $(2 \times 10) + (1 \times 1) + (0 \times \frac{1}{10}) + (0 \times \frac{1}{100}) + (1 \times \frac{1}{1000}) = 21.001$

(f) $(0 \times 1) + (3 \times \frac{1}{10}) + (0 \times \frac{1}{100}) + (4 \times \frac{1}{1000}) = 0.304$

(g) $(2 \times 1) + (3 \times \frac{1}{10}) = 2.3$

(h) $(8 \times 10) + (3 \times 1) + (4 \times \frac{1}{10}) + (3 \times \frac{1}{100}) = 83.43$

(i) $(0 \times 1) + (2 \times \frac{1}{10}) + (9 \times \frac{1}{100}) = 0.29$

(j) $(0 \times 1) + (4 \times \frac{1}{10}) + (1 \times \frac{1}{100}) + (3 \times \frac{1}{1000}) + (6 \times \frac{1}{10\ 000}) = 0.4136$

(k) $(2 \times 10) + (0 \times 1) + (0 \times \frac{1}{10}) + (1 \times \frac{1}{100}) = 20.01$

(l) $(0 \times 1) + (0 \times \frac{1}{10}) + (9 \times \frac{1}{100}) = 0.09$

(m) $(0 \times 1) + (1 \times \frac{1}{10}) + (0 \times \frac{1}{100}) + (3 \times \frac{1}{1000}) = 0.103$

(n) $(4 \times 1) + (3 \times \frac{1}{10}) = 4.3$

(0) $(6 \times 1) + (0 \times \frac{1}{10}) + (0 \times \frac{1}{100}) + (7 \times \frac{1}{1000}) = 6.007$

(p) $(0 \times 1) + (1 \times \frac{1}{10}) = 0.1$

(q) $(0 \times 1) + (0 \times \frac{1}{10}) + (1 \times \frac{1}{100}) + (2 \times \frac{1}{1000}) + (0 \times \frac{1}{10\ 000})$
$+ (4 \times \frac{1}{100\ 000}) = 0.01204$

(r) $(0 \times 1) + (2 \times \frac{1}{10}) + (7 \times \frac{1}{100}) = 0.27$

(s) $(1 \times 1) + (4 \times \frac{1}{10}) = 1.4$

(t) $(3 \times 10) + (0 \times 1) + (0 \times \frac{1}{10}) + (9 \times \frac{1}{100}) = 30.09$

. Write decimal numerals for each of the following:

HINT: *Use zeroes for place holders if necessary.*

(a) $(8 \times 100) + (7 \times 10) + (9 \times 1) + (8 \times \frac{1}{10}) + (9 \times \frac{1}{100}) =$ 879.89

(b) $(7 \times 1) + (3 \times \frac{1}{10}) =$ 7.3

(c) $(2 \times 1) + (3 \times \frac{1}{10}) + (8 \times \frac{1}{100}) =$ 2.38

(d) $(2 \times \frac{1}{10}) + (7 \times \frac{1}{1000}) =$ 0.207

(e) $(6 \times 1) + (4 \times \frac{1}{100}) =$ 6.04

(f) $(7 \times \frac{1}{1000}) =$ 0.007

(g) $(4 \times 10) + (3 \times \frac{1}{100}) =$ 40.03

(h) $(2 \times \frac{1}{10}) + (3 \times \frac{1}{100}) + (4 \times \frac{1}{1000}) =$ 0.234

(i) $(1 \times 10) + (5 \times \frac{1}{10}) + (5 \times \frac{1}{100}) + (7 \times \frac{1}{1000}) =$ 10.557

(j) $(9 \times \frac{1}{10}) =$ 0.9

DEVELOPMENT AND EXERCISES

STRAND: *Rational Numbers* LEVEL: *7*

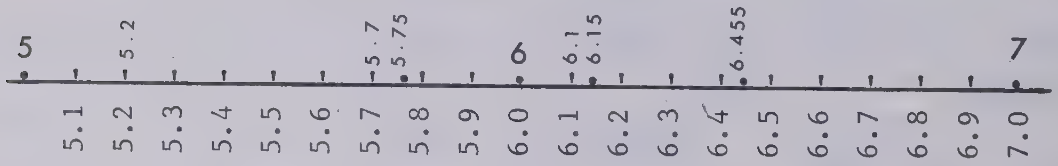
UNIT: *III* OBJECTIVE NUMBER: *C-4*

OBJECTIVE: *Order decimal numerals, using the symbols <, >, = .*

SUGGESTED DEVELOPMENT: 1. Number line. Present the following numbers:

5.7, 5.75, 5.2, 6.1, 6.15, 6.455

Draw a number line from 5 to 7, using tenths as the basis.



Place the above numbers on the line. Now the numbers can be ordered:

5.2, 5.7, 5.75, 6.1, 6.15, 6.455

Write: 5.2 < 5.7, 5.7 < 5.75, etc.

6.455 > 6.15, 6.1 > 5.2, etc.

5.20 = 5.2

2. Present the following numbers:

3.72, 6.8, 5.326, 3.08, 7

3.72	*(hundredths)*
6.8	*(tenths)*
5.326	*(thousandths)*
3.08	*(hundredths)*
7	*(ones)*

Add zeros after the digits following the decimal to thousandths.

3.720, 6.800, 5.326, 3.080, 7.000

Now you can arrange from smallest to largest.

3.080, 3.720, 5.326, 6.800, 7.000

Write: 3.080 < 3.720, 5.326 < 7.000

3.720 > 3.080, 6.800 > 3.080

3.72 = 3.720, etc.

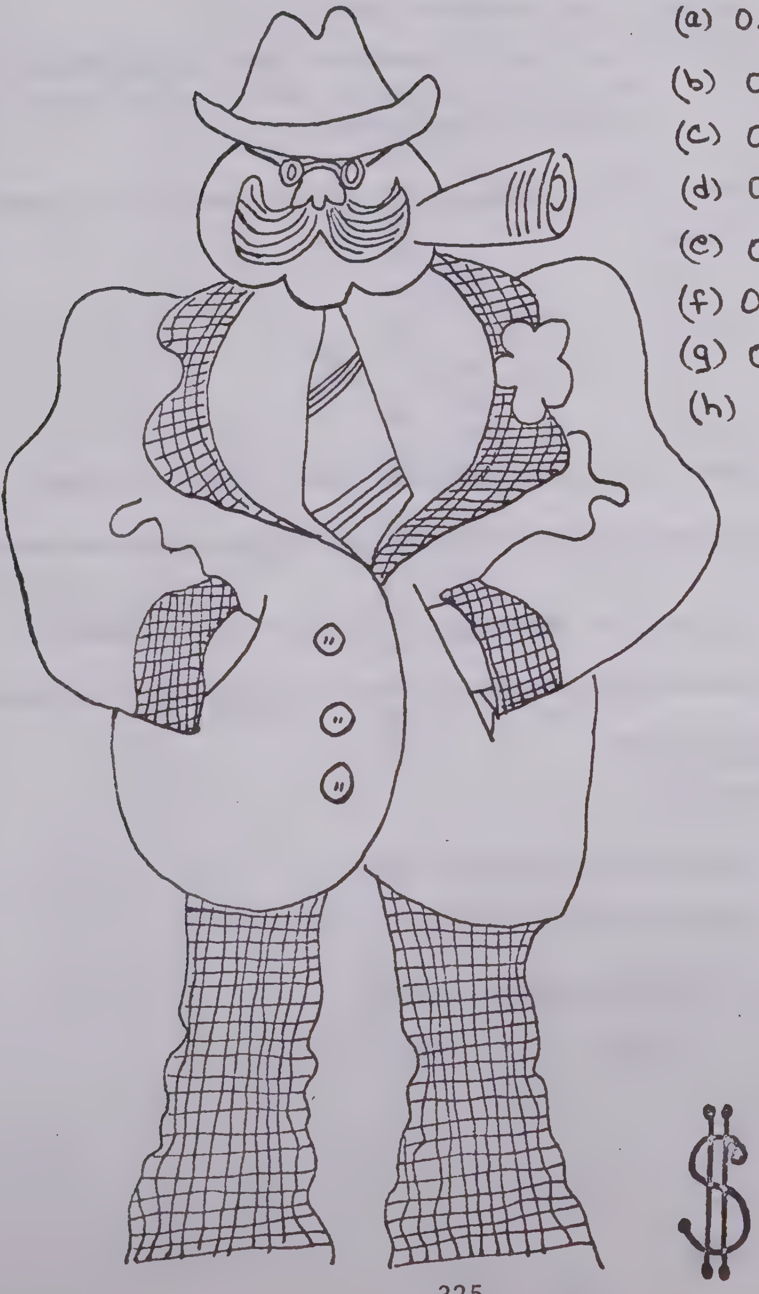
1. Make these statements true by using >, <, =.

(a) 0.380 > 0.038
(b) 1.14 < 1.4
(c) 0.053 = 0.0530
(d) 0.0017 > 0.000 17
(e) 0.3245 < 0.325
(f) 0.7768 > 0.776 09
(g) 4.805 < 4.8051
(h) 6.62 > 6.58
(i) 3.005 < 3.105
(j) 0.1043 > 0.010 43

2. List the numbers in order from least to greatest:

(a) 0.05, 0.1, 0.09
(b) 0.08, 0.06, 0.11
(c) 0.47, 0.43, 0.42
(d) 0.092, 0.089, 0.088
(e) 4.04, 0.44, 0.044, 0.404
(f) 0.56, 0.056, 5.6, 0.5060
(g) 9.2, 0.92, 0.902, 0.092
(h) 10.1, 0.0101, 0.101, 1.01

(a) 0.05, 0.09, 0.1
(b) 0.06, 0.08, 0.11
(c) 0.42, 0.43, 0.47
(d) 0.088, 0.089, 0.092
(e) 0.044, 0.404, 0.44, 4.04
(f) 0.056, 0.5060, 0.56, 5.6
(g) 0.092, 0.902, 0.92, 9.2
(h) 0.0101, 0.101, 1.01, 10.1



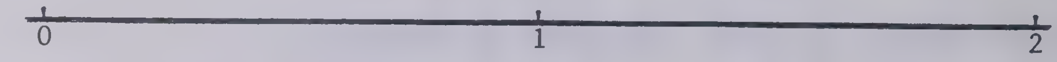
DEVELOPMENT AND EXERCISES

STRAND: *Rational Numbers*　　LEVEL: *7*

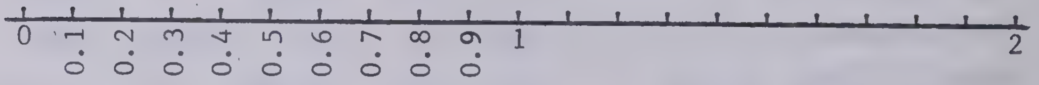
UNIT: *III*　　OBJECTIVE NUMBER: *C-5*

OBJECTIVE: *Demonstrate knowledge of decimals by plotting a given set on the number line. (Limit: 100th)*

SUGGESTED DEVELOPMENT: 1. Draw a line segment and label it as follows:



Divide the segment 0 - 1 into ten segments.



Extend this division from 1 - 2.

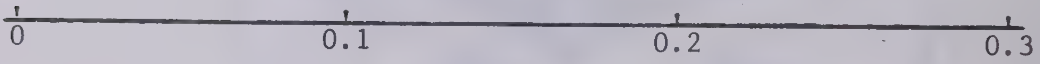
N.B. *Point out to students that if crowding occurs they might prefer to label every second point or every fifth point.*

Have the students find certain points on these lines.

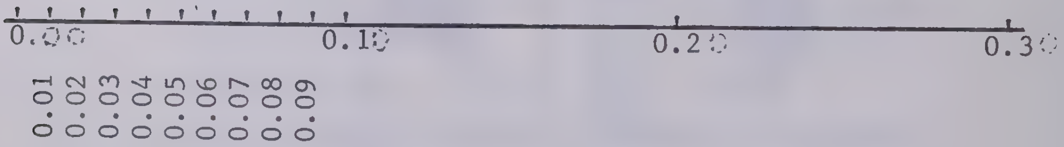
$\{0.1,\ 1.5,\ 1.7\}$

$\{0.3,\ 0.9,\ 1.2,\ 1.7\}$

2. Draw a line segment and label it as follows.



Divide the segment 0 - 0.1 into ten segments.

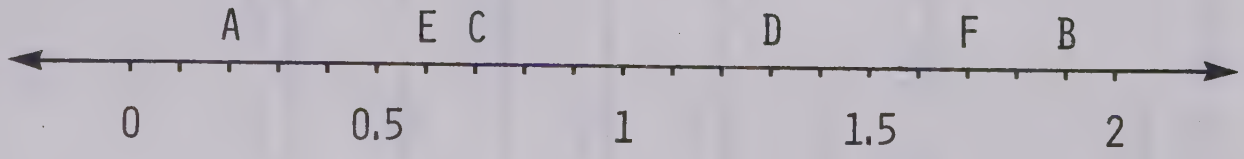


Extend this division from 0.1 - 0.2. Have the students find certain points on these lines.

$\{0.04,\ 0.06,\ 0.08\}$

$\{0.05,\ 0.12,\ 0.17,\ 0.2\}$

DECIMAL NUMBER LINE



WHAT DECIMAL NUMBER IS ASSOCIATED +
WITH EACH OF THE FOLLOWING

A =

B =

C =

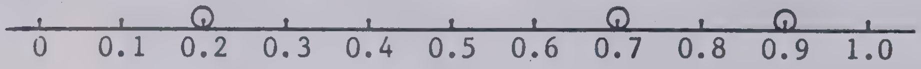
D =

E =

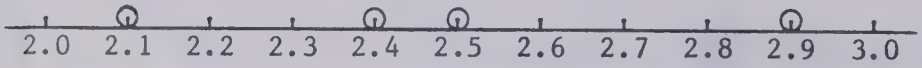
F =

EXAMPLES:

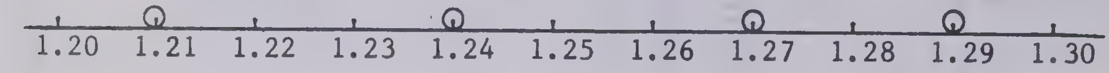
(a) {0.2, 0.7, 0.9}



(b) {2.4, 2.1, 2.9, 2.5}

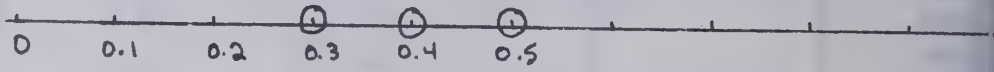


(c) {1.24, 1.27, 1.21, 1.29}

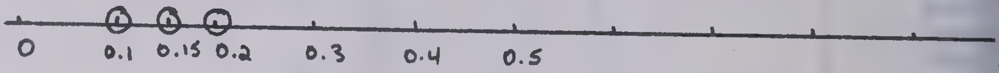


Draw number lines, and graph the following sets:

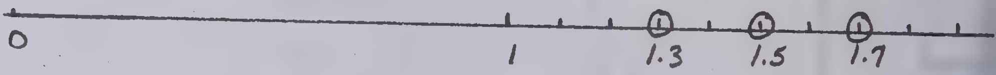
1. {0.3, 0.4, 0.5}



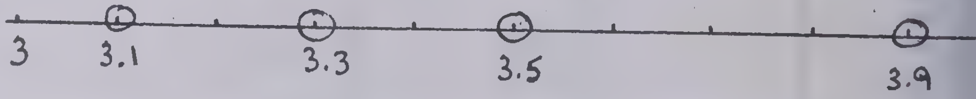
2. {0.10, 0.15, 0.20}



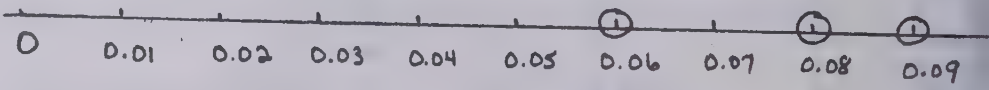
3. {1.7, 1.5, 1.3}



4. {3.1, 3.3, 3.5, 3.9}



5. {0.06, 0.08, 0.09}



DEVELOPMENT AND EXERCISES

TRAND: Rational Numbers LEVEL: 7

NIT: III OBJECTIVE NUMBER: C-6

BJECTIVE: Perform operations of addition and subtraction with decimal numerals.

SUGGESTED DEVELOPMENT: RULES FOR ADDING (OR SUBTRACTING) DECIMAL NUMBERS

1. Line up the decimal points of all numbers to be added (or subtracted).
2. Put in zero placeholders where necessary.
3. Add (or subtract).

*4. If no decimal appears in a number, the decimal is always understood to be at the right hand end of a whole number.

EXAMPLES:

1. 270.4 + 18.61 + 1046

$$\begin{array}{r} 270.40 \\ 18.61 \\ \underline{1046.00} \\ 1335.01 \end{array}$$

2. 476.4 - 19.031

$$\begin{array}{r} 476.400 \\ \underline{19.031} \\ 457.369 \end{array}$$

NOTE: A class of abler students may be interested in examining why decimal points must be aligned. You can use the idea of common denominators to satisfy these concerns.

SUBTRACTION OF DECIMALS

5.63 - 4.132	
5.63 4.132	1. LINE UP THE DECIMALS
5.630 4.132	2. PUT IN ZERO PLACE HOLDERS
5.630 4.132 1.498	3. SUBTRACT

SUBTRACTION
OF DECIMALS

1. 32 - 10.05

2. 16 - 13.32

3. 4.5 - 0.006

ADDITION
OF DECIMALS

1. 270.4 + 18.61 + 104.6 =

2. 476.4 + 19.031 =

3. 2.5 + 6 + 17.001 =

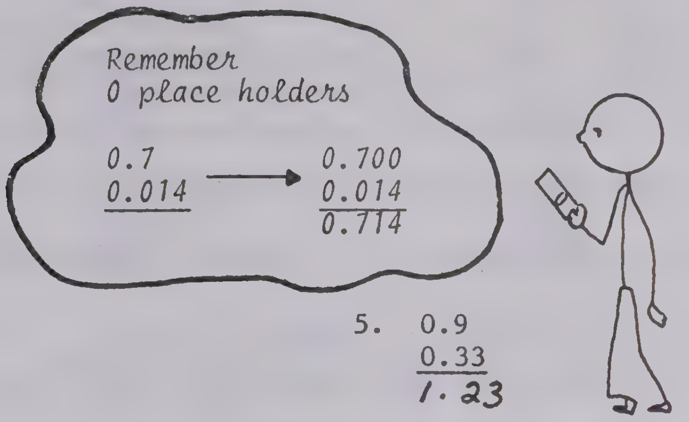
ADDITION OF DECIMALS

3.6 + .042 + 56.7	
3.6 .042 56.7	1. LINE UP THE DECIMAL POINTS
3.600 .042 56.700	2. PUT IN ZERO PLACE HOLDERS
3.600 .042 56.700 60.342	3. ADD

NOTE:

IF NO DECIMAL APPEARS, PUT ONE IN AT THE RIGHT END OF THE NUMBER

65 = 65.0



Remember
0 place holders

0.7 → *0.700*
0.014 → *0.014*
0.714

. Add:

1. 0.2
0.6
0.8

2. 0.37
0.91
1.28

3. 2.8
6.3
9.1

4. 0.5
0.22
0.72

5. 0.9
0.33
1.23

6. 2.1
3.0
5.1

7. 3.8
7.14
10.94

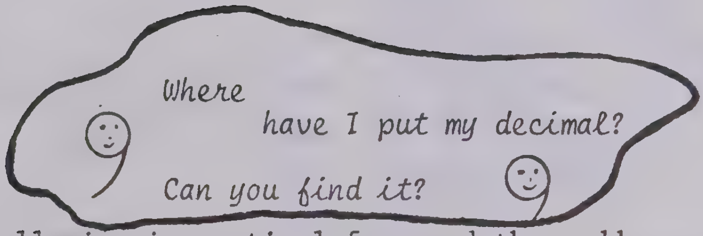
8. 0.9
1.35
2.25

9. 1.07
0.98
2.05

10. 0.63
6.214
6.844

11. 5.1
6.2
11.3

12. 7.1345
0.0123
7.1468



Where
have I put my decimal?
Can you find it?

Rewrite the following in vertical form and then add.

13. 0.2 + 0.57 = 0.77
14. 7 + 0.8 = 7.8
15. 3.87 + 0.6 = 4.47
16. 4.9 + 0.3 = 5.2
17. 7.09 + 0.93 + 2 = 10.02
18. 2.3 + 0.99 = 3.29
19. 0.1 + 0.2 + .03 = 0.33
20. 2.1 + 3.2 + 4.3 = 9.6
21. 3 + 2.6 + 3.1 = 8.7
22. 8 + 24 + 7.1 = 39.1

I. Subtract:

1. 3.1
1.7
1.4

2. 5.41
2.89
2.52

3. 6.5
3.91
2.59

4. 7.61
1.3
6.31

5. 6.1
2.63
3.47

6. 9.
6.31
2.69

7. 5.5
1.83
3.67

8. 12.61
9.27
3.34

9. 15.11
7.
8.11

10. 23.3
19.87
3.43

11. 8.1
4.3
3.8

12. 4.81
2.
2.81

Rewrite the following in vertical form and then subtract.

13. 0.6 - 0.29 = 0.31
14. 0.73 - 0.63 = 0.10
15. 19 - 3.97 = 15.03
16. 1.912 - 1 = 0.912
17. 4.73 - 2.1 = 2.63
18. 7 - 1.1 = 5.9
19. 1.1 - 0.8 = 0.3
20. 4.12 - 1.11 = 3.01
21. 17.34 - 2.86 = 14.48
22. 31.8 - 12 = 19.8

II. Fun time:

1 7	3	▨	2 2	5	▨	3 4
0	▨	4 1	0	▨	5 3	2
▨	6 2	1	▨	7 4	5	▨
8 1	0	▨	9 2	2	▨	10 1
▨	11 1	0	5	▨	12 2	2

YOU DO NOT HAVE TO PUT IN DECIMAL POINTS.

ACROSS

1.	Change to decimal $\frac{73}{100}$	.73
2.	5.3 + 17.7 + 2	25
3.	3 - 2.6	0.4
4.	$0.1 = \frac{?}{100}$	10
5.	15.3 - 14.98	0.32
6.	Write as a decimal $2\frac{1}{10}$	2.1
7.	$4.5 = \frac{?}{10}$	45
8.	$5.1 = \frac{51}{?}$	10
9.	0.6 + 1.6	2.2
10.	2 - 1.9	0.1
11.	$1\frac{5}{100}$ as a decimal	1.05
12.	5.2 - 4.98	0.22

DOWN

1.	$0.7 = \frac{\quad}{100}$	70
2.	1.3 + 0.7	2
3.	Which is greater, 0.36 or 0.42?	0.42
4.	8.3 + 2.7	11
5.	6 - 2.5	3.5
6.	$2\frac{1}{100}$ in decimal form.	2.01
7.	2.3 + 1.8 + 0.1	4.2
9.	5.2 - 4.95	0.25
10.	5.1 + 2.5 + 4.4	12

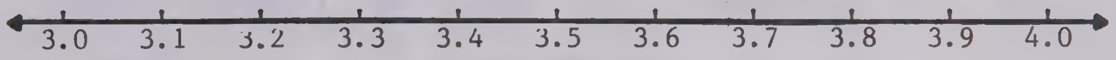
DEVELOPMENT AND EXERCISES

TRAND: *Rational Numbers* LEVEL: *7*

NIT: *III* OBJECTIVE NUMBER: *C-7*

BJECTIVE: *Round off decimal numerals. (Limit: 1000th).*

SUGGESTED DEVELOPMENT: Present the following number line graph:



Consider the number 3.2. Have students determine whether 3.2 is closer to 3.0 or 4.0.

Consider 3.4, 3.6, 3.8.

Consider 3.5. Lead students to determine the rules for rounding off. Use 5 for rounding off to next highest digit.

RULES:

1. *Determine place value required.*
2. *Consider digit to the right of required place.*
3. *If the value is 5 or greater add one to the digit in the required place.*
4. *If the value of the digit to the right is less than 5, the digit in the required place remains the same.*

EXAMPLES:

A) 0.914 to nearest thousandth is 0.914

B) 0.083 to nearest hundredth is 0.08

C) 3.35 to nearest tenths is 3.4

D) 19.98 to nearest tenth is 20.0

ROUNDING OFF DECIMALS

1. 342.1 TO THE NEAREST HUNDRED

2. 67.483 TO THE NEAREST TENTH

3. 42.518 TO THE NEAREST HUNDREDTH

Round off:

1. 13.41 (tenth) 13.4 2. 73.81 (tenth) 73.8

3. 12.617 (hundredth) 12.62 4. 9.890 (hundredth) 9.89

5. 9.995 (hundredth) 10.00 6. 64.0255 (thousandth) 64.026

7. 43.6 (ones) 44 8. 9.8765 (hundredth) 9.88

9. 32.146 (tenth) 32.1 10. 1.005 106 281 (thousandth) 1.005

1. Round off the following number to:

(a) tenth (b) hundredths

(c) thousandths

14.8187

(a) 14.8
(b) 14.82
(c) 14.819

DEVELOPMENT AND EXERCISES

STRAND: *Rational Numbers* LEVEL: 7

UNIT: *III* OBJECTIVE NUMBER: *C-8*

OBJECTIVE: *Multiply and divide decimal numerals by powers of 10.*

SUGGESTED DEVELOPMENT: Place the following questions on the blackboard:

15. x 10 = 15.0

15. x 100 = 15.00

15. x 1000 = 15.000

15. x 10 000 = 15. 0000

15.756 x 10 = 157.56

15.756 x 100 = 1575.6

15.756 x 1000 = 15 756.

15.756 x 10 000 = 157 560.

Multiplying by 10 moves the decimal point one place to the right.

Multiplying by 100 moves the decimal point two places to the right.

Multiplying by 1000 moves the decimal point three places to the right.

Multiplying by 10 000 moves the decimal point four places to the right.

Place this card on the blackboard.

15. ÷ 10 = 1.5

15. ÷ 100 = .15

15. ÷ 1000 = .015

15. ÷ 10 000 = .0015

1575.6 ÷ 10 = 157.56

1575.6 ÷ 100 = 15.576

1575.6 ÷ 1000 = 1.575 6

1575.6 ÷ 10 000 = .15 756

Let students discover the pattern for dividing a decimal by 10, 100, 1000, 10 000.

1. Multiply and show how many places the decimal point moves. The first one is done for you.

1) 25 x 10 = 250.0
2) 15.6 x 100 = 1560.0
3) 6.2 x 10 = 62.0
4) 3.24 x 10 = 32.4
5) 5.65 x 10 = 56.5
6) 2.86 x 100 = 286.0
7) 9.5 x 1000 = 9500.0
8) 0.86 x 10 = 8.6
9) 0.897 x 100 = 89.7
10) 5.24 x 1000 = 5240.0
11) 0.003 x 10 = 0.03
12) 0.003 x 100 = 0.3

2. Calculate the answers: The first one is done for you.

1) 16. ÷ 100 = 0.16
2) 16. ÷ 1000 = 0.016
3) 12.34 ÷ 10 = 1.234
4) $\frac{12.34}{100}$ = 0.1234
5) $\frac{184.3}{1000}$ = 0.1843
6) $\frac{5673.4}{100}$ = 56.734
7) $\frac{1000.01}{1000}$ = 1.00001
8) $\frac{1000.01}{10}$ = 100.001
9) 158.5 ÷ 10 = 15.85
10) 158.5 ÷ 1000 = 0.1585
11) 158.5 ÷ 10 000 = 0.01585
12) $\frac{0.15}{100}$ = 0.0015

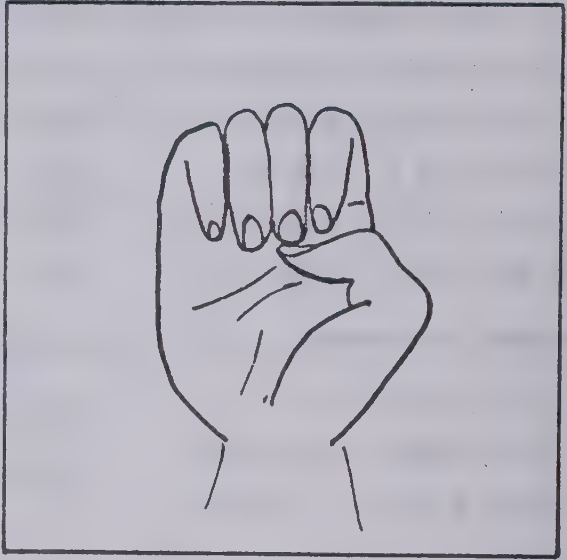
3.

FIND THE TITLE OF THE PICTURE

FIRST: Work out any problem below.

SECOND: Find the answer under the picture at the right.

THIRD: Place letter of question over answer.



H. 18 ÷ 10 = 1.8

E. 87.5 ÷ 10 = 8.75

S. 0.45 x 100 = 45

O. 3.14 ÷ 1000 = 0.003 14

T. 2.4 x 100 = 240

R. 9.7 ÷ 10 = 0.97

T. 0.625 x 10 = 6.25

W. 0.14 x 1000 = 140

E. 6.04 x 100 = 604

T. 0.375 x 100 = 37.5

P. 0.045 x 100 = 4.5

O. 25.4 ÷ 1000 = 0.0254

N. 19.5 ÷ 10 = 1.95

E. 25.6 ÷ 100 = 0.256

TITLE:

P	O	W	E	R
4.5	0.003 14	140	604	0.97

T	O	T	H	E
240	0.0254	6.25	1.8	0.256

T	E	N	S
37.5	8.75	1.95	45

REMEMBER!

A. In multiplication the decimal moves to the right.

A →

B. In division the decimal moves to the left.

← B

DEVELOPMENT AND EXERCISES

STRAND: *Rational Numbers* LEVEL: *7*

UNIT: *III* OBJECTIVE NUMBER: *C-9*

OBJECTIVE: *Perform operations of multiplication and division with decimal numerals.*

SUGGESTED DEVELOPMENT: A. *Rules for Multiplying Decimal Numbers*

1. Line up right end of the numbers.
2. Multiply ordinarily.
3. Count the number of digits to the right of the decimal in each number.
4. Count from the right end of the answer as many digits as you wanted in #3, and put the decimal there.

NOTE: *For abler students, teachers may be interested in explaining why counting the decimal places works. e.g. changing to fractions to satisfy these concerns.*

B. *Rules for Dividing Decimal Numbers*

1. Move the decimal in the divisor to the right end (if necessary).
2. Move the decimal in the dividend as many places to the right as you did in step 1.
3. Place the decimal on top directly above the decimal in the dividend.
4. Divide as usual.

NOTE: *Students should not write remainders in fractional form, but should round off quotients to a specified place value.*

1. Place the decimal point in the product. The first one is done for you.

(a) 0.2 x 4.1 = 0.82 two decimal places
(b) 0.3 x 0.04 = 12 0.012
(c) 0.5 x 0.5 = 25 0.25
(d) 0.15 x 0.3 = 45 0.045
(e) 3 x 0.53 = 159 1.59
(f) 8.1 x 0.01 = 81 0.081
(g) 9 x 0.6 = 54 5.4
(h) 8 x 0.07 = 56 0.56
(i) 23 x 0.04 = 92 0.92
(j) 0.7 x 0.04 = 28 0.028

REMEMBER!!
Multiply ordinarily
and then put in your decimal place.

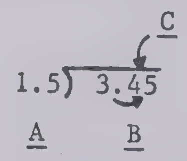
2. Find the following products.

(a) $\begin{array}{r} 3 \\ \underline{0.4} \\ 1.2 \end{array}$ (b) $\begin{array}{r} 12 \\ \underline{0.5} \\ 6.0 \end{array}$ (c) $\begin{array}{r} 3.6 \\ \underline{0.3} \\ 1.08 \end{array}$ (d) $\begin{array}{r} 4.8 \\ \underline{3.9} \\ 18.72 \end{array}$

(e) 2 x 0.6 = 1.2
(f) 10 x 0.8 = 8
(g) 0.2 x 0.5 = 0.10
(h) 0.03 x 0.8 = 0.024
(i) 1.3 x 0.4 = 0.52
(j) 9 x 0.3 = 2.7
(k) 3.3 x 0.02 = 0.066
(1) 12 x 0.003 = 0.036
(m) 60 x 0.5 = 30
(n) 0.0001 x 0.01 = 0.000001

3. Divide:

Steps to Follow:



A. *Move the decimal in the divisor to the right end, if necessary.*

B. *Move the decimal in the dividend as many places to the right as you did in the divisor.*

C. *Place the decimal on top directly above the decimal in the dividend.*

D. *Divide as usual.*

D $\begin{array}{r} 2.3 \\ 15\overline{)\,34.5} \\ \underline{30} \\ 45 \\ \underline{45} \\ 0 \end{array}$

3. (a) 24.3 ÷ 0.3 81
(b) 1.75 ÷ 0.5 3.5
(c) 5.4 ÷ 0.03 180
(d) 3.15 ÷ 0.005 630
(e) 10.584 ÷ 2.1 5.04
(f) 0.384 ÷ 0.4 0.96
(g) 0.297 ÷ 0.3 0.99
(h) 0.9936 ÷ 0.18 5.52
(i) 0.7626 ÷ 1.23 0.62
(j) 0.65 ÷ 2.5 0.26
(k) 564 ÷ 1.2 470

4. Do the following:

(a) 1.069 + 673.527 + 0.088 27 = 674.68427
(b) 51.273 - 49.08 = 2.193
(c) 6.08 x 0.23 = 1.3984
(d) 8740.08 ÷ 79.6 = 109.8
(e) 0.25 x 25 = 6.25
(f) 64.8 ÷ 18 = 3.6
(g) 72.13 + 0.869 + 14.0 + 1.004 = 88.003
(h) 1 - 0.999 43 = 0.00057
(i) 5.018 ÷ 26 = 0.193
(j) 6.8 + 3.798 + 678.1 + 0.0753 = 688.7733
(k) 4.18 x 0.006 = 0.02508
(1) 50 - 5.607 = 44.393

5. The words in parenthesis tell how you are to round off the quotient.

(a) 21.4 ÷ 9 (ones) 2
(b) 9.54 ÷ 30 (ones) 0
(c) 43.1 ÷ 7 (ones) 6
(d) 19.7 ÷ 0.6 (ones) 33
(e) 1.37 ÷ 0.04 (ones) 34
(f) 4.9 ÷ 3 (tenths) 1.6
(g) 8.11 ÷ 4 (tenths) 2.0
(h) 0.137 ÷ 0.09 (tenths) 1.5
(i) 6.8 ÷ 7 (tenths) 1.0
(j) 11.3 ÷ 0.8 (tenths) 14.1
(k) 6.77 ÷ 3 (hundredths) 2.26
(1) 9.12 ÷ 0.7 (hundredths) 13.03
(m) 9.8 ÷ 1.2 (hundredths) 8.17
(n) 9.17 ÷ 4 (hundredths) 2.29
(o) 8.09 ÷ 0.6 (hundredths) 13.48

6. Rewrite in vertical form and multiply.

I. (a) 3.75 x 1.13 4.2375
(b) 0.345 x 0.2 0.0690
(c) 9.8 x 0.0024 0.02352
(d) 3.7 x 0.02 0.074
(e) 0.0056 x 0.000 25 0.0000014
(f) 1.274 x 0.713 0.908362
(g) 92.47 x 8.2 758.254
(h) 17.47 x 2.351 41.07197
(i) 26 x 2.41 62.66
(j) 6.214 x 3.5 21.749
(k) 325.1 x 0.45 146.295
(1) 19.37 x 0.124 2.40188
(m) 8.4 x 9.15 76.86
(n) 16.2 x 25 405
(o) 9.84 x 241 2371.44
(p) 650 x 12.7 8255
(q) 1274 x 0.6 764.4
(r) 1.005 x 8 8.04
(s) 22.04 x 8.2 180.728
(t) 90.71 x 12.3 1115.733
(u) 75.146 x 0.006 0.450876

II. (a) 1.284
x3.16 4.05744

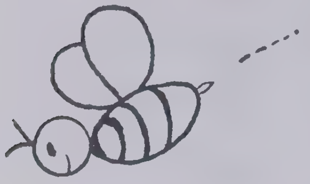
(b) 8.21
x0.614 5.04094

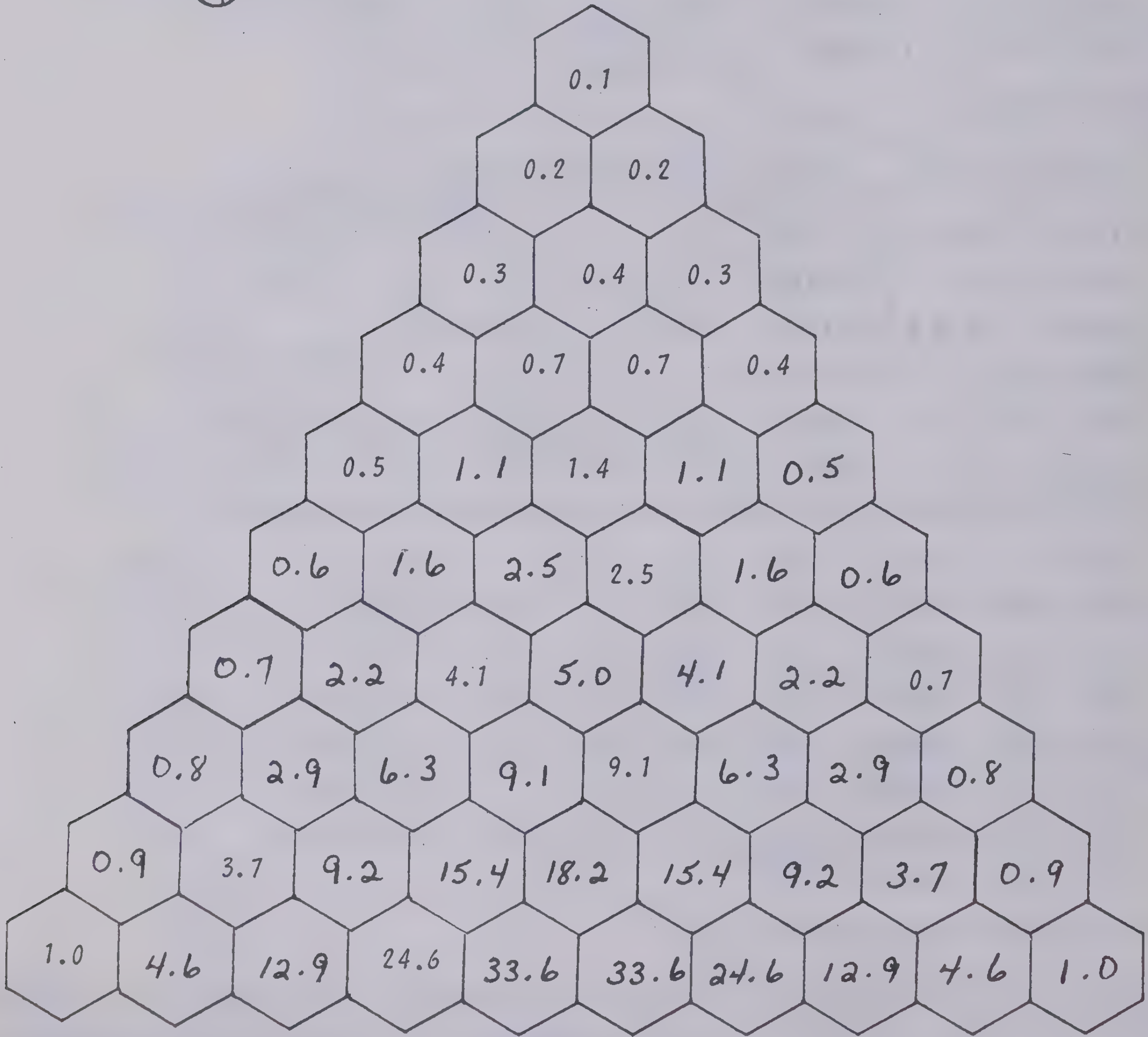
(c) 3.2
x8.64 27.648

(d) 0.2
x6.47 1.294

(e) 3.26
8.54 27.8404

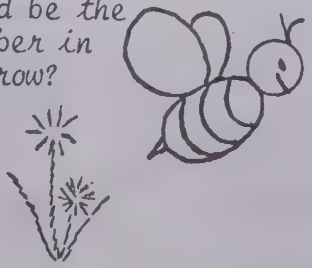
7. *Find a pattern and then complete the figure.*

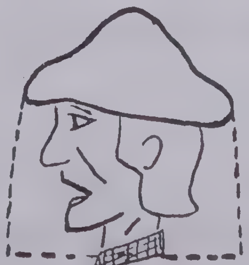




0.1
0.2 0.2
0.3 0.4 0.3
0.4 0.7 0.7 0.4
0.5 1.1 1.4 1.1 0.5
0.6 1.6 2.5 2.5 1.6 0.6
0.7 2.2 4.1 5.0 4.1 2.2 0.7
0.8 2.9 6.3 9.1 9.1 6.3 2.9 0.8
0.9 3.7 9.2 15.4 18.2 15.4 9.2 3.7 0.9
1.0 4.6 12.9 24.6 33.6 33.6 24.6 12.9 4.6 1.0

3.0 *If this continued what would be the first number in the 30th row?*





What would be the first number in the 100th row? 10.0

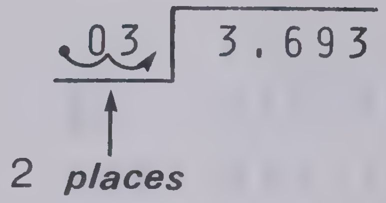
MULTIPLICATION OF DECIMALS

	4.3×5.6
1. MULTIPLY AS IF THE NUMBERS HAVE NO DECIMALS	43 56 258 215 2408
2. COUNT THE NUMBER OF DIGITS TO THE RIGHT OF THE DECIMAL IN EACH NUMBER	4.3×5.6 2 *digits*
3. PLACE THE DECIMAL IN THE PRODUCT WITH THE SAME NUMBER OF DIGITS TO THE RIGHT OF THE DECIMAL AS IN STEP 2	$4.\underline{3} \times 5.\underline{6} = 24.\underbrace{08}_{2\ \textit{digits}}$ 2 *digits*

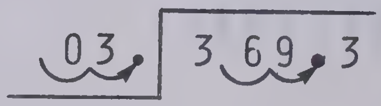
DIVISION OF DECIMALS

1. MOVE THE DECIMAL IN THE DIVISOR TO THE RIGHT END

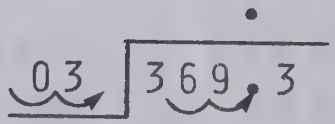
3.693 ÷ .03



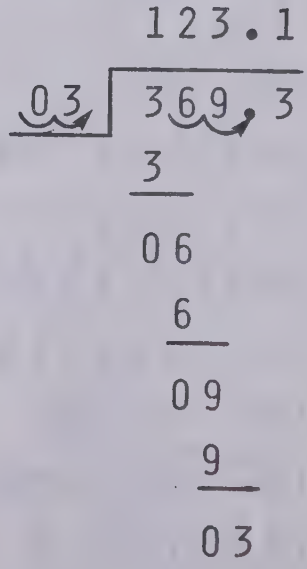
2. MOVE THE DECIMAL IN THE DIVISION THE SAME NUMBER OF PLACES



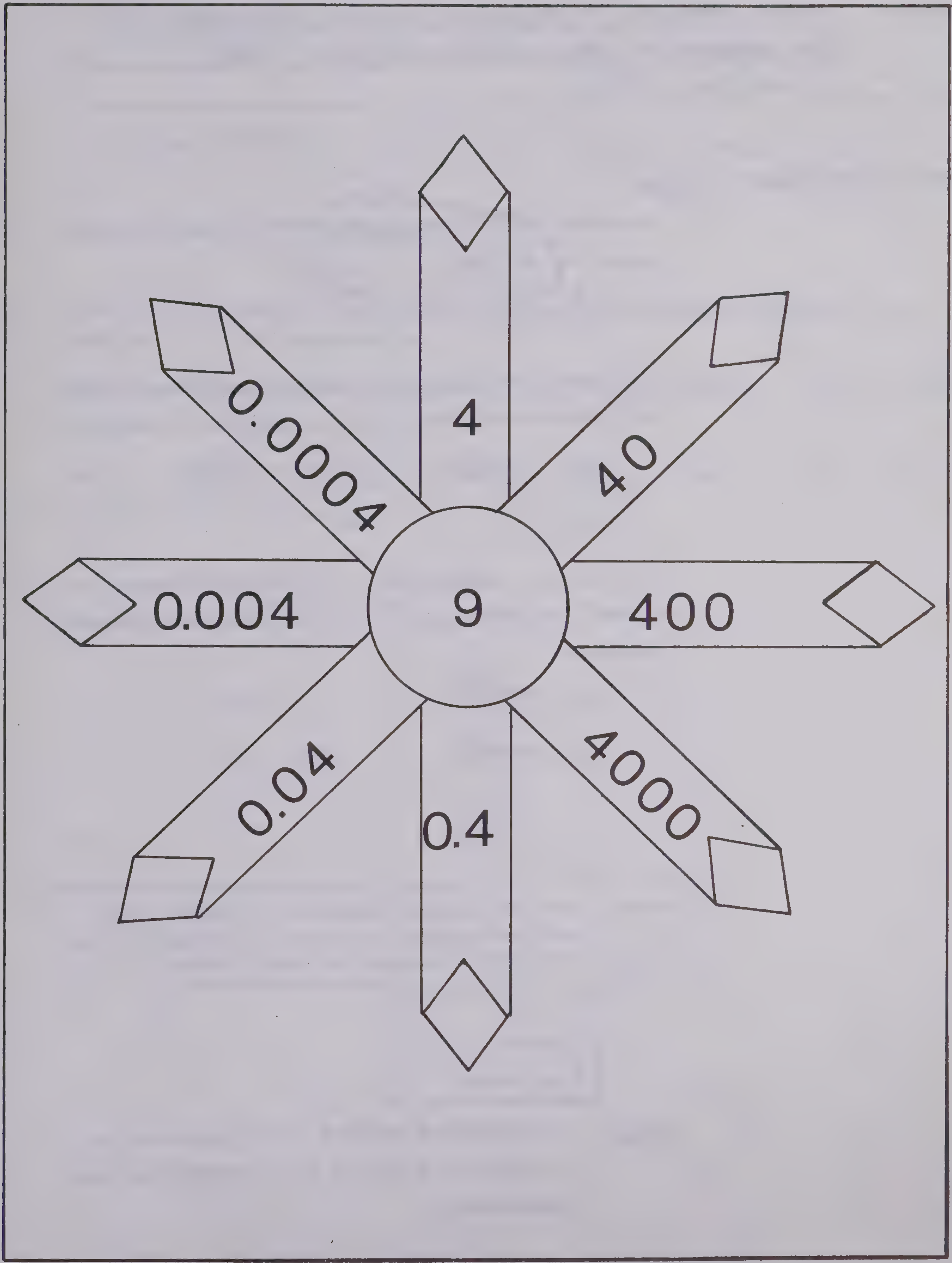
3. PLACE THE DECIMAL IN THE QUOTIENT



4. DIVIDE



Find the Values on the Whirling Wheel by Addition, Subtraction, Multiplication & Division



DEVELOPMENT AND EXERCISES

STRAND: Rational Numbers
LEVEL: 7
UNIT: III
OBJECTIVE NUMBER: C-10
OBJECTIVE: Convert fractions to decimals by dividing the numerator by the denominator. Note repeating decimals. (Limit: denominators of 100).

SUGGESTED DEVELOPMENT: Review

1. Any whole number can be expressed as an infinitely repeating decimal.

$$17 = 17.\overline{0}$$
$$1 = 1.\overline{0}$$

2. The line that separates numerator and denominator in a fraction expresses the operation of division.

(a) $\frac{1}{2} = 1 \div 2$
(c) $\frac{2}{3} = 2 \div 3$
(b) $\frac{7}{8} = 7 \div 8$

If $\frac{7}{8}$ has the meaning of $7 \div 8$, we can change $\frac{7}{8}$ to a decimal by dividing $7.\overline{0}$ by 8 by using our division algorithm.

(a) $$\begin{array}{r} 0.875 \\ 8\overline{)\,7.000} \end{array} \qquad \therefore\ \frac{7}{8} = 0.875$$

(b) $$\begin{array}{r} .66 \\ 3\overline{)\,2.00} \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array} \qquad \therefore\ \frac{2}{3} = 0.\overline{6}$$

Note: That the maximum number of places in the quotient before it begins to repeat is one less than the value of the divisor.

ex. $\frac{1}{7} = 0.142857$

divisor = 7

# of places = 6

Rule: An equivalent decimal for a fraction can be found by dividing the numerator by the denominator.

1. Convert these fractions to decimals.

(a) $\frac{4}{5}$ 0.8 (b) $\frac{3}{8}$ 0.375 (c) $\frac{3}{4}$ 0.75 (d) $\frac{9}{20}$ 0.45

(e) $\frac{5}{8}$ 0.625 (f) $\frac{13}{40}$ 0.325 (g) $\frac{11}{16}$ 0.6875 (h) $\frac{7}{16}$ 0.4375

(i) $\frac{3}{80}$ 0.0375 (j) $\frac{17}{80}$ 0.2125 (k) $\frac{1}{40}$ 0.025 (l) $\frac{5}{16}$ 0.3125

Remember
$4 \div 5 = \frac{4}{5} = 0.8$

2. Change the following fractions to repeating decimals by dividing the numerator by the denominator.

(a) $\frac{1}{3}$ $0.\overline{3}$ (b) $\frac{3}{11}$ $0.\overline{27}$ (c) $\frac{4}{15}$ $0.2\overline{6}$ (d) $\frac{7}{36}$ $0.19\overline{4}$

(e) $\frac{5}{6}$ $0.8\overline{3}$ (f) $\frac{1}{7}$ $0.\overline{142857}$ (g) $\frac{2}{3}$ $0.\overline{6}$ (h) $\frac{7}{3}$ $2.\overline{3}$

DEVELOPMENT AND EXERCISES

STRAND: *Rational Numbers* LEVEL: *7*

UNIT: *III* OBJECTIVE NUMBER: *C-11*

OBJECTIVE: *Convert decimals to fractions (Limit: repeating zero and thirds).*

PREREQUISITE: Place value of decimals

SUGGESTED DEVELOPMENT: 1. Review the chart for decimal system.

hundreds	tens	ones	.	tenths	hundredths	thousandths
5	2	7	.	3	6	8

2. Review the expanded form of decimals

$$3.268 = 3 + \frac{2}{10} + \frac{6}{100} + \frac{8}{1000}$$

3. Finally, express the fraction as a numerator over a multiple of 10.

$$= 3 + \frac{200 + 60 + 8}{1000}$$

$$3.268 = 3\,\frac{268}{1000}$$

Further examples:

$$15.02 = 15 + \frac{0}{10} + \frac{2}{100}$$

$$= 15 + \frac{2}{100}$$

$$= 15\,\frac{2}{100}$$

$$0.007 = \frac{0}{10} + \frac{0}{100} + \frac{7}{1000}$$

$$= \frac{7}{1000}$$

$$3.72 = 3 + \frac{7}{10} + \frac{5}{100}$$

$$= 3 + \frac{70 + 5}{100}$$

$$= 3 + \frac{75}{100}$$

$$= 3\,\frac{75}{100}$$

RULE OF THUMB

Put as many zero's in your denominator as there are decimal places.

e.g. $6.87 = 6\,\frac{87}{100}$

$7.683 = 7\,\frac{68}{100}$

I. Write the following decimals as fractions.

(a) 3.16 $3\frac{4}{25}$ (b) 4.36 $4\frac{9}{25}$ (c) 0.165 $\frac{33}{200}$ (d) 0.01 $\frac{1}{100}$

(e) 20.3 $20\frac{3}{10}$ (f) 7.8 $7\frac{4}{5}$ (g) 12.98 $12\frac{49}{50}$ (h) 12.3 $12\frac{3}{10}$

(i) 1.05 $1\frac{1}{20}$ (j) 5.005 $5\frac{1}{200}$ (k) 8.08 $8\frac{2}{25}$ (l) 5.4 $5\frac{2}{5}$

(m) 0.001 $\frac{1}{1000}$ (n) 2.875 $2\frac{7}{8}$ (o) 23.6 $23\frac{3}{5}$ (p) 19.684 $19\frac{171}{250}$

II. Write the answer to each of the following questions in decimal form. Locate the answers on the number line and use the letters associated with the answers to answer the riddle.

How do you move after an elephant sits on you?

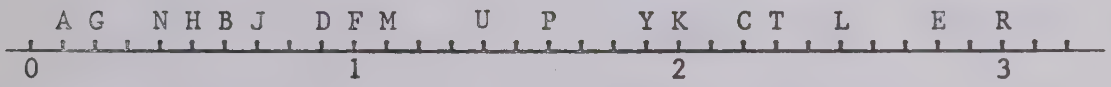
B Y A M B U L A N C E

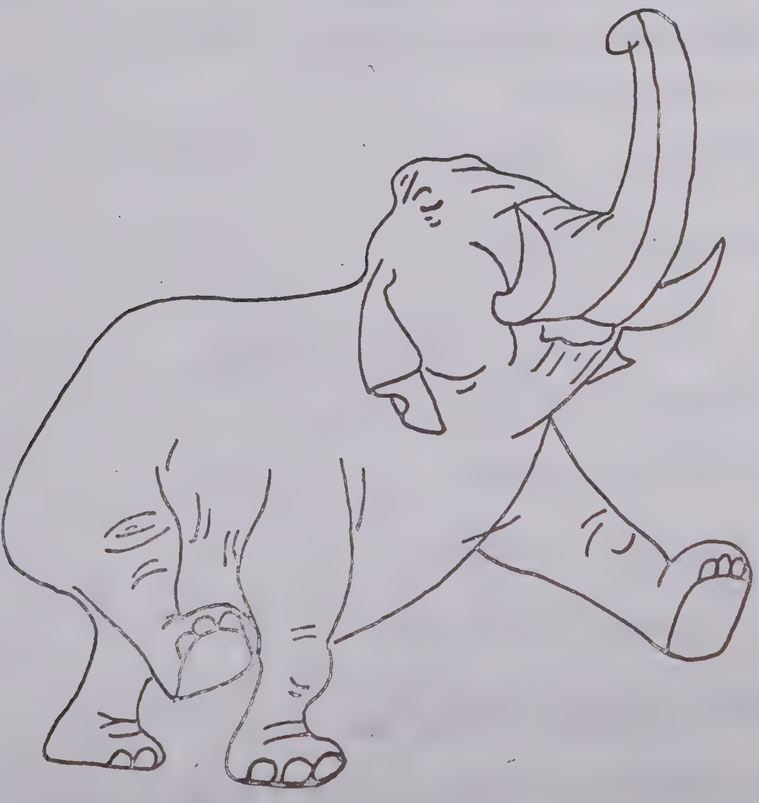
(1) $\frac{3}{5} =$ 0.6 (2) $2 - 0.1 =$ 1.9 (3) $2.74 - 2.64 =$ 0.10

(4) $0.5 + 0.6 =$ 1.1 (5) $1.2 \div 2 =$ 0.6 (6) 7×0.2 1.4

(7) $2\frac{1}{2} =$ 2.5 (8) $\frac{1}{2} \times \frac{1}{5} =$ 0.1 (9) $2 \times 0.1 + 0.2$ 0.4

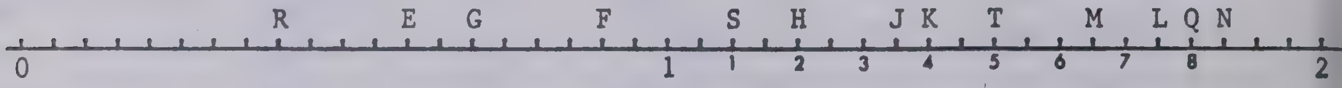
(10) $1.5 + 0.7 =$ 2.2 (11) $7 \times \frac{2}{5} =$ 2.8





REVIEW EXERCISES: OBJECTIVE NO. C-11

Use the number line below to answer questions 1 to 6.



G 1. The decimal 0.7 is associated with letter

K 2. The decimal fraction 1.4 is associated with letter

N 3. The decimal fraction 1.85 is associated with letter

4. The decimal fraction associated with letter 'R' is

b (a) 4/10 (b) 0.4 (c) 0.5 (d) 5/10

5. The decimal fraction associated with letter 'L' is

c (a) 1.85 (b) 1.70 (c) 1.75 (d) 1.80

6. The decimal fraction associated with letter 'M' is

b (a) 1.70 (b) 1.65 (c) 1.60 (d) 1.55

7. 3.542 written in expanded notation would be

(a) $3(1000) + 5(100) + 4(10) + 2(1)$

(b) $3(10) + 5(1) + 4(\frac{1}{10}) + 2(\frac{1}{100})$

d

(c) $3(1) + 5(1) + 3(\frac{1}{10}) + 2(\frac{1}{100})$

(d) $3(1) + 5(\frac{1}{10}) + 4(\frac{1}{100}) + 2(\frac{1}{1000})$

8. Which is the expanded form for 50.802?

(a) $5(10) + 1(8) + 1(2)$

(b) $5(10) + 8(10) + 2(1000)$

d

(c) $5(10) + 8(\frac{1}{10}) + 2(\frac{1}{100})$

(d) $5(10) + 0(1) + 8(\frac{1}{10}) + 0(\frac{1}{100}) + 2(\frac{1}{1000})$

9. What is an expanded form for 0.505 05 ?

(a) $5(10) + 5(100) + 5(1000)$

(b) $5(\frac{1}{10}) + 5(\frac{1}{1000}) + 5(\frac{1}{100\ 000})$

b

(c) $5(10) + 5(1000) + 5(100\ 000)$

(d) $5(\frac{5}{1000}) + 5(\frac{1}{10\ 000}) + 5(\frac{1}{100\ 000})$

10. Which is the decimal for $5(100) + 3(10) + 5(\frac{1}{10}) + 4(\frac{1}{100})$?

c (a) 5354 (b) 53.050 04 (c) 530.54 (d) 530 + 54

11. Which is the decimal for $8(100) + 0(10) + 7(1) + 0(\frac{1}{10}) + 6(\frac{1}{100}) + 0(\frac{1}{1000}) + 5(\frac{1}{10\ 000})$?

d (a) 87.65 (b) 807.065 (c) 87.6005 (d) 807.0605

12. Which is the decimal for three hundred and five tenths?

b (a) 305.0 (b) 300.5 (c) 305.5 (d) 300.05

13. Which is the decimal for 7 tens, 5 tenths, and 15 hundredths?

d (a) 70.515 (b) 7.515 (c) 70.501 500 (d) 70.65

14. 18.4 + 5.7 = n

d (a) 241 (b) 23.1 (c) 18.97 (d) 24.1

15. 204.35 + 0.058 = n

a (a) 204.408 (b) 2.044 08 (c) 262.35 (d) .262 35

16. 3.05 + 40.5 + 205.69 = n

c (a) 212.79 (b) 248.24 (c) 249.24 (d) 915.69

17. 17.4 - 13.8 = n

a (a) 3.6 (b) 4.2 (c) 4.4 (d) 4.6

18. 7.64 - 7.467 = n

a (a) 0.173 (b) 0.183 (c) 0.227 (d) 0.287

19. 1.438 + 8.425 = n

d (a) 7.987 (b) 6.987 (c) 7.097 (d) 9.863

20. 84 x 0.047 =

c (a) 0.039 48 (b) 0.3948 (c) 3.948 (d) 39.48

21. 0.068 x 0.05 = n

a (a) 0.0034 (b) 0.000 34 (c) 0.034 (d) 0.34

22. 5.4 x 6.007 = k

c (a) 5.4063 (b) 3.2778 (c) 32.4378 (d) 32 437.8

23. 3.4 ÷ 0.02 = n

c (a) 17 (b) 1.7 (c) 170 (d) 0.170

24. 5.76 ÷ 0.08 = n

a (a) 72 (b) 720 (c) 0.72 (d) 7.2

25. 1.005 ÷ 2.01 = n

b (a) 0.4 (b) 0.5 (c) 4.6 (d) 4.7

26. 0.9304 ÷ 0.8 = h

b (a) 1.162 (b) 1.163 (c) 0.116 (d) 0.117

27. 8.46 x 100 = k

c (a) 84 (b) 840 (c) 846 (d) 0.0846

28. 64.3 x 100 000 = k

c (a) 64 300 000. (b) 64 300. (c) 6 430 000. (d) 643 105.

29. 74.3 ÷ 1000 = k

d (a) 74 300. (b) 0.000 743 (c) 0.743 (d) 0.0743

30. 8.45 ÷ 100 000 = k

c (a) 845 000. (b) 0.008 45 (c) 0.000 084 5 (d) 0.000 845

31. What is a decimal for $8\frac{7}{100}$?

b (a) 8.7 (b) 8.07 (c) 0.87 (d) 8.007

32. What is a decimal for $\frac{49}{10}$?

c (a) 0.49 (b) 0.049 (c) 4.9 (d) 49.0

33. What is a decimal for $\frac{17}{20}$?

d (a) 0.17 (b) 0.68 (c) 0.085 (d) 0.85

34. What is a fractional numeral for 0.008?

c (a) $\frac{8}{125}$ (b) $\frac{1}{1000}$ (c) $\frac{1}{125}$ (d) $\frac{8}{1250}$

35. Which is the fractional numeral for 0.0807?

b (a) $\frac{807}{1000}$ (b) $\frac{807}{10\ 000}$ (c) $\frac{87}{1000}$ (d) $\frac{807}{100}$

36. Which is a fractional numeral for 24.7?

c (a) $\frac{247}{1000}$ (b) $\frac{247}{10\ 000}$ (c) $\frac{247}{10}$ (d) $\frac{247}{100}$

37. Which is less than 0.0707?

b (a) 0.7007 (b) 0.070 07 (c) 0.08 (d) 0.077

38. 6.52 ____ 6.5028

a (a) > (b) < (c) =

39. What is 16.5934 rounded off to thousandths?

d (a) 16.595 (b) 16.594 (c) 16.59 (d) 16.593

40. What is a decimal for $\frac{1}{3}$?

b (a) 0.33 (b) $0.\overline{3}$ (c) 0.03 (d) 0.333 33

DEVELOPMENT AND EXERCISES

STRAND: *Rational Numbers* LEVEL: *7*

UNIT: *III* OBJECTIVE NUMBER: *C-12*

OBJECTIVE: *Use symbols and notation illustrated below. (Use fractions and decimals).*

(a) Multiplication (i) $0.25 \times 0.1 = (0.25)(0.1)$

(ii) $0.3y = (0.3)(y) = 0.3(y)$

(b) Division

(i) $\frac{0.5}{0.4} = 0.5 \div 0.4 = 0.4\overline{)0.5} = 0.5(\frac{1}{0.4})$

(ii) $\frac{0.2y}{0.50} = \frac{0.2}{0.5}(y) = 0.2y \div 0.5 = \frac{0.2}{0.5}(y)$

SUGGESTED DEVELOPMENT: (a) MULTIPLICATION:

The symbols for multiplication are x, .,( )( ).

Use the same example and show all symbols for multiplication.

NOTE: *In SI the . will not be used for multiplication.*

x is not a good symbol to use when literal coefficients are involved.

EXAMPLE: 0.35×0.2

$(0.35)\ (0.2)$

$0.35 \cdot 0.2$

$0.35(0.2)$

$(0.35)0.2$

Use simple examples:

$0.3y$ means any of: $(0.3)(y)$, $0.3(y)$

(b) DIVISION:

The symbols for division are ÷, $\overline{)}$, —— .

Once again, examples using the same numbers and different notation would be effective here.

Equivalent statements
$$\left\{\begin{array}{l} 0.2 \div y \\ \frac{0.2}{y} \\ \begin{array}{r} 0.5 \\ y\overline{)0.2} \end{array} \end{array}\right.$$

EXERCISES: OBJECTIVE NO. C-12

1. Write the correct number in the correct position.

(a) $0.7 \times 0.9 = 0.7(0.9)$ (b) $1.6 \times 4.2 = 1.6(4.2)$

(c) $2.4 \times 0.9 = 2.4(0.9)$ (d) $0.05(0.062) = 0.05 \times 0.062$

(e) $0.56(0.009) = 0.56 \times 0.009$

2. Place each number in the correct position.

(a) $\frac{0.5}{0.4} = 0.5 \div 0.4 = 0.4\overline{)0.5}$ (b) $\frac{1.2}{5}$ $1.2 \div 5 = 5\overline{)1.2}$

(c) $\frac{0.05}{0.1} = 0.05 \div 0.1 = 0.1\overline{)0.05}$ (d) $\frac{0.092}{0.02} = 0.092 \div 0.02 = 0.02\overline{)0.092}$

(e) $\frac{12.56}{0.2} = 12.56 \div 0.2 = 0.2\overline{)12.56}$

3. MATCHING - Place the letter of the expression in column 'B' in the appropriate blank beside column 'A'.

COLUMN A		COLUMN B
1. $5y$	F	A. $\frac{16}{0.4}$
2. $5(0.8)$	J	B. $3 \div 0.7$
3. $1 \div 0.7$	H	C. $25(1.5)$
4. 1.2×0.4	E	D. $\frac{0.48}{3.6}$
5. $16 \div 0.4$	A	E. $0.4(1.2)$
6. $\frac{3}{0.7}$	B	F. $(5)(y)$
7. 0.4×0.5	I	G. $5(3y)$
8. 25×1.5	C	H. $0.7\overline{)1}$
9. $5(3y)$	G	I. $(0.4)(0.5)$
10. $3.6\overline{)0.48}$	D	J. 5×0.8

4. Write the following statements in any other way. Note: Answers may vary

(a) $0.5(0.6)$ (b) $0.5 \div 0.6$ (c) 0.5×6 (d) $0.6\overline{)5}$

(e) $0.3 \div y$ (f) $y \div 0.3$ (g) $0.3y$ (h) $\frac{0.4}{y}$

(i) $\frac{y}{0.4}$ (j) $0.4(y)$ (k) $y(0.4)$

(a) 0.5×0.6 (b) $\frac{0.5}{0.6}$ (c) $0.5(6)$ (d) $\frac{5}{0.6}$

(e) $\frac{0.3}{y}$ (f) $\frac{y}{0.3}$ (g) $0.3 \times y$ (h) $0.4 \div y$

(i) $y \div 0.4$ (j) $0.4 \times y$ or $0.4y$ (k) $0.4y$ or $y \times 0.4$

DEVELOPMENT AND EXERCISES

STRAND: *Rational Numbers* LEVEL: *7*
UNIT: *III* OBJECTIVE NUMBER: *C-13*
OBJECTIVE: *Evaluate expressions by substituting for the variable.*
(Limit: Fractions , decimals and one variable).

SUGGESTED DEVELOPMENT: 1. *DEFINITION:* *A numerical expression is a name or symbol of a number*

$4 + 2$

$3.7 + 2.3$

0.15×40

These numerical expressions are different ways of writing the value of 6. When we find a single number named, it is called the *value* of the expression.

2. *DEFINITION:* *An expression which contains a variable is called a variable expression.*

e.g. 1) $3.5a + 4.7$ is a variable expression because it contains the variable "a".

2) $0.6b + c + 4.5d$

3) $z - y$

In such variable expressions, we cannot find a number named by the expression unless we are given a value for the variable. Once we are given the value for the variable, we can find the value of the expression. This is called *evaluating* an expression.

(a) $5.3a$

Replace "a" by 0.45

$5.3(.45) = 2.385$

NOTE: *Explain use of brackets when replacing variable by number.*

(b) $0.6b + c + 4.5d$ $b = 3.2$

$0.6(3.2) + 0.8 + 4.5(3)$ $c = 0.8$

$1.92 + .80 + 13.50$ $d = 3$

$= 16.22$

(c) $z - y$ $z = 23.5$

$23.5 - 15.67$ $y = 15.67$

$= 7.83$

SUBSTITUTION

EXPRESSION	*REPLACE "x" BY*	*REPLACEMENT*	*VALUE*
$3x$	6	$3(6)$	18
$2x + 0.4$	5.2	$2(5.2) + 0.4$	10.8
$0.4x + 6$	2.4		
$3x + 0.02$	0.5		
$3x + 5x$	$\frac{4}{15}$		
$1.6 + 2x$	0.01		

RAT 7 OBJ. C-13

1. Evaluate each variable expression. The replacement set for "x" is $\{0.3,\ 1.4,\ 0.325\}$

   (a) y + 7.5 (b) 5(y) (c) 3(y) - 0.51 (d) 75.3 - 10(y)
   7.8, 8.9, 7.825 1.5, 7, 1.625 0.39, 3.69, 0.465 72.3, 61.3, 72.05

2. Evaluate each expression given the value of "a" is $\frac{3}{5}$ and the value of "b" is $\frac{3}{4}$.

   (a) 5a + 2 (b) 2a - 1 (c) $3b + 2\frac{1}{2}$ (d) $2a + 3\frac{2}{5}$
   5 $\frac{1}{5}$ $4\frac{3}{4}$ $4\frac{3}{5}$

3. (a) Replace "a" by 0.09 in: 0.3 + a + 2a 0.57
   (b) Replace "b" by 5.7 in: 6b - 0.53 33.67
   (c) Replace "c" by 0.36 in: c + c - 0.25 0.47
   (d) Replace "y" by $\frac{4}{5}$ in: 5y + y $4\frac{4}{5}$
   (e) Replace "m" by 0.105 in: 15m - 0.59 x 2 0.395

4. Complete the following chart:

	Expression	Replace "a" by	Replacement	Value
(a)	15a	$\frac{2}{3}$	$15 \times \frac{2}{3}$	= 10
(b)	3 + a	$12\frac{3}{5}$	$3 + 12\frac{3}{5}$	= $15\frac{3}{5}$
(c)	(a)(2.6)	0.23	(0.23)(2.6)	= 0.598
(d)	3.5a - 15.658	12.3	3.5(12.3) - 15.658	= 27.392
(e)	a + 12.67 + a	5.9	5.9 + 12.67 + 5.9	= 24.47
(f)	$\frac{3}{4}a + \frac{6}{7}$	$\frac{4}{7}$	$\frac{3}{4}\left(\frac{4}{7}\right) + \frac{6}{7}$	= $1\frac{2}{7}$

5. (a) Evaluate (15b + 2c) when b = 2.4 and c = 0.5 37
   (b) Evalute (6b - c) when b = $1\frac{5}{6}$ and c = $\frac{1}{4}$ $10\frac{3}{4}$

*6. Evaluate the following expression:
   (3c + .5b) - (0.01c + 1.2b) when c = 4.5 and b = 0.3 13.245

DEVELOPMENT AND EXERCISES

STRAND: *Rational Numbers* LEVEL: 7

UNIT: *III* OBJECTIVE NUMBER: C-14

OBJECTIVE: *Solve the following conditions (Limit: Fractions and decimals).*

PREREQUISITE SKILLS: Formalized approach to solving conditions.

Review meaning of related and equivalent statements.

SUGGESTED DEVELOPMENT: Use condition of the forms

a) $y + A = B$

b) $Ay = B$

c) $Ay + B = C$

d) $Ay + By = C$

I. Related Number Sentences

(a) $\boxed{x + a = b}$

Statement	① $3 + 2 = 5$	② $x + 2 = 5$	③ $2 + x = 5$
Related Statement	① $3 = 5 - 2$	② $x = 5 - 2$	③ $x = 5 - 2$

To solve a condition of the type $x + a = b$, write the related condition. i.e. $x = b - a$

(b) $\boxed{ax = b}$

Statement	① $2(3) = 6$	② $2x = 6$	③ $x(2) = 6$
Related Statement	① $3 = 6 \div 2$	② $x = 6 \div 2$	③ $x = 6 \div 2$

To solve a condition of the type $ax = b$, write the related condition. i.e. $x = b \div a$.

(c) $\boxed{\frac{x}{a} = \frac{b}{c}}$

$\frac{2}{4} \times \frac{8}{16}$

$2 \times 16 = 4 \times 8$

$2(16) = 32$

$16 = 32 \div 2$

$\frac{x}{4} = \frac{5}{8}$

$x(8) = 4 \times 5$

$x(8) = 20$

$x = 20 \div 8$

(c) (Cont'd.)

$$\frac{4}{x} = \frac{8}{5}$$

$$4(5) = x(8)$$

$$20 = 8x$$

$$\frac{20}{8} = x$$

To solve a condition of the type $\frac{x}{a} \times \frac{b}{c}$

First find the cross products $x(c) = ab$,
then find the solution $x = ab \div c$.

See method 2 for final step.

*(d) $\boxed{ax + = c}$

Statement	$2(3) + 4 = 10$	$2x + 4 = 10$
Related Statements	$2(3) = 10 - 4$	$2x = 10 - 4$
	$3 = (10 - 4) \div 2$	$x = (10 - 4) \div 2$

To solve a condition of the type $ax + b = c$, write the related statements and follow previous procedure.

*(e) $\boxed{ax + bx = c}$

$2(5) + 4(5) = 30$	$2x + 4x = 30$
$6(5) = 30$	$6x = 30$
$5 = 30 \div 6$	$x = 30 \div 6$

If we have to solve a condition of the type $ax + bx = c$, then the solution is the same as for $x = c \div (a + b)$.

After combining like terms, follow procedure as for type "b".

II. Balance Approach

a) $y + A = B$ $y + A - A = B - A$ $y = B - A$ e.g. $y + 0.2 = 0.3$ $y + 0.2 - 0.2 = 0.3 - 0.2$ $y = 0.1$	e.g. $y + \frac{1}{2} = \frac{7}{8}$ $y + \frac{1}{2} - \frac{1}{2} = \frac{7}{8} - \frac{1}{2}$ $y = \frac{7}{8} - \frac{1}{2}$ $y = \frac{7}{8} - \frac{4}{8}$ $y = \frac{3}{8}$
b) $y - A = B$ $y - A + A = B + A$ $y = B + A$ e.g. $y - 0.2 = 0.5$ $y - 0.2 + 0.2 = 0.5 + 0.2$ $y = 0.7$	e.g. $y - \frac{1}{4} = \frac{5}{8}$ $y - \frac{1}{4} + \frac{1}{4} = \frac{5}{8} + \frac{1}{4}$ $y = \frac{5}{8} + \frac{2}{8}$ $y = \frac{7}{8}$
c) $Ay = B$ $\frac{Ay}{A} = \frac{B}{A}$ $y = \frac{B}{A}$ $0.2(y) = 0.06$ $\frac{0.2y}{0.2} = \frac{0.06}{0.2}$ $y = 0.3$	e.g. $\frac{1}{2} y = 4$ $\frac{2}{1} \times \frac{1}{2} y = 4 \times \frac{2}{1}$ $y = 4 \times \frac{2}{1}$ $y = 8$ NOTE: *Review use of reciprocal.*
d) $\frac{y}{A} = B$ $\frac{y}{A} \times A = B \times A$ $y = B \times A$ e.g. $\frac{y}{0.2} = 0.8$ $\frac{y}{0.2} \times 0.2 = 0.8 \times 0.2$ $y = 0.16$	

e)

$$Ay + B = C$$
$$Ay + B - B = C - B$$
$$Ay = C - B$$
$$\frac{Ay}{A} = \frac{C - B}{A}$$
$$\therefore \quad y = \frac{C - B}{A}$$

e.g.

$$0.2y + 0.6 = 0.66$$
$$0.2y + 0.6 - 0.6 = 0.66 - 0.6$$
$$0.2y = 0.06$$
$$\frac{0.2y}{0.2} = \frac{0.06}{0.2}$$
$$y = 0.3$$

---

*f)

$$Ay + By = C$$
$$(A + B)y = C$$
$$\frac{(A + B)y}{(A + B)} = C \div (A + B)$$

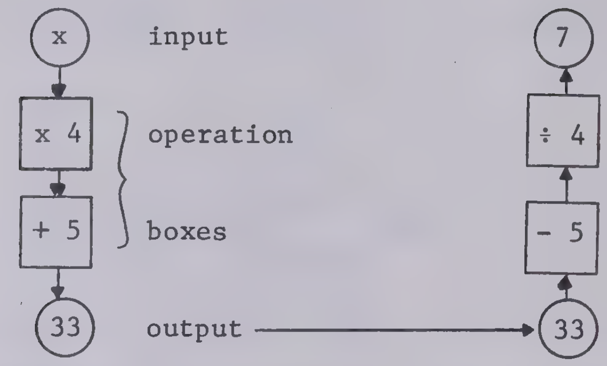
e.g.

$$0.3y + 0.2y = 0.05$$
$$(0.3 + 0.2)y = 0.05$$
$$\frac{(0.5)y}{(0.5)} = \frac{0.05}{(0.5)}$$
$$y = 0.1$$

NOTE: *See whole number developments for additional approaches to solving conditions.*

III. Flow Chart Approach

EXAMPLE: $4x + 5 = 33$

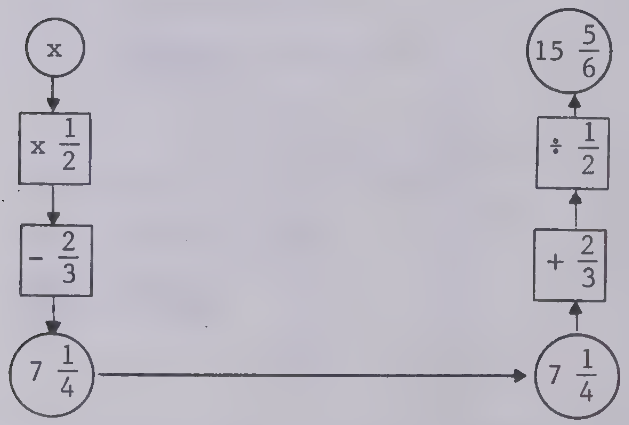


Therefore,

$x = 7$

To solve, reverse the flow chart and use opposite operation.

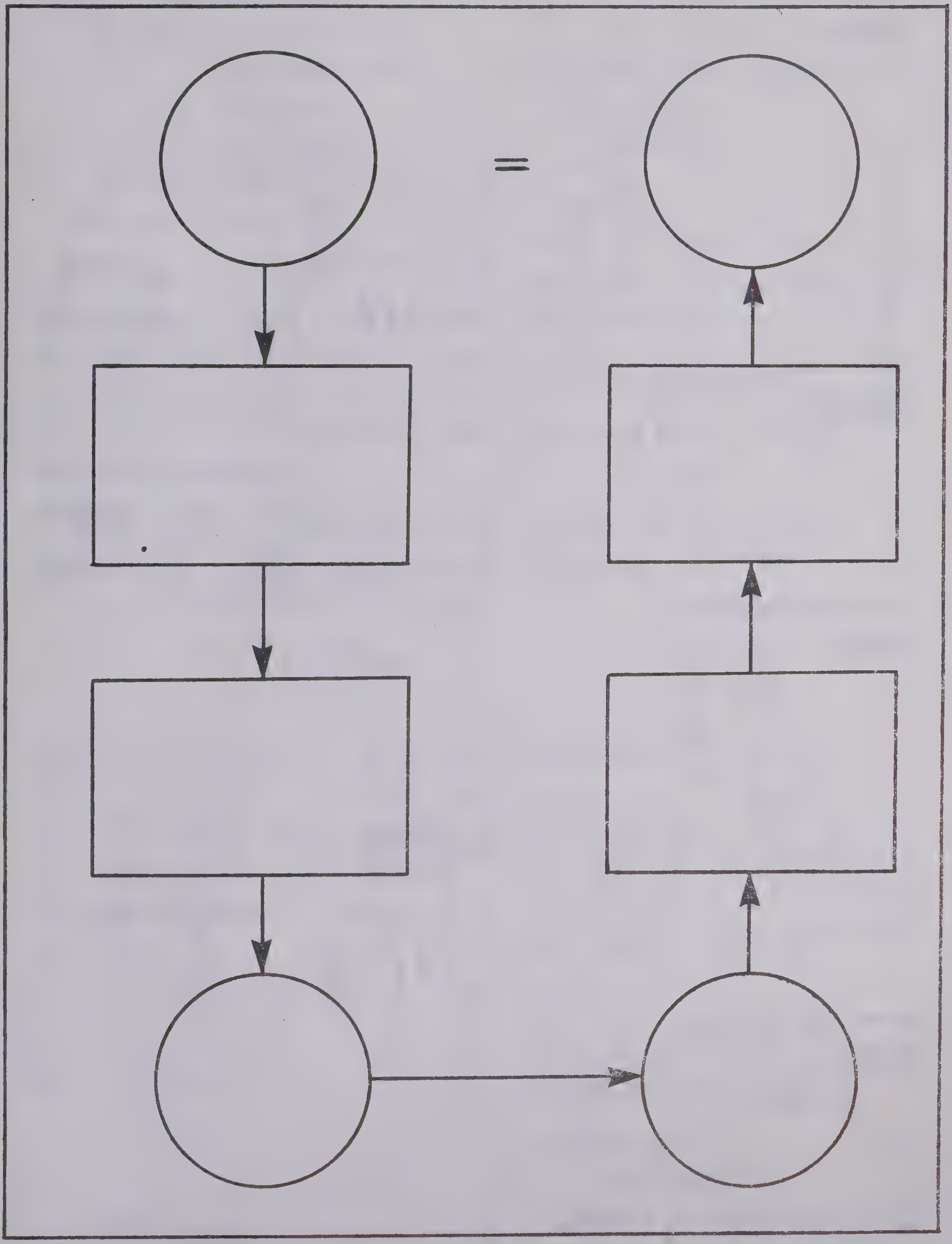
EXAMPLE: $\frac{1}{2}x - \frac{2}{3} = 7\frac{1}{4}$



Therefore,

$x = 15\frac{5}{6}$

FLOW CHART
CONDITION SOLVING



RAT 7 OBJ. C-14

EXERCISES: OBJECTIVE NO. C-14

1. Solve the following:

EXAMPLE: $y + 0.77 = 2.5$

$$y + 0.77 - 0.77 = 2.5 - 0.77$$
$$y = 2.5 - 0.77$$
$$y = 1.73$$

a) $y + 0.5 = 1.5$ $y = 1$

b) $0.3 + y = 4.1$ $y = 3.8$

c) $n + 0.25 = 1.75$ $n = 1.5$

d) $0.31 + n = 7.4$ $n = 7.09$

e) $0.328 + y = 0.328$ $y = 0$

f) $y + \frac{1}{4} = 2\frac{1}{2}$ $y = 2\frac{1}{4}$

g) $\frac{2}{3} + y = 6\frac{1}{3}$ $y = 5\frac{2}{3}$

h) $y + 1\frac{1}{5} = 7\frac{2}{5}$ $y = 6\frac{1}{5}$

2. Solve the following:

EXAMPLE: $y - 1.7 = 2$

$$y - 1.7 + 1.7 = 2 + 1.7$$
$$y = 3.7$$

a) $a - 0.6 = 0.12$ $a = 0.72$

b) $a - 0.18 = 0.36$ $a = 0.54$

c) $a - 2.1 = 9.1$ $a = 11.2$

d) $a - 0.126 = 3.24$ $a = 3.366$

3. Solve the following:

EXAMPLE: $0.3y = 2.7$

$$\frac{0.3y}{0.3} = \frac{2.7}{0.3}$$
$$y = \frac{2.7}{0.3}$$
$$y = 9$$

a) $0.2y = 0.4$ $y = 2$

b) $n(0.06) = 0.9$ $n = 15$

c) $0.5y = 1$ $y = 2$

d) $0.25r = 1.25$ $r = 5$

e) $y(0.006) = 0.06$ $y = 10$

f) $\frac{2}{3}y = 4$ $y = 6$

g) $y \times \frac{3}{4} = 2\frac{1}{2}$ $y = 3\frac{1}{3}$

h) $1\frac{1}{2}y = 1\frac{1}{2}$ $y = 1$

4. Solve the following:

EXAMPLE: $y \div 0.15 = 0.3$

$$\frac{y}{.15} \times 0.15 = 0.3 \times 0.15$$
$$y = 0.3 \times 0.15$$
$$y = 0.045$$

a) $y \div 0.2 = 0.3$ $y = 0.06$

b) $\frac{y}{1.5} = 2$ $y = 3$

c) $\frac{y}{6.66} = 2$ $y = 13.32$

d) $y \div 3 = 0.33$ $y = 0.99$

e) $y \div 12 = 0.3$ $y = 3.6$

f) $\frac{y}{0.01} = 0.1$ $y = 0.001$

g) $y \div \frac{1}{2} = \frac{1}{4}$ $y = \frac{1}{8}$

5. Solve the following:

EXAMPLE: $6y + 0.4 = 14.8$

$$6y + 0.4 - 0.4 = 14.8 - 0.4$$

$$6y = 14.8 - 0.4$$

$$6y = 14.4$$

$$\frac{6y}{6} = \frac{14.4}{6}$$

$$y = \frac{14.4}{6}$$

$$y = 2.4$$

(a) $0.3y + 0.4 = 0.4$ $y = 0$ (e) $0.3y + 0.03 = 3$ $y = 9.9$

(b) $0.5y + 0.25 = 0.75$ $y = 1$ (f) $\frac{1}{2}y + \frac{1}{5}y = \frac{4}{5}$ $y = 1\frac{1}{7}$

(c) $0.2y + 0.2 = 0.4$ $y = 1$

(d) $0.75y + 1.25 = 4.25$ $y = 4$ (g) $1\frac{1}{4}y + 1\frac{1}{2} = 4$ $y = 2$

6. Solve the following:

EXAMPLE: $1.5y + 0.15y = 8.25$

$$(1.5 + 0.15)y = 8.25$$

$$\frac{(1.5 + 0.15)y}{(1.5 + 0.15)} = \frac{8.25}{(1.5 + 0.15)}$$

$$y = \frac{8.25}{(1.65)}$$

$$y = 5$$

(a) $0.2y + 0.4y = 6$ $y = 10$ (e) $1.2y + 3.3y = 9$ $y = 2$

(b) $0.25y + 0.75y = 2.5$ $y = 2.5$ (f) $\frac{1}{2}y + \frac{1}{4}y = 3$ $y = 4$

(c) $0.3n + 0.25n = 5.5$ $n = 10$

(d) $0.48r + 0.02r = 5.05$ $r = 10.1$ (g) $1\frac{1}{5}y + \frac{2}{5}y = \frac{2}{5}$ $y = \frac{1}{4}$

7. Solve the following:

(a) $0.8y = 1.6$ $y = 2$ *(f) $0.3y = 1.5$ $y = 5$

*(b) $0.02y + 0.1 = 0.4$ $y = 15$ (g) $n + 0.4 = 8.1$ $n = 7.7$

*(c) $0.4n + 0.6n = 12$ $n = 12$ (h) $0.7r = 2.8$ $r = 4$

*(d) $7.5n + 0.3 = 15.3$ $n = 2$ (i) $n - 4.8 = 1.3$ $n = 6.1$

*(e) $3r + 0.001 = 3.001$ $r = 1$ (j) $y + 7.8 = 9.02$ $y = 1.22$

DEVELOPMENT AND EXERCISES

STRAND: Rational Numbers LEVEL: 7

UNIT: III OBJECTIVE NUMBER: C-15

OBJECTIVE: Verify solutions to conditions by substitution.

SUGGESTED DEVELOPMENT: Substitution is one method of checking the accuracy of students work. The letters of algebra represent numbers:

$$0.2y + 0.04 = 0.8$$
$$0.2y = 0.8 - 0.04$$
$$0.2y = 0.76$$
$$y = 0.76 \div 0.2$$
$$y = 3.8$$

Take 3.8 and substitute it for y in the original condition.

LHS:	RHS:
$0.2(3.8) + 0.04$	0.8
$0.76 + 0.04$	
0.8	
LHS =	RHS

Replacing y with 3.8 gives us a true statement
Therefore 3.8 must be the solution for the condition.

$$\frac{y}{0.3} = 1.8$$
$$\frac{y \times .3}{0.3} = 1.8 \times 0.3$$
$$y = 1.8 \times 0.3$$
$$y = 5.4$$

LHS:	RHS:
$\frac{5.4}{0.3}$	1.8
$5.4 \div 0.3$	
$1.8 =$	1.8

Therefore y = 5.4 is the correct solution.

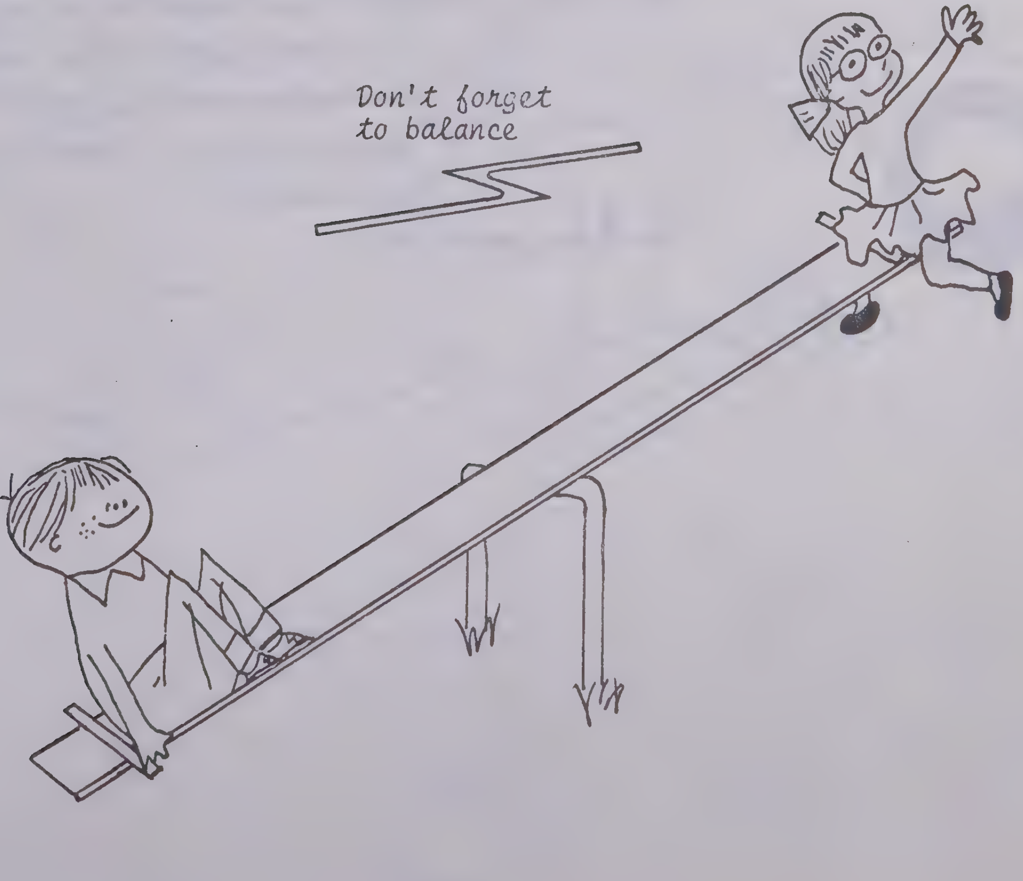
EXERCISES: OBJECTIVE NO. C-15

Solve the following conditions and check your work by substitution. The first one is done for you.

1. EXAMPLE:

	L.H.S.	R.H.S.
$y + 0.15 = 1.5$		
$y + 0.15 - 0.15 = 1.5 - 0.15$	$y + 0.15$	1.5
$y = 1.5 - 0.15$	$1.35 + 0.15$	
$y = 1.35$	1.5	
	$\therefore$ LHS =	RHS

2. $n + 0.25 = 0.5$ $n = 0.25$
3. $0.5k = 10$ $K = 20$
4. $0.2n = 0.5$ $n = 2.5$
5. $0.18n = 1.8$ $n = 10$
*6. $0.2n + 1.2n = 5.6$ $n = 4$
*7. $3y + 0.02 = 0.05$ $y = 0.01$
8. $0.25n = 0.5$ $n = 2$
9. $3.6n + 1.2 = 8.4$ $n = 2$
*10. $2.7y + 0.4y = 0.62$ $y = 0.2$
11. $\frac{1}{2}y = 2\frac{1}{4}$ $y = 4\frac{1}{2}$
12. $y + 1\frac{1}{3} = 3\frac{2}{3}$ $y = 2\frac{1}{3}$
13. $3\frac{1}{4}y = 26$ $y = 8$
14. $2\frac{1}{2}y + 2\frac{1}{2}y = 10\frac{1}{2}$ $y = 2\frac{1}{10}$
15. $2\frac{2}{5}y + \frac{3}{10} = \frac{7}{10}$ $y = \frac{1}{6}$



DEVELOPMENT AND EXERCISES

STRAND: Rational Numbers

LEVEL: 7

UNIT: III

OBJECTIVE NUMBER: C-16

OBJECTIVE: Write English sentences for mathematical sentences.

SUGGESTED DEVELOPMENT: Review the meaning of a mathematical sentence:

(1) numbers: 0.03, 0.2, 0.006
(2) operations: x, +, -, ÷
(3) relations: =

0.03 x 0.2 = 0.006 is a mathematical sentence. Such as sentence can also be stated in English:

0.03 multiplied by 0.2 is equal to 0.006

n ÷ 2.5 = 0.1 would be:

some number divided by 2.5 is equal to 0.1.

Students are not very creative as far as writing sentences other than those involving "the number". The teacher should emphasize the use of sentences involving actual objects and quantities.

For the sentence:

$$6x + 12 = 72$$

the students should be encouraged to write problems like:

> A student spent 72¢ on school supplies. He bought 6 pencils and an eraser. The eraser cost 12¢. What was the cost of each pencil?

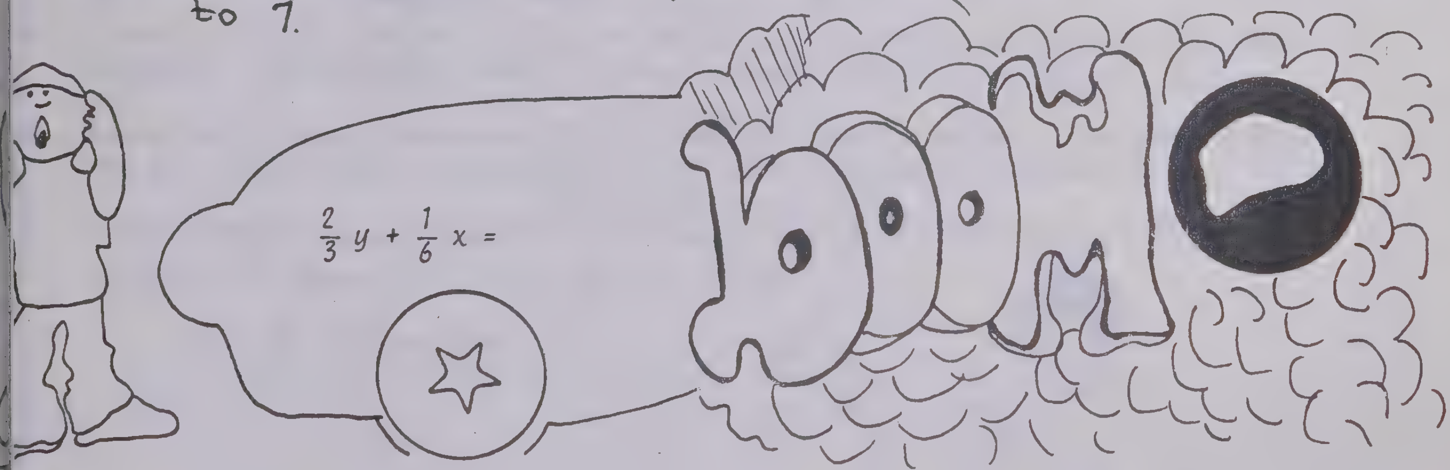
Encourage Creativity.

Write English sentences for the following:

1. $35.89 + 0.25 = n$
2. $0.5 + y = 0.7$
3. $4 \times 0.465 = n$
4. $0.15n = 15.6$
5. $n \div 5 = 0.7$
6. $0.03y + 0.02y = 2.50$
7. $y + \frac{1}{2} = \frac{3}{4}$
8. $y - \frac{1}{3} = \frac{5}{6}$
9. $\frac{2}{3} y = 9$

*10. $\frac{1}{4} y + 2 = 7$

1. The sum of 35.89 and 0.25 equals some number
2. the sum of 0.5 and some number is equal to 0.7
3. the product of 4 and 0.465 equals some number
4. The product of a number and 0.15 is equal to 15.6
5. A number divided by 5 is equal to 0.7
6. The sum of the product of 0.03 and a number and 0.02 and the number is equal to 2.50.
7. The sum of a number and $\frac{1}{2}$ is equal to $\frac{3}{4}$
8. The difference of a number and $\frac{1}{3}$ is equal to $\frac{5}{6}$
9. The product of $\frac{2}{3}$ and a number is equal to 9.
10. The sum of 2 and the product of $\frac{1}{4}$ and a number is equal to 7.



DEVELOPMENT AND EXERCISES

STRAND: Rational Numbers LEVEL: 7
UNIT: III OBJECTIVE NUMBER: C-17
OBJECTIVE: Write mathematical sentences for English sentences.

SUGGESTED DEVELOPMENT: One of the most important mathematical skills is being able to take words and translate them into mathematical symbols.

In order to do this, the students must be given numerous examples so that they will identify such key words as difference, product, of, is, rate, between, etc.

EXAMPLE: (1) What number is half of 242?

Recognition: half $\frac{1}{2}$

of multiplication

To solve, write $n = \frac{1}{2} \times 242$.

(2) What number added to 25 equals 218?

Recognition: added to +

To solve, write $n + 25 = 218$.

NOTE: You may wish to have the class write mathematical sentences for the problems in objective 18 as a prelude to solving the problems.

Write mathematical sentences for:

(DO NOT SOLVE)

1. The cost of a shirt is 5 more than 8 dollars.
$8 + 5 = n$
2. Eight more than a number is 14.
$n + 8 = 14$
3. Four times a number is 36.
$4n = 36$
4. Six times a number is 12 more than that number.
$6n = n + 12$
5. When a number is multiplied by 3 and its result added to 12, the answer is 35.
$12 + 3n = 35$
6. A boy had some marbles, but gave half of them to his sister. If he now has 23 marbles, how many did he have at first?
$n - \frac{1}{2}n = 23$
7. Dick caught a certain number of frogs. His friend Harry caught 3 times as many as he did. Together they caught 64 frogs. How many frogs did Dick catch?
$n + 3n = 64$
8. Four girls had stamp collections. If there were 268 stamps altogether, how many did each girl have. $4n = 268$ or $n + n + n + n = 268$
9. If a ball player could hit three times as many home runs next month as this month, he would hit 5 fewer than 35. How many did he hit last month?
$3n = 35 - 5$
10. A super rabbit jumps 4 metres every jump. The total distance he covers is 320 metres. How many jumps does he take? $4n = 320$
11. A snail, Pokey, crawled 2.7 cm one morning. His friend crawled 0.3 of that distance. How far did Pokey's friend, creepy, crawl? $2.7 \div 0.3 = n$
12. Susie has a large container of milk available for a recipe. She uses 0.75 ℓ and has 0.33 ℓ left. How much milk did she start with originally?
$n - 0.75 = 0.33$
13. A man's yard was landscaped by putting flowers and shrubs on $\frac{1}{4}$ of it and grass on $\frac{3}{7}$ of it. How much is left for the house?
$1 - \frac{1}{4} - \frac{3}{7} = n$

DEVELOPMENT AND EXERCISES

STRAND: *Rational Numbers* LEVEL: *7*
UNIT: *III* OBJECTIVE NUMBER: *C-18*
OBJECTIVE: *Solve problems using an organized approach (Limit: decimals and fractions)*

SUGGESTED DEVELOPMENT: The teacher will decide upon the approach the students will use. Below is a suggested method:

Tom lives $2\frac{1}{3}$ blocks from the school.

John lives 4 times as far. How many blocks from school does John live?

1) *Define the Variable* *(Underline relevant facts)*	Let y = The number of blocks John lives from the school.
2) *Write a Condition:*	$y = 2\frac{1}{3} \times 4$
3) *Solve the condition*	$y = 9\frac{1}{3}$
4) *Answer the question using correct units*	John lives $9\frac{1}{3}$ blocks from the school.
5) *Verify the solution to the condition and determine if it satisfies the problem.*	CHECK: $2\frac{1}{3} \times 4 = 9\frac{1}{3}$ $9\frac{1}{3} = 9\frac{1}{3}$

NOTE: *There are problems in this section which lend themselves to a two-step approach. You may wish to develop some format.*

1. Joan bought 3.5 kg of apples, 0.75 kg of grapes, and 1.3 kg of bananas. How much fruit did she buy in all? 5.55 Kg.

2. How many wooden floor blocks 8.75 cm in length will fit end-to-end on a floor 2100 cm long? 240

3. The price of turkeys increased by $0.06 per kg. What would be the price increase for a 5.5 kg turkey? 33 cents

4. A steel plate is 0.56 cm thick. A bracket to be bolted to the plate is 0.24 cm thick. How much of 1.50 cm bolt will project if one washer 0.125 cm thick and a nut 0.25 cm thick are used? (HINT: draw a diagram) 0.325 cm

5. Share $1256.48 equally among 8 people. How much will each receive? $157.06

6. What length of steel rod will be used to make 5256 bicycle spokes, each each 24.75 cm long? 130086 cm.

7. A telephone wire contracts in the winter by 1.325 cm in every 100 m of its length. Find the amount of contraction in 1760 m of the wire. 23.32 cm.

8. A turtle, Bert, eats 8 g of hamburger a day. A second turtle, Ernie, eats 9.4 g of hamburger a day. How much hamburger would Bert and Ernie consume in a week? 121.8 g

9. A car travels 5.5 km on a litre of gasoline. How many kilometres will it travel on 18 litres of gasoline? 99 Km

10. A girl bought 3.75 m of material at $2.95 per m. How much change should she receive from a $20 bill? $8.94

11. A machinist cuts 15 pieces each 3.95 cm long from a steel rod that is 120 cm long. What length of the steel rod remains? 60.75 cm

12. A refrigerator sells for $395 cash, or $50 down and 12 payments of $33.29 each. How much is saved by paying cash? $54.48

13. A piece of meat costs $9.36. The price of the meat was $0.96 per kg. What number of kg did the piece of meat weigh? 9.75 Kg

14. Six students were practicing using a micrometer. A micrometer is a very accurate measuring instrument. Each student measured the thickness of a block of metal. The readings obtained were as follows: 0.9984 cm, 1.0006 cm, 0.9998 cm, 0.9995 cm, 1.0002 cm, and 1.0002 cm. What was the average of the readings? 0.9998 cm

15. Ed bought some white popcorn and 0.2 kg of yellow popcorn. He bought 0.75 kg of popcorn in all. How much white popcorn did Ed buy? 0.55 Kg

16. Mercury is about 13.6 times as heavy as water. A bottle holds 0.65 kg of water. About what mass of mercury would this bottle hold? 8.84 Kg

17. An engineer was using metal plates that were 0.17 cm thick. How many plates of this thickness did he need to bolt together to give a total thickness of 2.55 cm? 15

18. If one notebook is 15 mm thick, how many notebooks may be stacked in a desk 90 mm high? 6

19. A motorist calculated that he got 5.5 km for each litre of gas. How many litres of gas would be needed to go 80 km? 14.5 ℓ

20. A wrapper in a store uses 1.75 m of paper and 3.3 m of string for one package. How much paper and how much string would she use to wrap 16 similar parcels?
Paper : 28 m String : 52.8 m

21. What was the total rainfall in a week during which it rained 1.5 cm on Monday, 1.3 of a cm on Tuesday, 0.9 of a cm on Wednesday, and was dry the rest of the week? 3.7 cm

22. Terry mailed four letters that weighed a total of 185 g. One letter weighed 95 g, another weighed 25 g and a third weighed 30 g. How much did the fourth letter weigh? 35 g

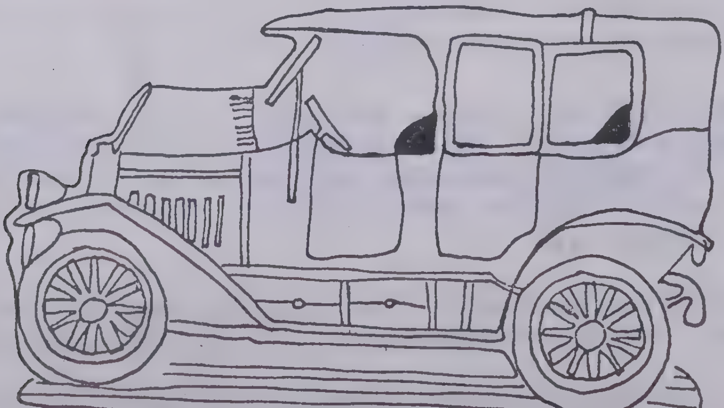
23. From a sheet of tin 1.5 m long, two lengths were cut, one 0.5 m long, the other 0.45 m long. How much of the sheet was left? 0.55 m

24. If you increase "n" by $1\frac{3}{4}$ and then diminish this sum by $\frac{3}{8}$, the result is $3\frac{1}{2}$. Find "n". $2\frac{1}{8}$

25. The movie theatre has 300 balcony seats and 540 orchestra seats. At a certain performance $\frac{2}{3}$ of the balcony and $\frac{5}{6}$ of the orchestra seats were occupied. How many seats were unoccupied? 190

26. An 80 litre tank is to be filled with gasoline by repeated pourings from a can holding 2.5 litres. How many times must the can be used? 32

27. A giant-size bottle of liquid detergent has a selling price of 81¢ for 1000 ml. The detergent also comes in a smaller size retailing at 53¢ for 550 ml. Which is the better buy? FIRST ONE



REVIEW EXERCISES: OBJECTIVE NO. C-18

WORD SEARCH

Z	N	X	W	C	R	O	T	A	N	I	M	O	N	E	D	L	A
S	M	V	B	N	O	I	T	A	R	E	P	O	B	N	H	I	O
M	J	L	A	N	O	I	T	A	R	J	L	A	R	E	M	U	N
U	D	A	P	A	U	N	H	T	E	N	I	N	K	C	A	P	S
L	U	M	Q	D	E	W	O	R	E	P	O	R	P	R	T	Z	M
T	A	I	R	M	I	N	M	Y	E	I	G	O	T	B	D	A	T
I	E	C	G	N	R	U	N	E	T	D	I	E	Y	O	R	X	E
P	H	E	O	S	T	O	H	C	O	N	R	W	I	X	A	L	R
L	S	D	M	R	M	W	A	U	T	U	K	O	V	D	D	P	C
I	E	D	I	M	P	R	L	D	I	V	I	S	I	O	N	R	S
C	V	I	O	T	T	I	F	E	D	L	F	V	T	U	A	E	G
A	E	C	N	B	O	E	C	R	H	I	G	E	A	S	T	B	R
T	N	I	U	S	N	U	M	E	R	A	T	O	R	L	S	M	A
I	T	S	M	I	S	C	I	R	R	F	F	I	A	O	E	U	R
O	H	A	L	P	E	C	X	D	E	U	E	N	O	X	W	N	Y
N	A	B	F	W	R	H	E	I	T	T	O	V	P	N	O	X	T
P	D	I	G	B	A	O	D	V	J	E	M	A	D	R	L	N	O
Q	R	X	T	Q	R	U	P	A	R	E	N	T	H	E	S	E	S
T	B	L	V	E	G	L	H	E	K	D	H	T	F	I	F	P	B
W	A	N	N	O	I	T	C	A	R	F	O	T	H	I	R	D	A

There are at least 36 words associated with rational numbers. See how many you can find.

denominator
operation
rational
numeral
nineth
proper
division
numerator
parentheses
fifth
fraction
third

number
lowest
standard
reduce
half
mixed
decimal
basic
seventh
multiplication
order
equivalent

addition
reciprocal
term
tenth
improper
segment
common
subtraction
line
point
quarter
expand

1st - Work out the problems below

2nd - Find the answer under the blanks below

3rd - Place the letter of question over the answer

WHAT HAPPENS WHEN YOU FINISH THIS PUZZLE?

A Fred, the pet frog, ate a total of 24.5 mealworms over a two week period. If he averaged 1.5 mealworms a day for the second week how many mealworms did he average per day for the first week? 2

E $\frac{1}{8} = a$ $a = 0.125$

G $y + 39.67 = 69.81$ $y = 30.14$

H $y - 38.649 = 2.817$ $y = 41.466$

I $10 - 9.999 = n$ $n = 0.001$

M $3.763y = 69.2392$ $y = 18.4$

N 2.375 = what fraction $2\frac{3}{8}$

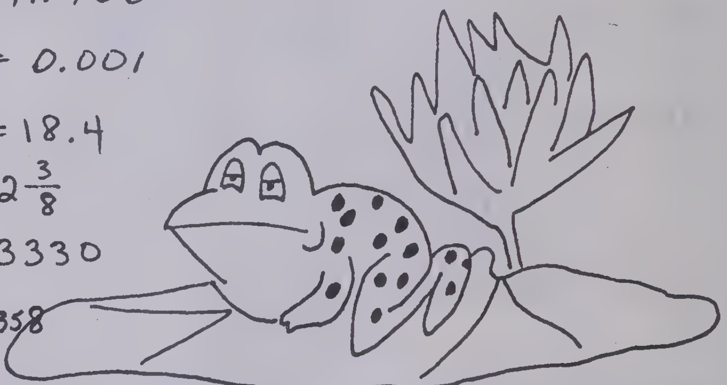
O $9.99 \div 0.003 = n$ $n = 3330$

T $y \div 21.46 = 2.3$ $y = 49.358$

U $3.64y + 8.31y = 156.545$ $y = 13.1$

Y $41.23 \times 0.34 = n$ $n = 14.0182$

X $46.379 \div 1000$ 0.046379



Y	O	U	M	A	Y
14.0182	3330	13.1	18.4	2	14.0182

G	O	O	N	T	O
30.14	3330	3330	$2\frac{3}{8}$	49.358	3330

T	H	E	N	E	X	T
49.358	41.466	0.125	$2\frac{3}{8}$	0.125	0.046379	49.358

U	N	I	T
13.1	$2\frac{3}{8}$	0.001	49.358

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